Phase transitions in real time following a quantum quench

Anatoli Polkovnikov,
Boston University

Luca D'Alessio BU
J. Dalibard ENS
K. Gunter ENS
Markus Heyl Munich (LMU)
Stefan Kehrein Gottingen
Ludwig Mathey Hamburg

Workshop on Quantum Simulations with Ultracold Atoms**, Trieste**
07/2012

Outline.

1. High temperature expansion – Short time expansion.
   a) Breakdown of short time perturbation theory and phase transition in time for a quench in a quantum Ising chain. (with M. Heyl and S. Kehrein)
   b) (Many body) energy localization transition in periodically driven systems and breakdown of Magnus expansion. (with L. D'Alessio).

2. Dynamics and the renormalization group: Kosterlitz-Thouless phase transition following a quench (with L. Mathey 2010, also R. Vosk and E. Altman, 2012).

3. Universality of quantum and classical dynamics (C. De Grandi et. al. 2011, M. Kolodrubetz et. al. 2011, A. Chandran et. al. 2012, C.-W. Liu, A.P., A. Sandvik in progress, E. Dalla Torre, A. P., E. Demler in progress).

High temperature expansion and Lee-Yang (Fisher) zeros.

Equilibrium: all information about observables is contained in the partition function

$$Z = \int dX dP \exp[-\beta(H_0(X,P) + H_{int}(X,P))] = Z_0 \langle \exp[-\beta H_{int}(X,P)] \rangle_0$$

High temperature (small interactions): can use high temperature expansion

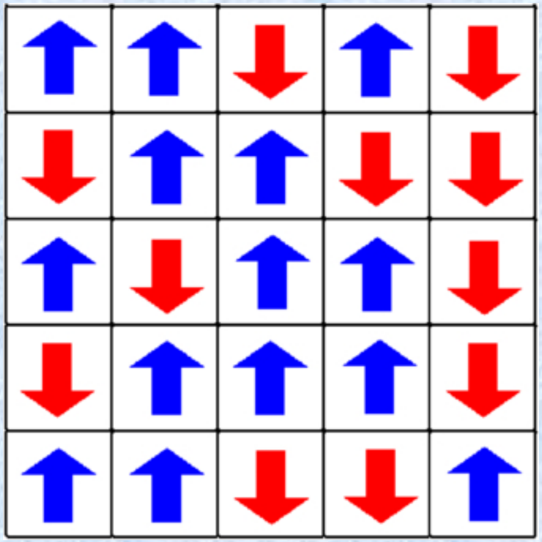
$$F = -T \log(Z) = F_0 - T \sum_{n\geq 1} \frac{(-1)^n \beta^n}{n!} \langle H_{int}^n \rangle_{0,c}$$

Phase transitions: free energy becomes non-analytic function of temperature (tuning parameter). The high temperature expansion breaks down.

Lee-Yang theorem (1952): understood non-analyticity through the condensation of zeros of the partition function in the complex plane.

$$H = -J\sum_{\langle ij\rangle} s_i s_j - h\sum_j s_j, \qquad z = \exp[-2\beta h]$$

$$Z_N = \sum_{\{s_i\}} \exp[-\beta H] = z^{-N/2}\sum_{n=0}^{N} P_n z^n = z^{-N/2} P_0 \prod_{i=1}^{N}(1 - z/z_i)$$

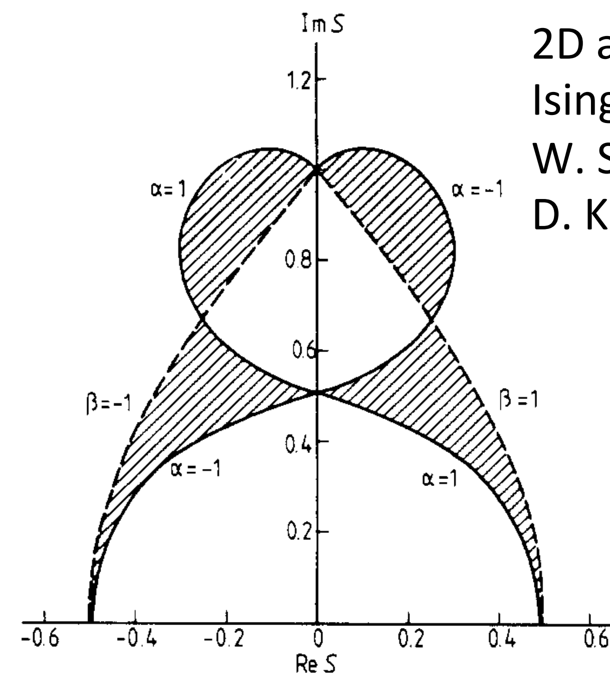


Lee-Yang: all zeros $\boldsymbol{z_i}$ are complex. They condense near real axis at the phase transition. Taylor expansion breaks.

M. Fisher (1965). Extension of these ideas to the high temperature expansion. Consider *h=0*.

$$Z_N = \mathrm{e}^{-\beta J d N}\sum_r P_r \exp[-2\beta J r]$$

Singularities develop in the complex temperature (coupling) plane: breakdown of the Taylor expansion



2D anisotropic Ising model
W. Saarloos and D. Kurtze (1984)

Partition function is the generator of moments of (interaction energy) energy

$$F = -T\log(Z) = F_0 - T\sum_{n\geq 1}\frac{(-1)^n\beta^n}{n!}\langle H_{int}^n\rangle_{0,c}$$

Analogue of the partition function for quench dynamics: Loschmidt echo – generator of the moments of work W (A. Silva, 2008)

$$H = H_0 + \delta H\Theta(t),\ L(t) = \langle \exp[iH_0 t]\exp[-iHt]\rangle_0$$

$$L(t) = \langle \exp[-iWt]\rangle_0 \ \Rightarrow\ f(t) = \log[L(t)] = \sum_n \frac{(-i)^n t^n}{n!}\langle W^n\rangle_c$$

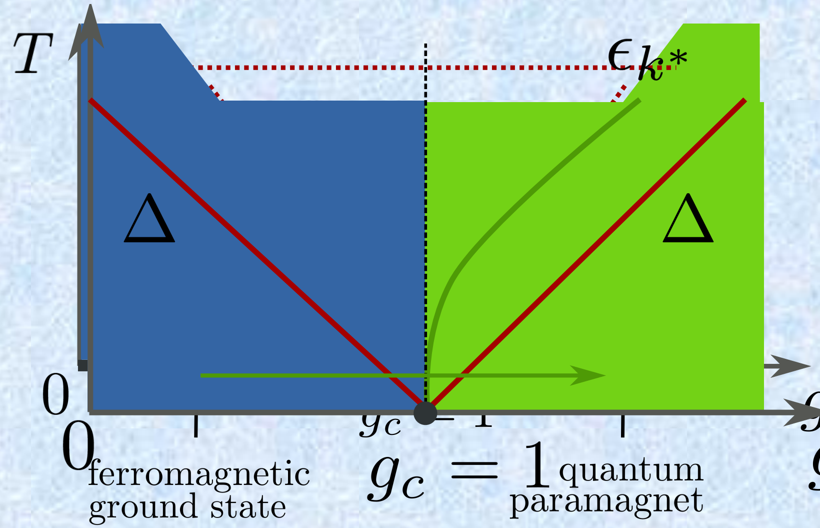
Close analogy between equilibrium physics and quench dynamics:
Partition function – Loschmidt echo,
Inverse temperature – time,
high temperature expansion – short time expansion.

Can we have phase transitions in time?

Dynamical phase transition in the transverse field Ising model

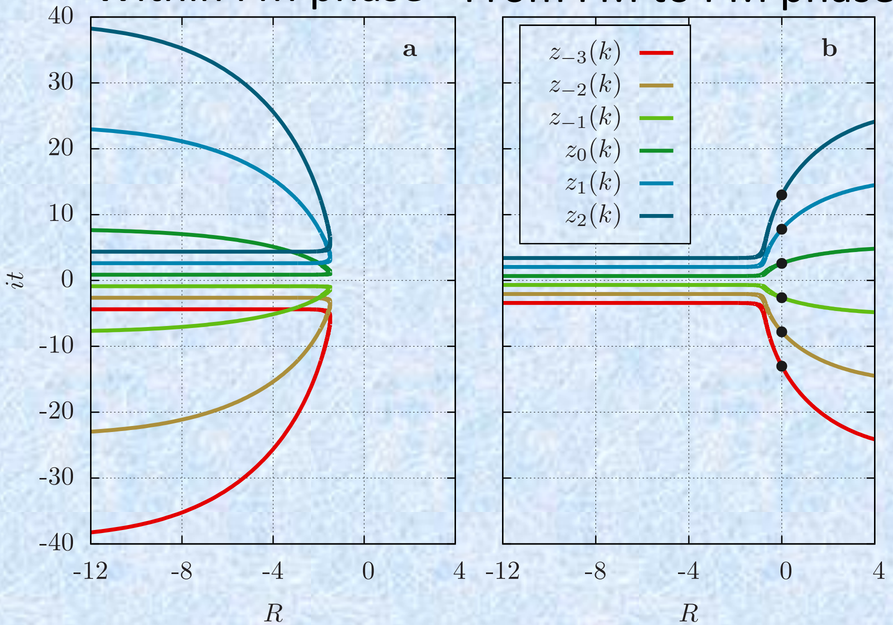
(M. Heyl, A. P., S. Kehrein, arXiv: 1206.2505)

$$H = -\sum_i \sigma_i^z \sigma_{i+1}^z + g \sum_i \sigma_i^x$$



Quench from FM ground state to PM phase. Study:

$$L(z) = \mathrm{e}^{E_0 z} \langle \psi_0 | \mathrm{e}^{-H(g_1) z} | \psi_0 \rangle$$

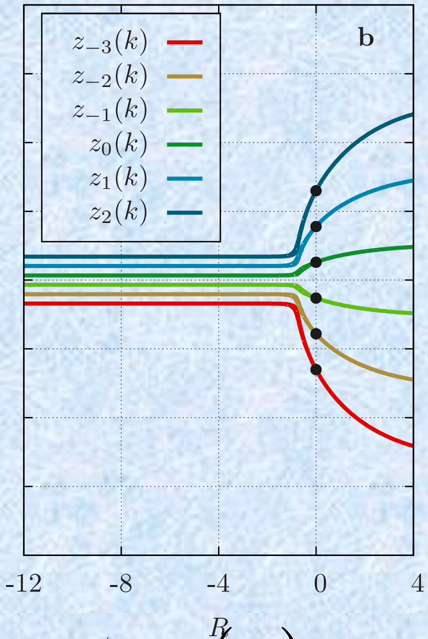


Fisher zeros crossing real time axis implies

$$f(t) = -1/N \log L(t)$$

is nonanalytic in time. Breakdown of short time expansion.

Phase transition in time!

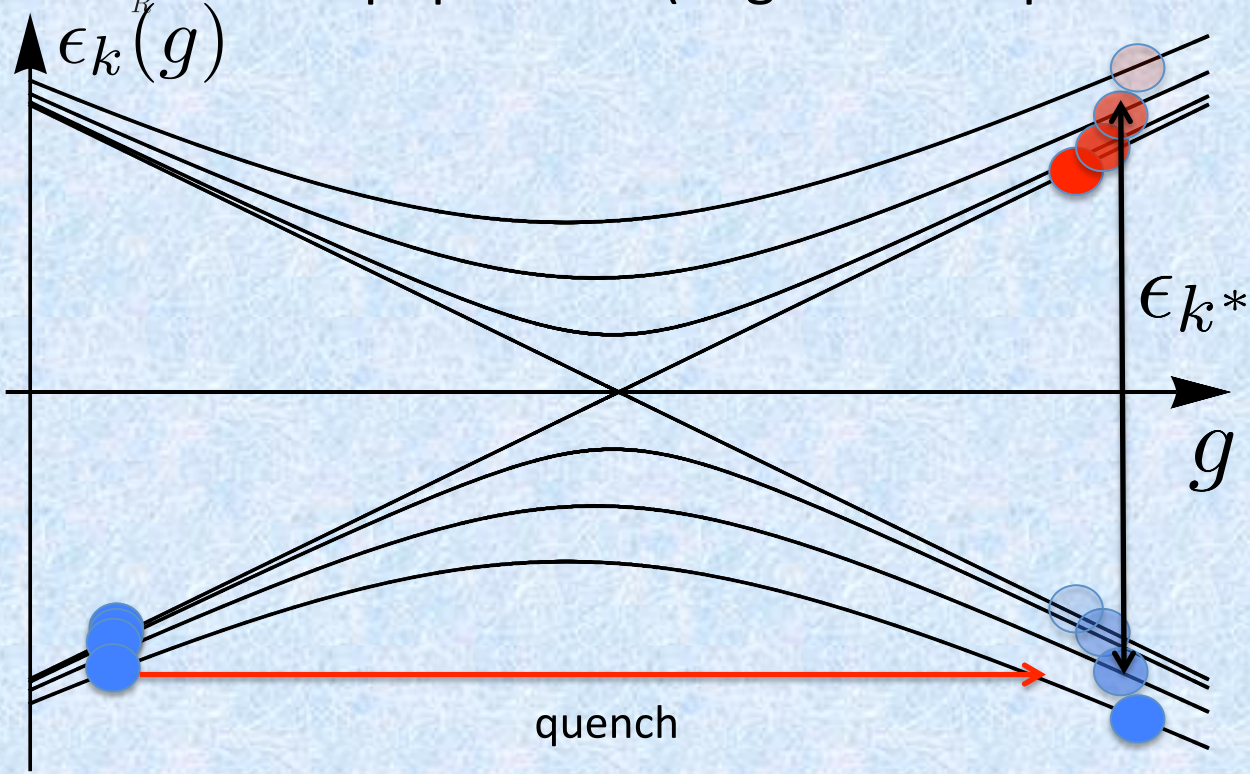


Emergent time (energy) scale, not the gap

$$t^* = \frac{\pi}{\epsilon_{k^*}} \qquad \cos k^* = \frac{1 + g_0\, g_1}{g_0 + g_1}$$

$$\epsilon_k(g) = \sqrt{(g - \cos k)^2 + \sin^2 k}$$

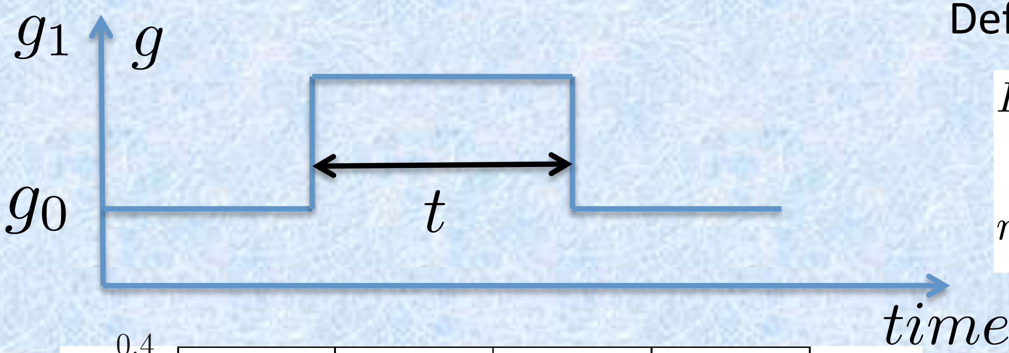
Physical origin of the transition: emergence of inverse population (negative temperature) for some modes



Observing the dynamical phase transition

Expectation values of of common observables are analytic in time.

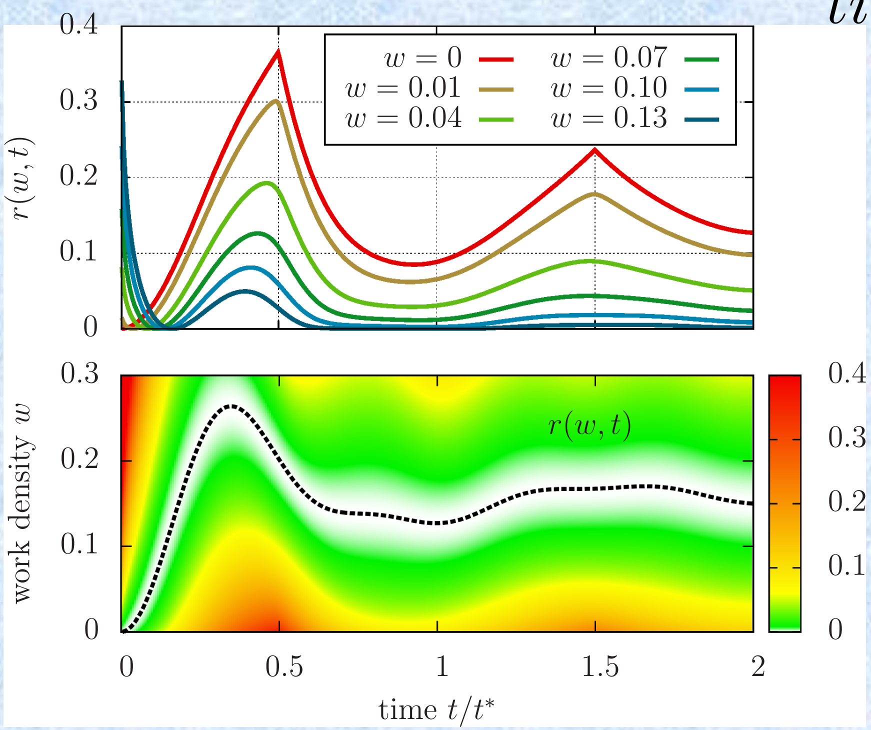
Physically *L(t)*=0 implies a state orthogonal to the initial state. Expect Small return probability for a double quench.



Define work probability after a double quench

$$P(w,t) = \sum_n |\langle n|\psi(t)\rangle|^2 \delta(E_n - E_0 - w)$$

$$r(w,t) = -\frac{1}{N}\log(P(w,t)), \quad r(0,t) = 2\Re[f(t)]$$



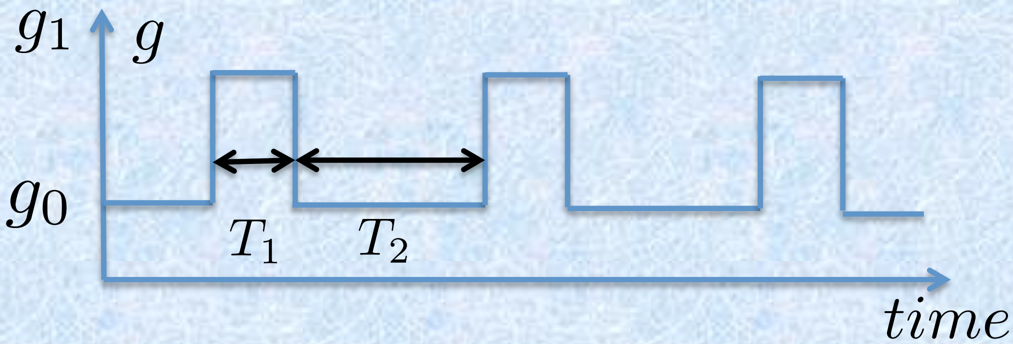
w plays the role of temperature. At zero work recover quantum phase transition.

General idea: can use post-selection as a non-equilibrium cooling. I.e. analyze only experiments with $w < w^*$.

Expect quantum critical behavior of post-selected observables as $w^* \to 0$.

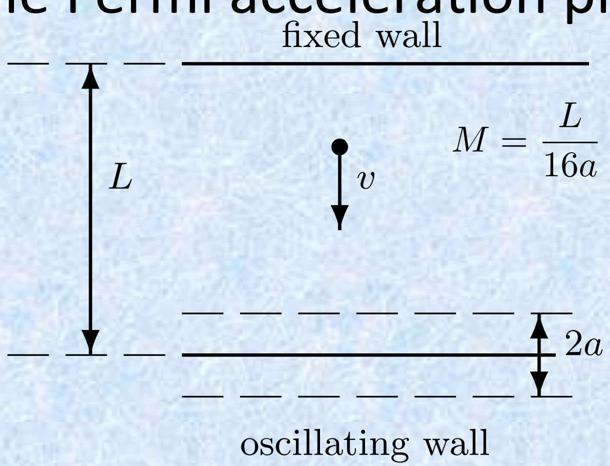
Energy localization transition in periodically driven systems and break down of (short time) Magnus expansion. (with L. D'Alessio)

Instead of a single quench consider a periodic sequence of pulses:



What is the long time limit in this system?

Fermi-Ulam problem (prototype of the Fermi acceleration problem).



(G. M. Zaslavskii and B. V. Chirikov, 1964
M. A. Liberman and J. Lichenberg 1972)

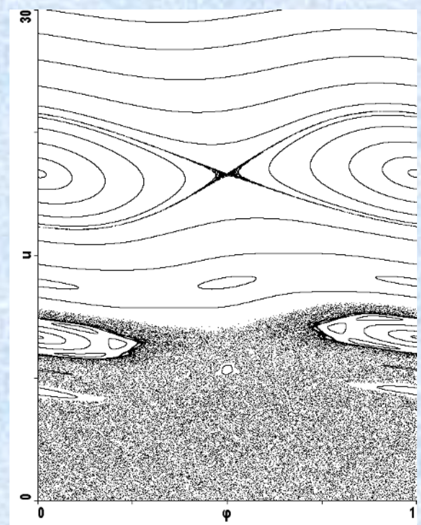


Fig. 7.14. Poincaré phase space section for a harmonic wall oscillation with $M = 20$. Iterations of several selected trajectories.

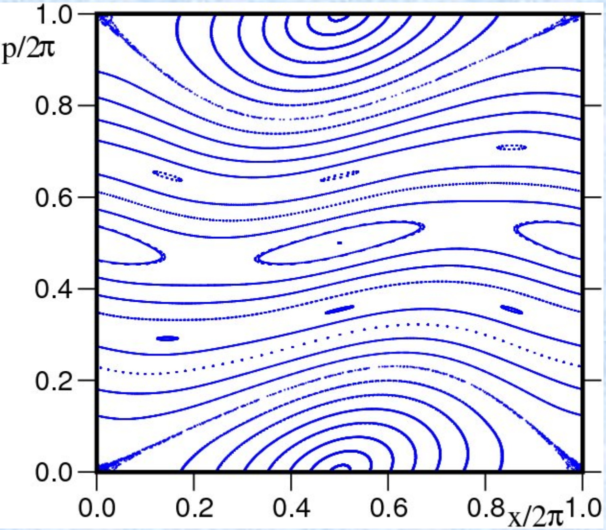
Small energies: chaos and diffusion. Large energies – periodic motion. Energy stays localized within the chaotic region.

Stochastic motion – infinite acceleration.

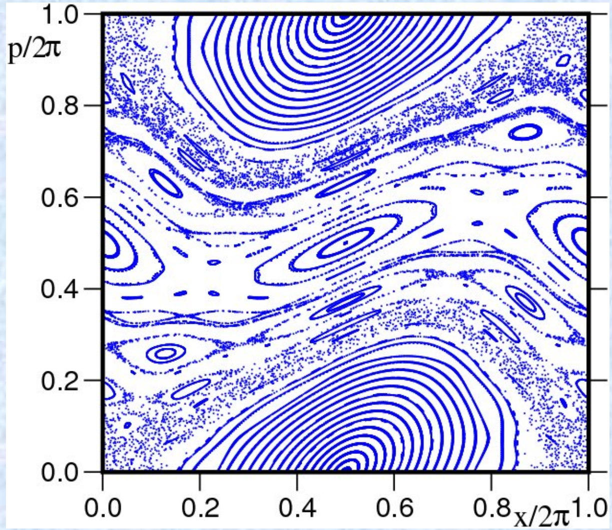
Kicked rotor (realization of standard Chirikov map)

$$H(p,x,t) = \frac{p^2}{2} + K\cos(x)\sum_n \delta(t-n)$$

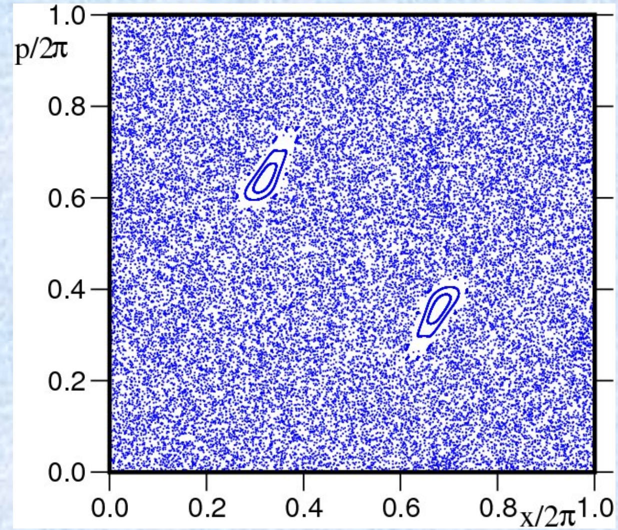
Transition from regular (localized) to chaotic (delocalized) motion as K increases. Chirikov, 1971



K=0.5



$K=K_g=0.971635$



K=5

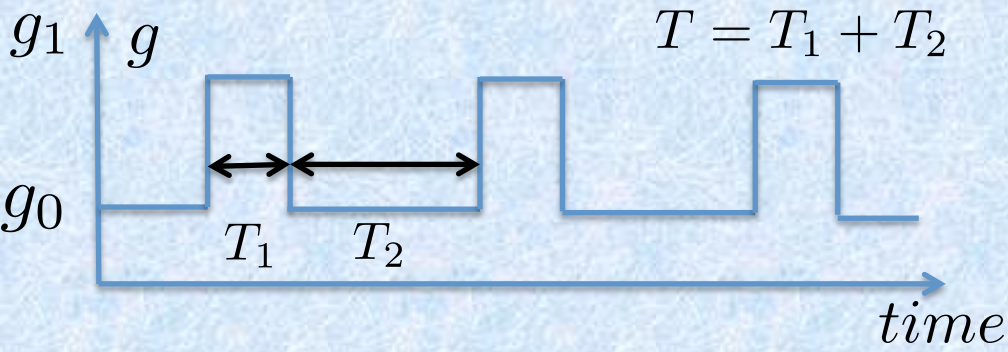
(images taken from scholarpedia.org)

Delocalization transition at $K_c \cong 1.2$ (B. Chirikov (1979)).

Quantum systems: (dynamical) localization due to interference even in the chaotic regime (F. Izrailev, B. Chirikov, … 1979).

What about periodically driven ergodic systems in thermodynamic limit?

Example: take an optical lattice and start shaking it in time with a small amplitude



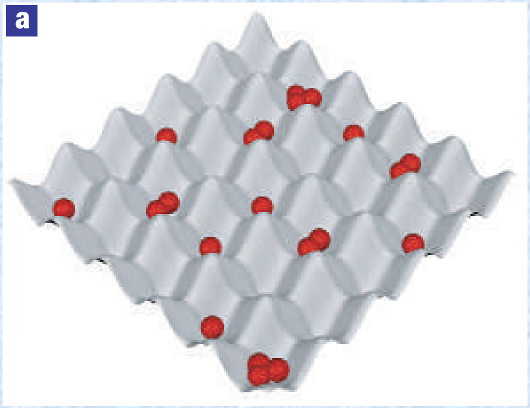


Image: I. Bloch, Nature Physics, 2004

Wait for a long time, follow the energy.

Expectations:

1. Long period: system constantly absorbs energy until reaching infinite temperature
2. Short period: quench to a time average potential – finite energy increase: energy localization.

Is there an energy localization transition or a crossover?

Wave function (density matrix) after n-periods

$$|\psi(nT)\rangle = [U(T)]^n\,|\psi_0\rangle, \quad U(T) = \exp[-iH_2T_2]\exp[-iH_1T_1] = \exp[-iH_FT]$$

$$|\psi(nT)\rangle = \exp[-iH_FnT]|\psi_0\rangle$$

Time evolution is like a single quench to the Floquet Hamiltonian

$H_F = H_0 + V$, $V = H_F - H_0$.
If V is small and local expect that the energy $\langle H_0\rangle$ is localized.

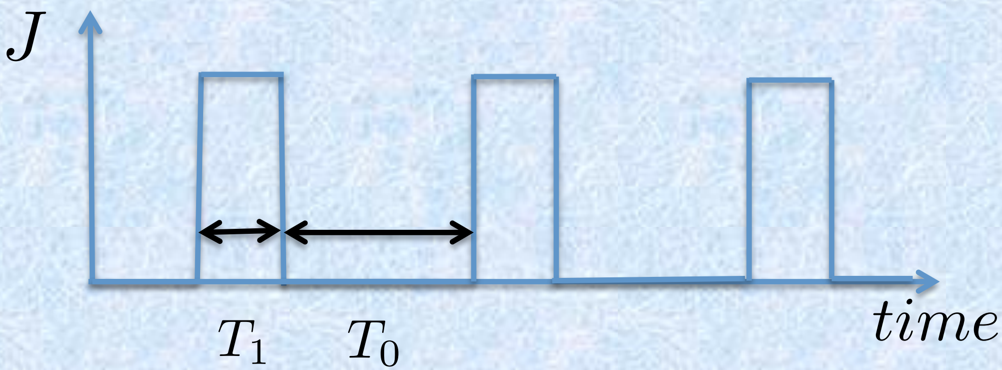
Magnus expansion:

$$H_F = \frac{i}{T}\log[\exp[-iH_1T_1]\exp[-iH_2T_2]] = \frac{1}{T}\int_0^T dtH(t) - \frac{i}{2T}\int_0^T dt_1\int_0^T dt_2\,[H(t_1),H(t_2)] + \dots$$

- Each term in the expansion is extensive and local (like in high temperature expansion)
- Higher order terms are suppressed by the period T but become more and more non-local.
- Competition between suppression of higher order term and their non-locality – similar to many-body localization.
- The expansion is well defined classically if we change commutators to the Poisson brackets.

Specific model: classical or quantum spin chain

$$H = -h\sum_j s_j^z - J\left[g\sum_j s_j^z s_{j+1}^z + \sum_j (s_j^+ s_{j+1}^- + s_j^- s_{j+1}^+)\right]$$



Start in the ground state of the noninteracting system. Follow the noninteracting energy.

Analytically tractable limit: $T_1 \to 0$ Classical limit: commutators -> Poisson brackets.

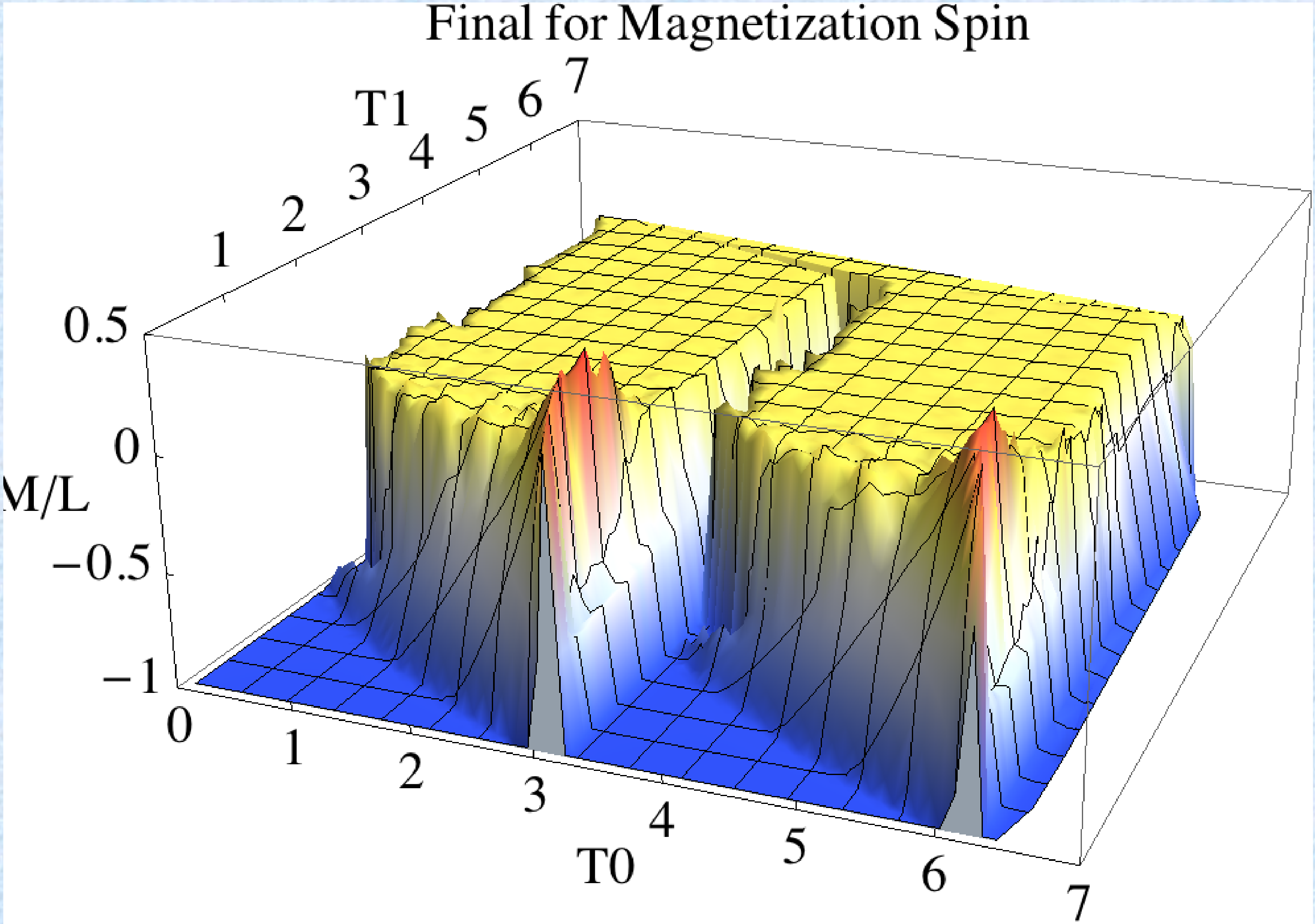
$$\log[\exp[X]\exp[Y]] = X + Y + \frac{1}{2}[X,Y] + \frac{1}{12}[X,[X,Y]] + \ldots + O(Y^2)$$

$$X = ihT_0\sum_j s_j^z, \quad Y = iT_1 J\left[g\sum_j s_j^z s_{j+1}^z + \sum_j (s_j^+ s_{j+1}^- + s_j^- s_{j+1}^+)\right]$$

$$H_F = \overline{H} + J(g-1)\frac{T_1}{2(T_1+T_0)}\left(hT_0\cot(hT_0) - 1\right)\sum_j (\sigma_j^z\sigma_{j+1}^z - \sigma_j^y\sigma_{j+1}^y) + \ldots + O(J^2T_1^2)$$

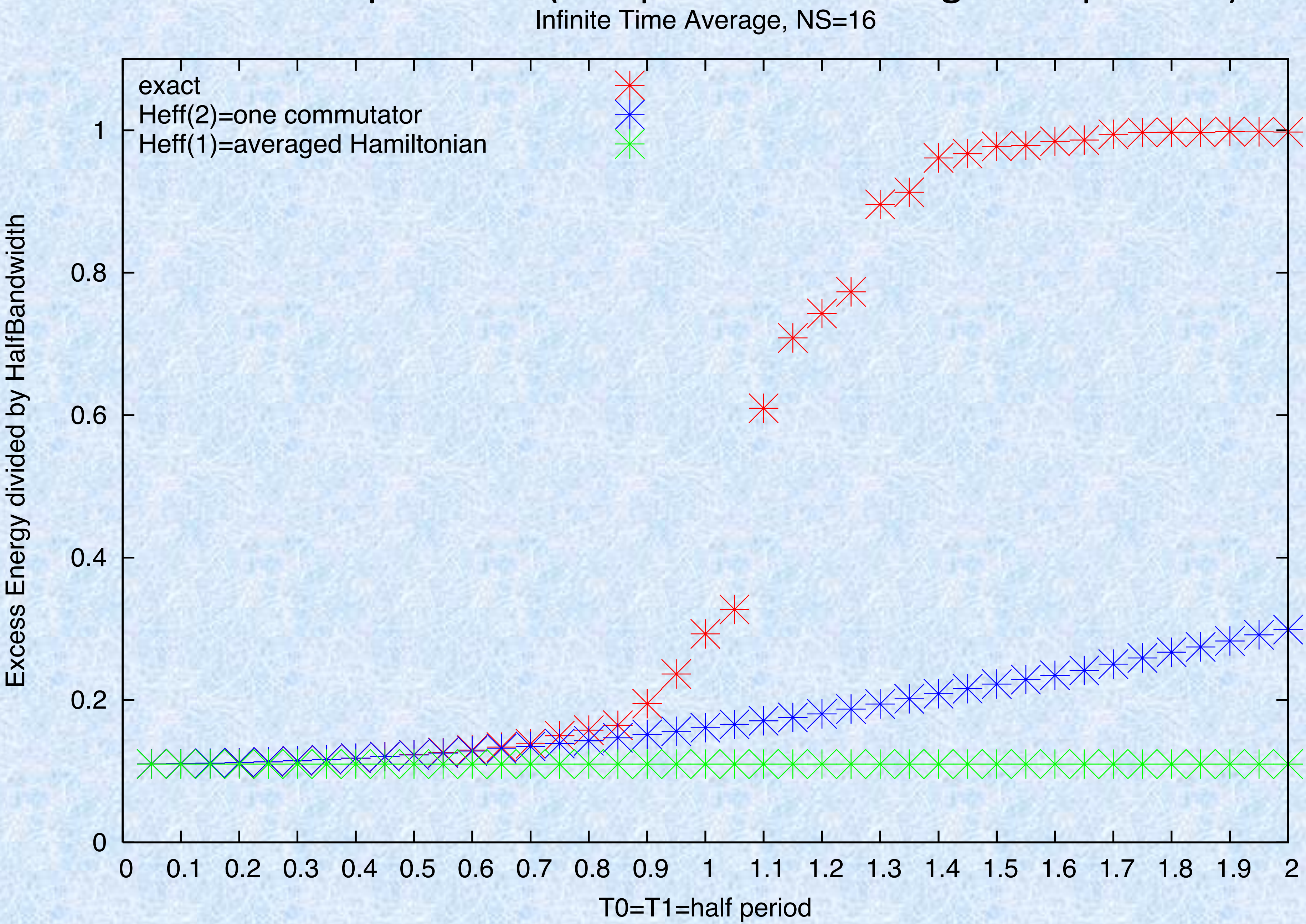
Singularity (phase transition?) at $hT_0 = \pi$

Simulations: classical spin chain



Strong evidence for (many-body) localization transition in energy space

Quantum spin chain (comparison with Magnus expansion)



Temporal simulations of a quantum spin chain.

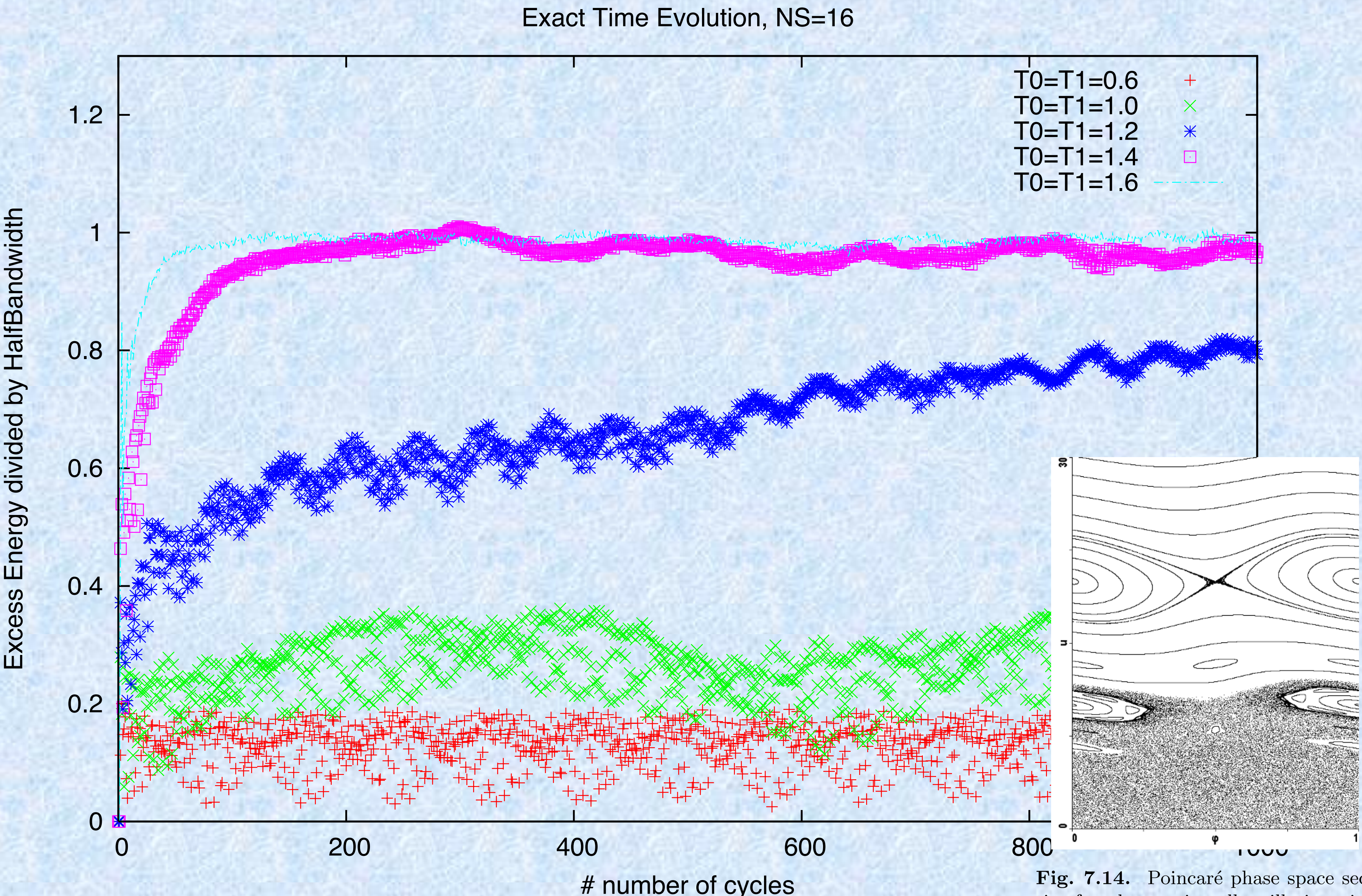
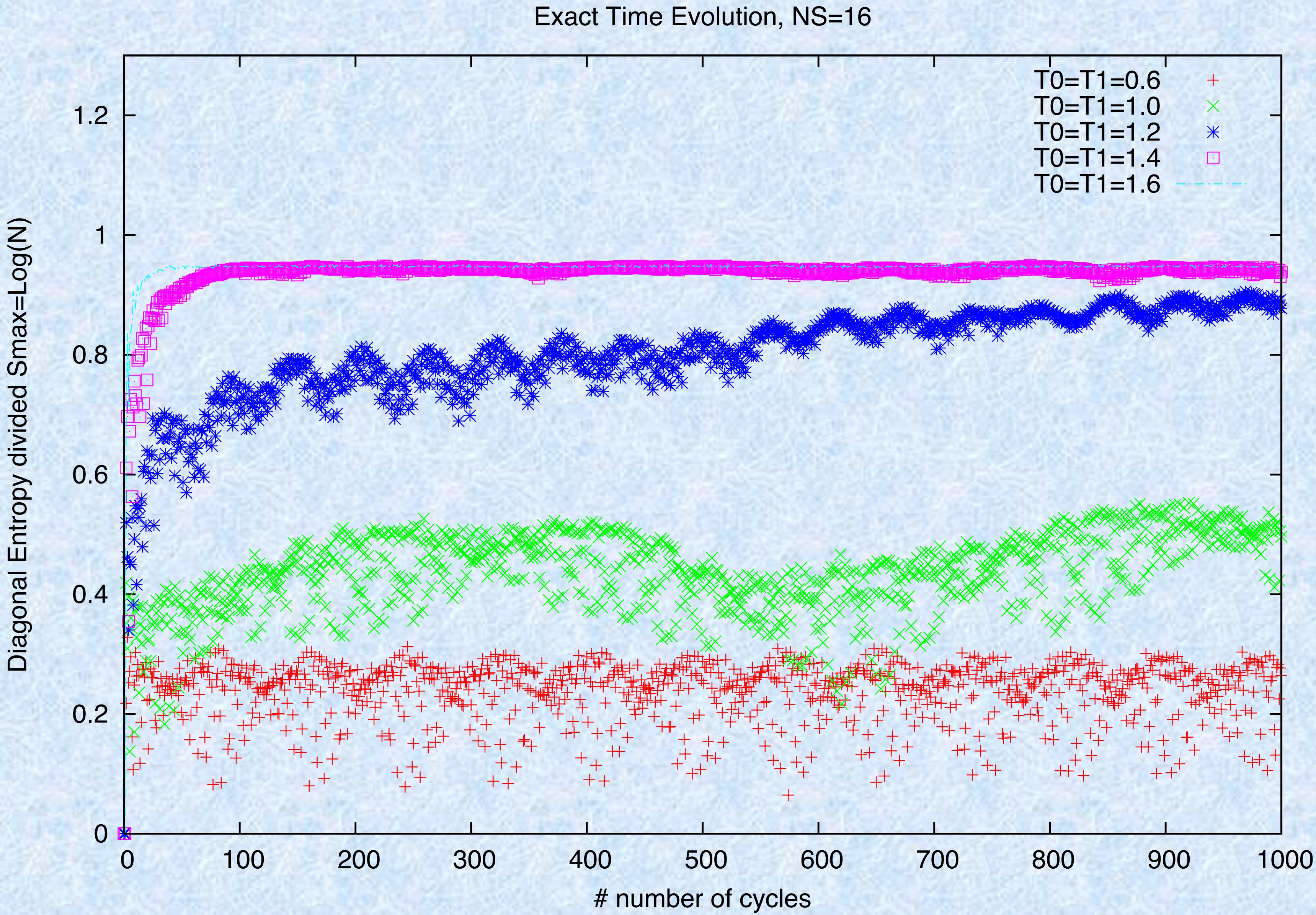


Fig. 7.14. Poincaré phase space section for a harmonic wall oscillation with $M = 20$. Iterations of several selected trajectories.

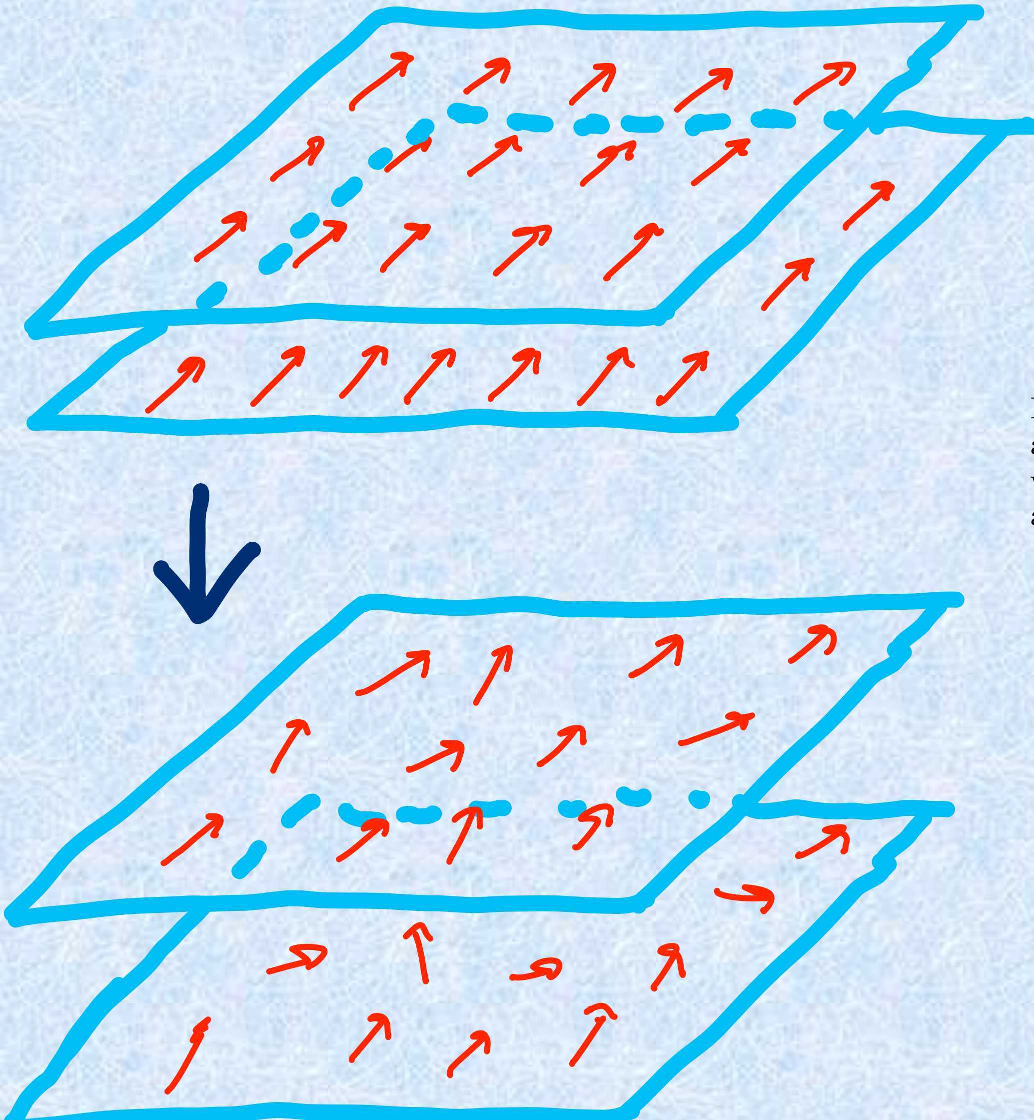
Entropy (log of number if occupied states)



Potential implications for driven dissipative systems

Thermalization forllowing 2D quench as an RG process

(together with L. Mathey, K. Gunter, J. Dalibard)



Realization with 2 component bosons

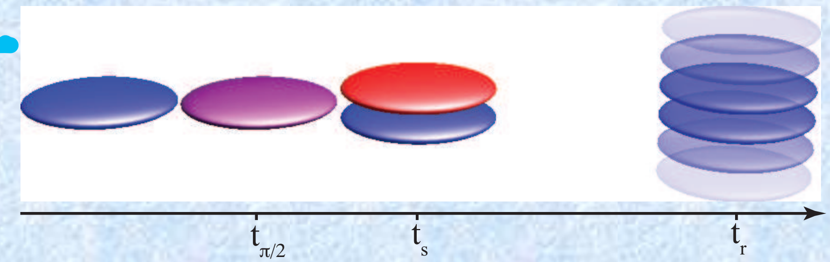
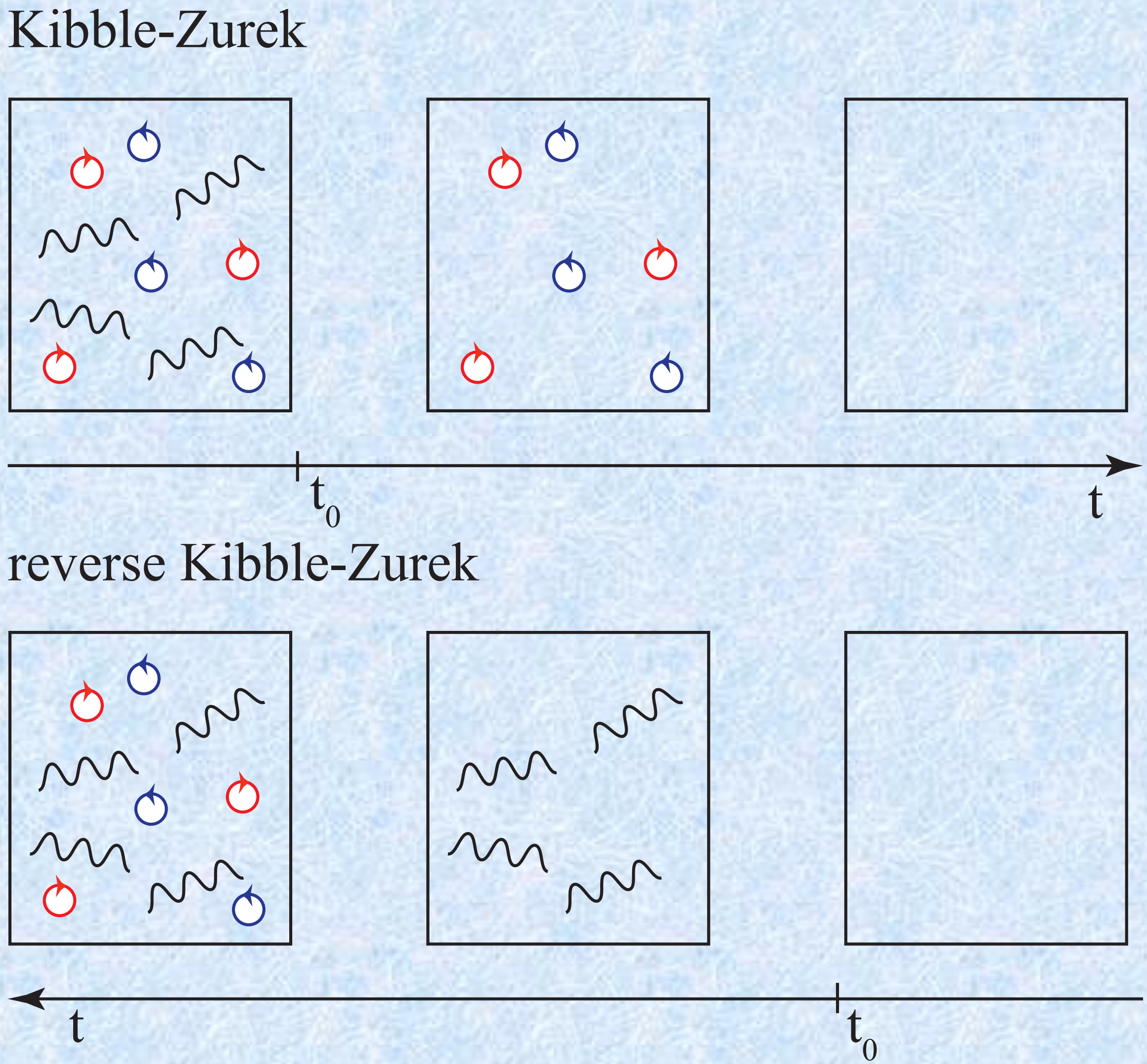


FIG. 1: We prepare a 2D atom cloud in state 1 (blue), and apply a $\pi/2$ pulse at $t_{\pi/2}$. We apply a field gradient at t_s, which separates state 1 and 2 (red) spatially. We release the atoms at time t_r and measure their interference properties.

Due to number phase uncertainty small phase fluctuations lead to large number fluctuations.

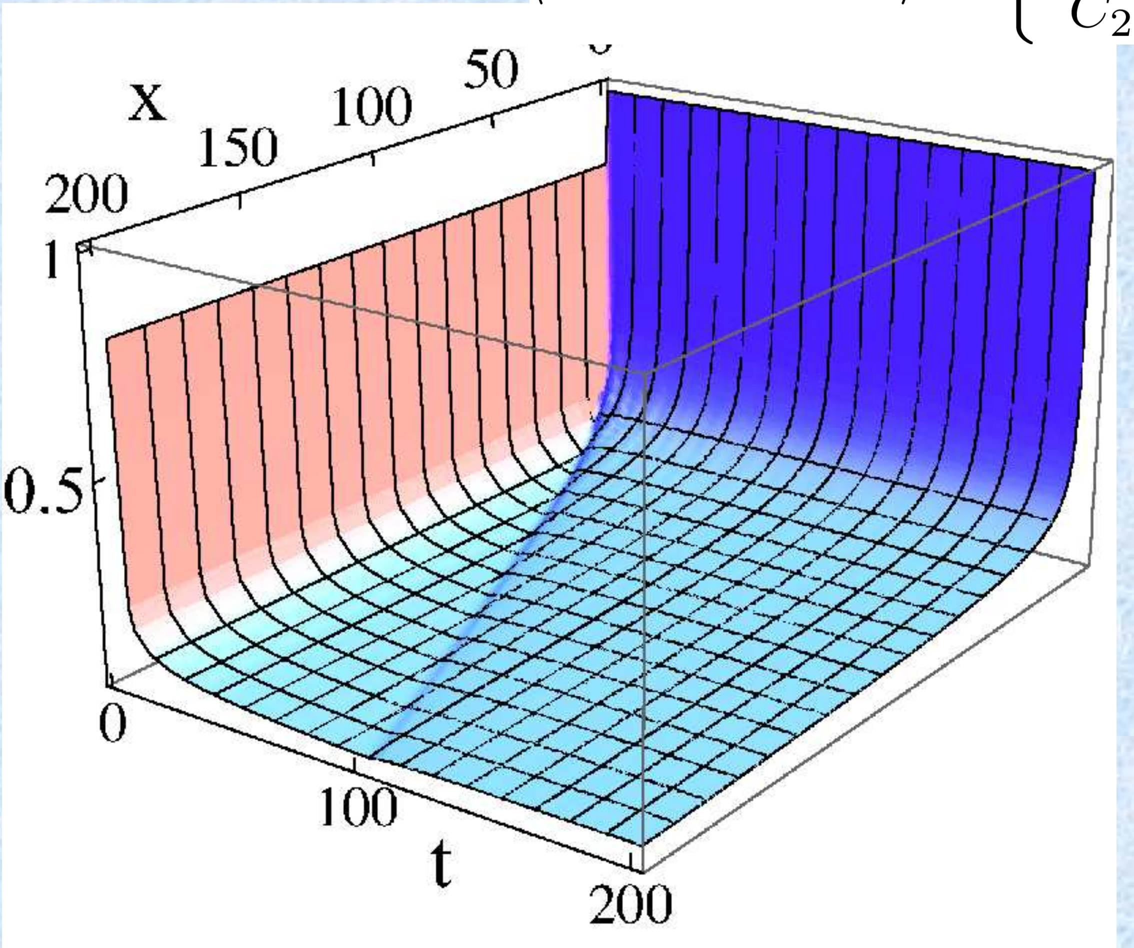
Reverse Kibble-Zurek mechanism



Expect smaller number of topologically protected defects than in equilibrium

Initial prethermalization via light cone dynamics

$$\langle e^{i(\phi(x,t)-\phi(0,t))}\rangle \approx \begin{cases} C_1|x|^{-T^*/4} & x \lesssim 2vt \\ C_2|t|^{-T^*/4} & x \gtrsim 2vt \end{cases}$$

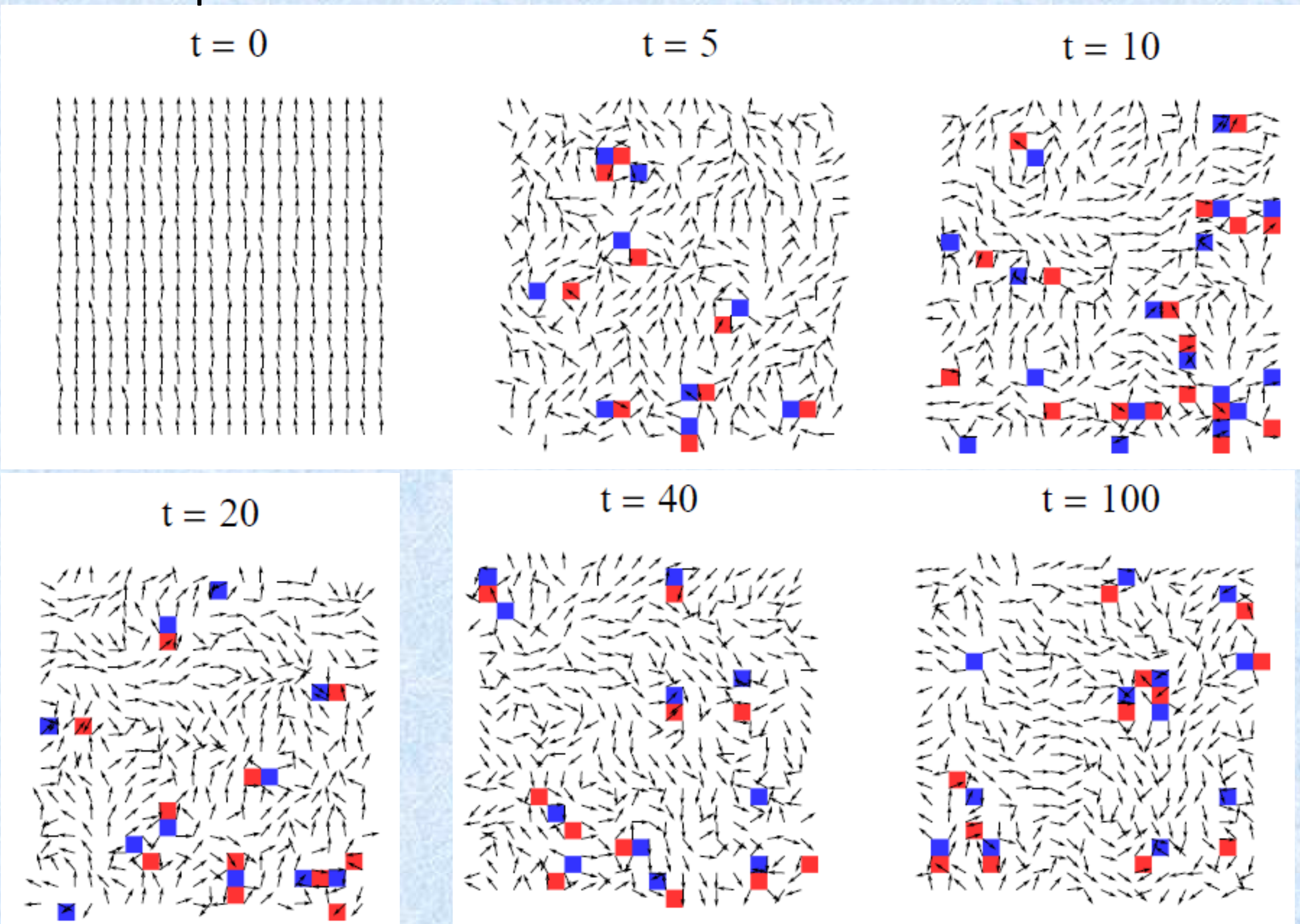


T^* depends on initial conditions,

$T^* \sim U/J$ for the Hubbard model

Long-time limit: vortex unbinding above KT temperature

Relative phase



Need to solve nonlinear Hamiltonian equations of motion:

$$\frac{d}{dt}p = \lambda T \partial_x^2 \theta + \frac{gT}{a^2}\sin\theta.$$
$$\frac{d}{dt}\theta = \mu T p.$$

Idea: split *p* and θ into low-momentum and high momentum sectors

$$\theta = \theta^{>} + \theta^{<}$$

Use perturbative approach to treat $\theta^{>}$
Time average over fast oscillating modes
Follow equations of motion for $\theta^{<}$

Additional subtlety: need to follow equations of motion for the energy.

Overall formalism very similar to the adiabatic perturbation theory.

Result: flow equations for couplings, very similar to usual KT form

$$\frac{dg}{dl} = \left(2 - \frac{1}{4\pi\lambda}\right)g$$
$$\frac{d\lambda}{dl} = \alpha\frac{g^2}{\lambda}.$$

Flow parameter l is the (exponent) of the real time!

Quantum RG dynamics (R. Vosk and E. Altman, 2012)

Recover for this problem two scenarios of relevant (normal) and irrelevant (superfluid) vortices with exponentially divergent time scale.

For this problem equilibrium = thermodynamics emerges as a result of the renormalization group process

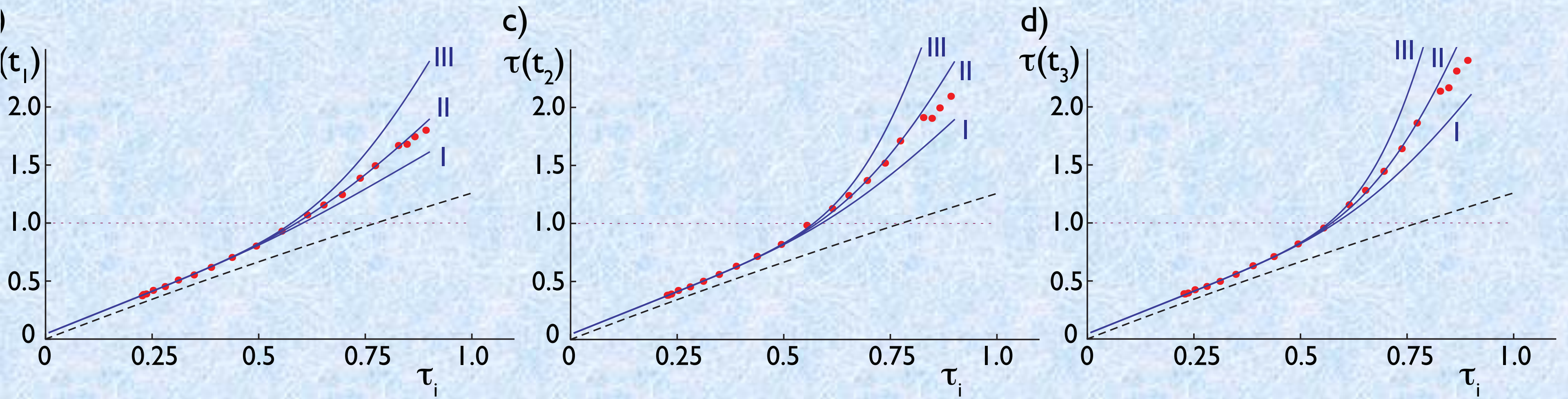
RG is a semigroup transformation (no inverse). Lost information in the time averaging of fast modes.

Comparison with numerical simulations

Interference: measure

$$\langle e^{i\phi(r)-i\phi(0)} \rangle \sim \frac{1}{r^{\tau/4}}$$

Fit of the RG prediction (at different times) and the numerical simulations



Conclusions

- Close analogy between high temperature expansion and short time expansion.

- Possibility of phase transitions in time as a result of breakdown of short time expansion: Fisher zeros in Loschmidt echo and dynamical (many-body) energy localization transition.

- Relaxational dynamics can be thought of as a renormalization process in time. Mounting number of examples.

- Universality of quench and slow dynamics (another evidence for RG).