Novel scenarios for ultra-cold lattice gases: zig-zag lattices and periodically modulated interactions

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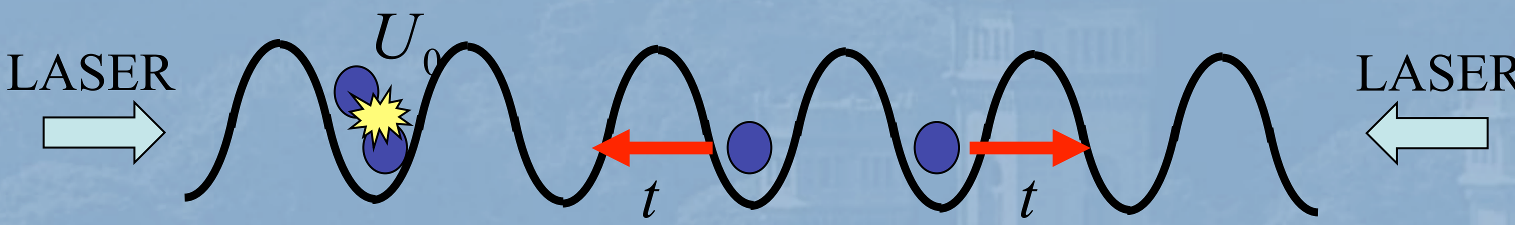
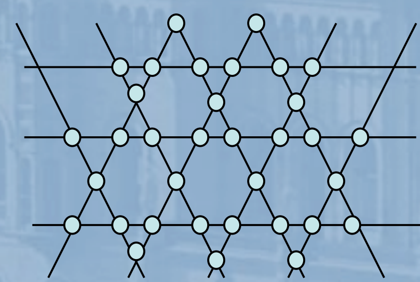
Control of atoms in optical lattices

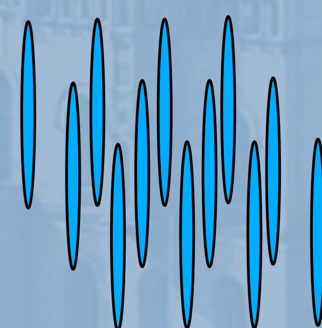


Lattice geometry

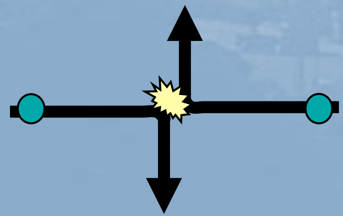


Internal structure

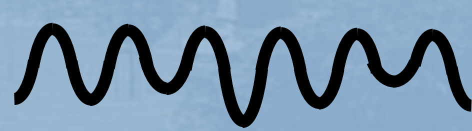
Low dimensions



Interactions
(Feshbach resonances,
dipolar interactions)



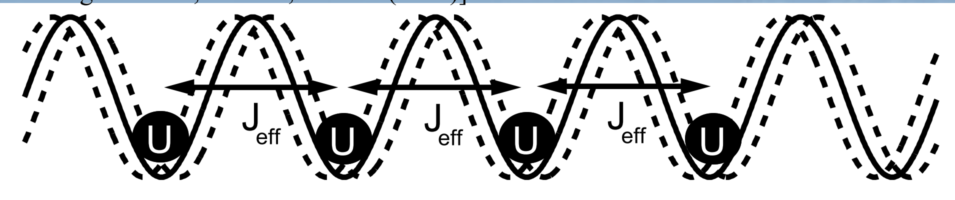
Disorder



<u>Also variable in real time!</u>

Control of atoms in OLs: periodic modulations

[From Lignier et al., PRL **99**, 220403 (2007)]



Periodic lattice modulations offer new possibilities for engineering lattice models with cold gases

$$\hat{H}_0 = -J\sum_{\langle i,j\rangle}(\hat{c}_i^\dagger\hat{c}_j+\hat{c}_j^\dagger\hat{c}_i)+\frac{U}{2}\sum_j\hat{n}_j(\hat{n}_j-1)+K\cos(\omega t)\sum_j j\hat{n}_j$$

For a strong shaking, ω>>U,J

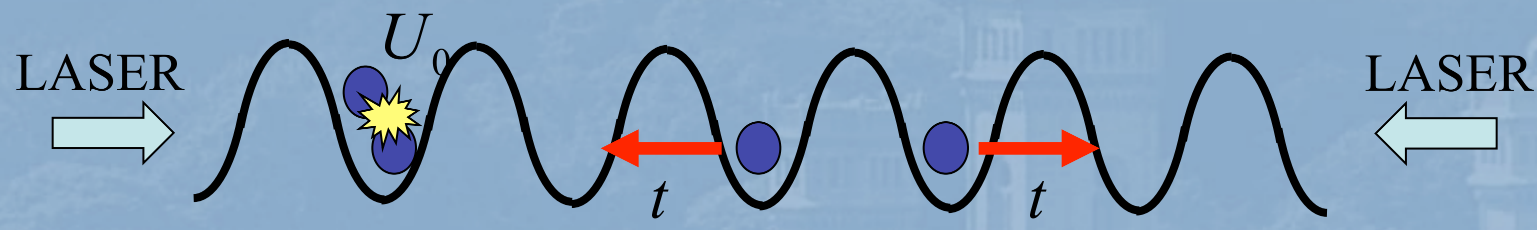
Modified hopping which may even change sign!

$$J_{eff}=JJ_0\left(\frac{K}{\hbar\omega}\right)$$

$$\hat{H}_{eff}=-J_{eff}\sum_{<i,j>}\left(\hat{c}_i^+\hat{c}_j+h.c.\right)+\frac{U}{2}\sum_i\hat{n}_i(\hat{n}_i-1)-\mu\sum_i\hat{n}_i$$

[Eckardt et al., PRL **95**, 260404 (2005); Lignier, et al., PRL **99**, 220403 (2007); Kierig et al., PRL **100**, 190405 (2008); Zenesini et al., PRL **102**, 100403 (2009); Struck et al., Science **333**, 996 (2011)]

Control of atoms in OLs: One and two-particle hard core



If U_0 is large enough one may forbid double occupation per site (hard-core regime)

$$\left(b_i^+\right)^2 = 0$$

In the presence of strong 3-body losses one may induce an effective 2-body hard-core
[Daley et al., PRL **102**, 040402 (2009)]

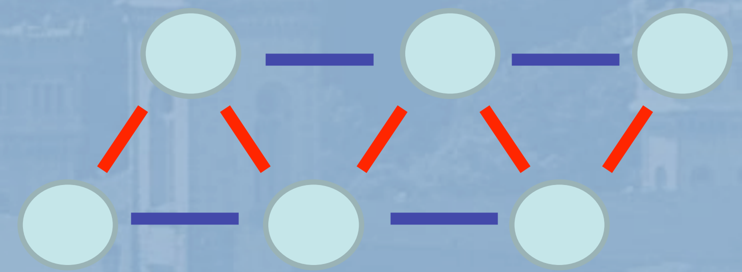
$$\left(b_i^+\right)^3 = 0$$

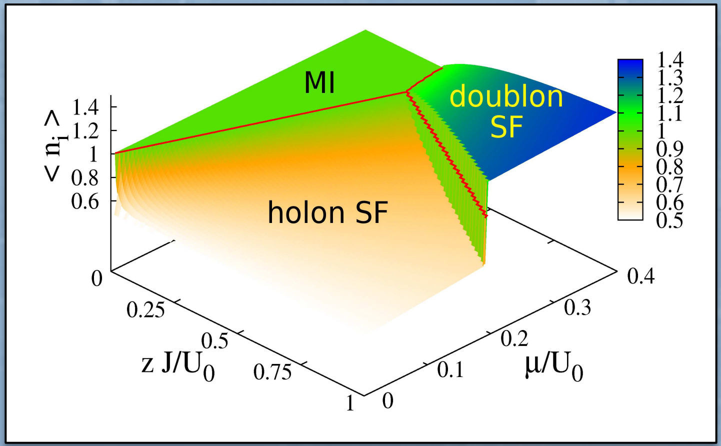
Recent experiment in Innsbruck [Mark et al., PRL **108**, 215302 (2012)]

This talk

Ultra-cold bosons
in zig-zag optical lattices

[Greschner et al., arXiv:1202.5386]

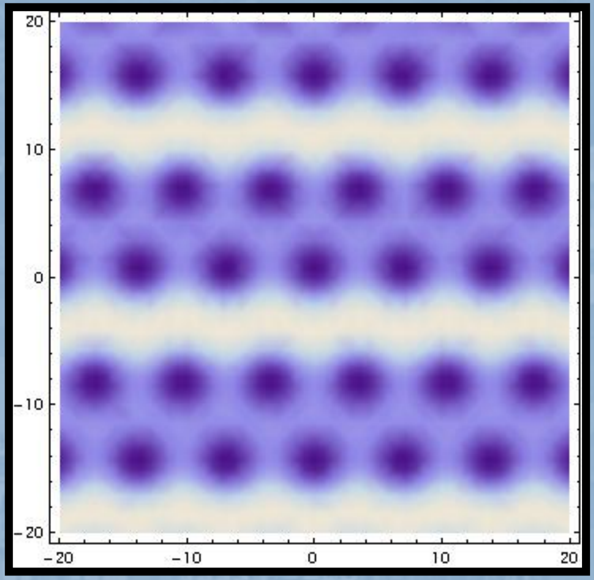


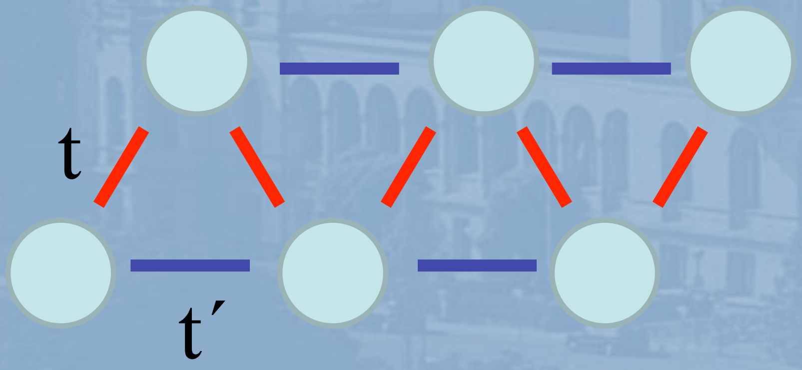


Atoms with periodically modulated interactions

Ultra-cold gases in zig-zag optical lattices

Zig-zag optical lattices may be obtained e.g. by overimposing a triangular lattice [Becker et al., NJP **12**, 065025 (2010)] and a superlattice of doubled period

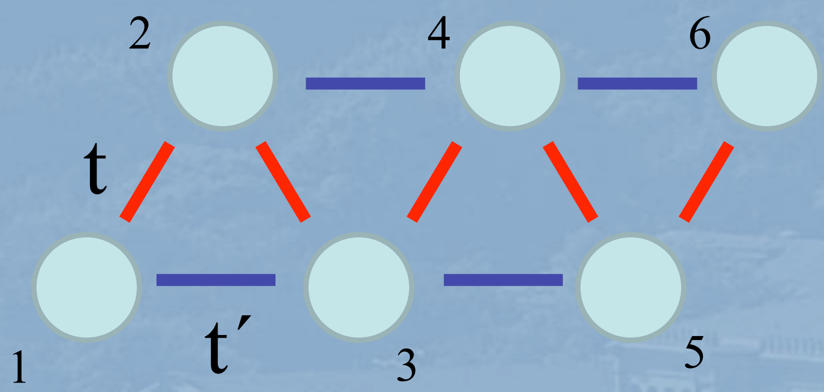




The value of t and t´ may be controlled independently by elliptical shaking (also their signs!) [Struck et al., Science **333**, 996 (2011)]

We shall consider the case where both t,t ‘<0 (AF coupling)

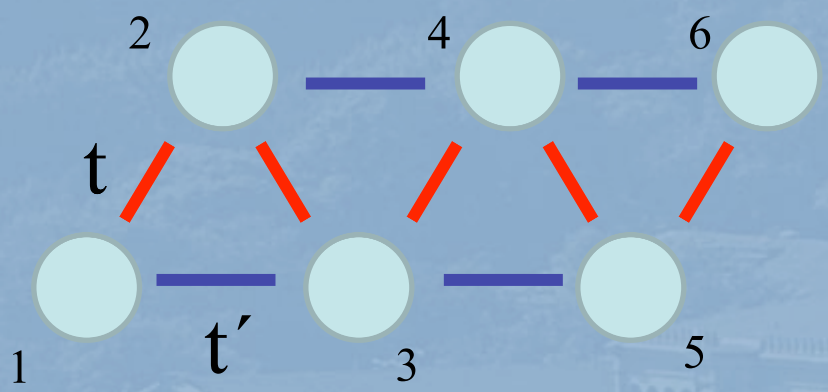
Bosons in zig-zag optical lattices



$$H = \sum_i \left[|t| b_i^+ b_{i+1} + |t'| b_i^+ b_{i+2} + H.c. \right]$$

$$+ \frac{U_0}{2} \sum_i n_i (n_i - 1) + \sum_i \left[U_1 n_i n_{i+1} + U'_1 n_i n_{i+2} \right]$$

Frustrated AF spin-1 chains



Assuming bosons with mean occupation n=1, we introduce the pseudo-spin

$$S_i^z = 1 - n_i \quad \begin{Bmatrix} |n=0\rangle \Rightarrow |m=+1\rangle \\ |n=1\rangle \Rightarrow |m=0\rangle \\ |n=2\rangle \Rightarrow |m=-1\rangle \end{Bmatrix}$$

The system resembles to a large extent a frustrated AF spin-1 chains with uniaxial single-ion anisotropy

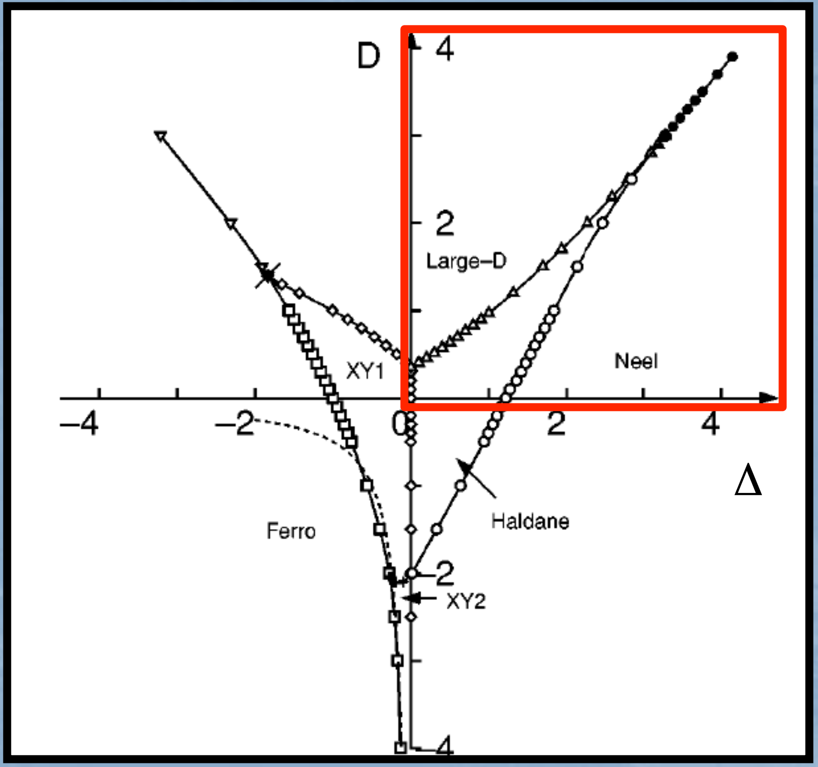
$$H = J\sum_i \left[\left(\vec{S}_i \cdot \vec{S}_{i+1}\right)_\Delta + j\left(\vec{S}_i \cdot \vec{S}_{i+2}\right)_{\Delta'} + D\left(S_i^z\right)^2 \right]$$

$$J = 2|t| \qquad j = t'/t \qquad D = U_0/4|t|$$

$$\Delta = U_1/2|t| \qquad \Delta = U'_1/2|t'|$$

AF spin-1 chains

$$H = J\sum_i \left[S_i^x \cdot S_{i+1}^x + S_i^y \cdot S_{i+1}^y + \Delta S_i^z \cdot S_{i+1}^z + D\left(S_i^z\right)^2 \right]$$



[Chen, Hida and Sanctuary, PRB **67**, 104401 (2003)]

Haldane phase …0+0…0-0…0+0…0-0…

(„diluted AF order")

String order: $\lim_{|i-j|\to\infty} \left\langle -S_i^z \exp\left[i\pi \sum_{l=i+1}^{j-1} S_l^z \right] S_j^z \right\rangle \neq 0$

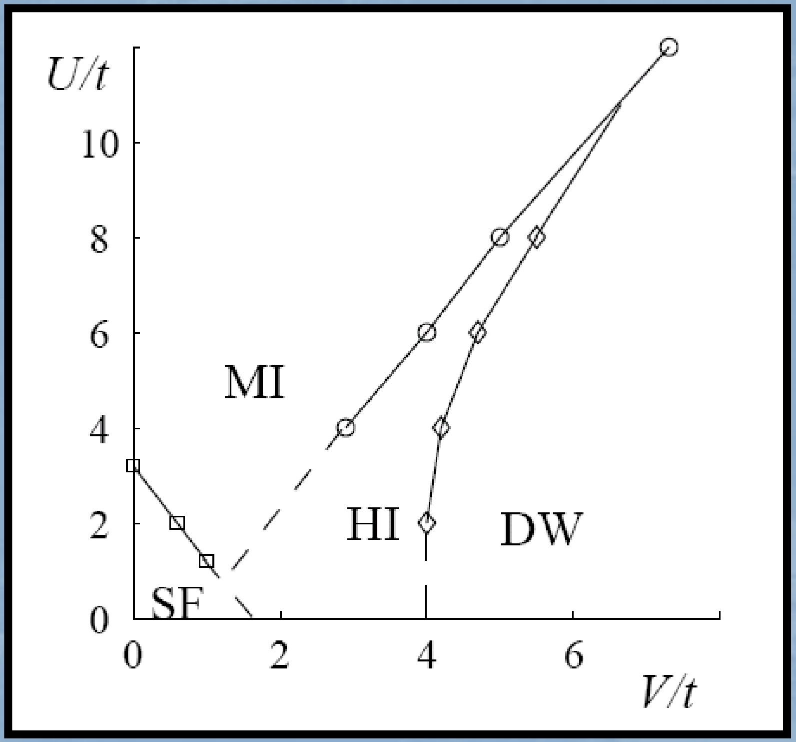
Large-D phase …0+-0…0-+0…0-+0…

Parity order: $\lim_{|i-j|\to\infty} \left\langle \exp\left[i\pi \sum_{l=i+1}^{j-1} S_l^z \right] \right\rangle \neq 0$

Néel phase …+-+-+-+-+-+-+-+-…

1D polar gases in optical lattices: Haldane-insulator phase

$$H = -t\sum_i \left[b_i^+ b_{i+1} + H.c.\right] + \frac{U_0}{2}\sum_i n_i(n_i - 1) + U_1 \sum_i n_i n_{i+1}$$



[Dalla-Torre, Berg and Altman, PRL **97**, 260401 (2006)]

Haldane-insulator ...101...121...101...121...

String order: $O_s^2 \equiv \lim_{|i-j|\to\infty} \left\langle \delta n_i \exp\left[i\pi \sum_{l=i+1}^{j-1} \delta n_l\right] \delta n_j \right\rangle \neq 0$

Mott-insulator ...1021...1201...1201...

Parity order: $O_p^2 \equiv \lim_{|i-j|\to\infty} \left\langle \exp\left[i\pi \sum_{l=i+1}^{j-1} \delta n_l\right] \right\rangle \neq 0$

Experiment [Endres et al., Science **334**, 200 (2011)]

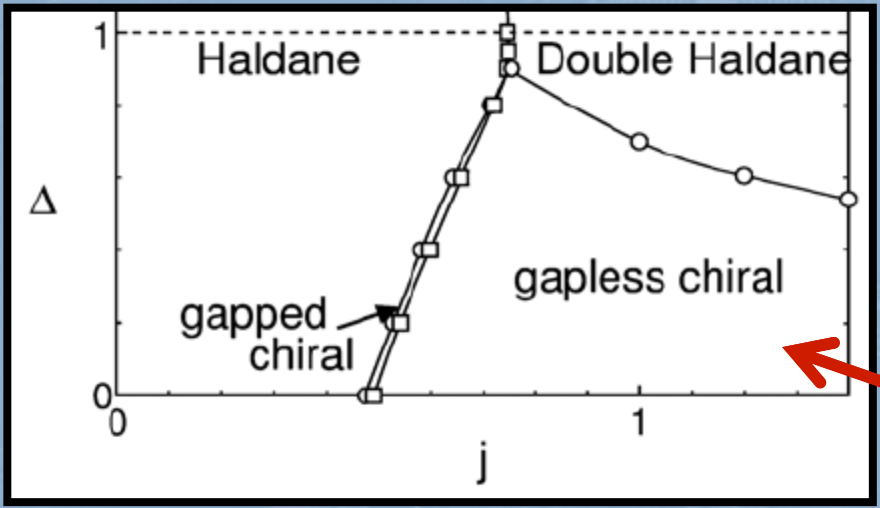
Density wave ...02020202020202...

Frustrated AF spin-1 chains

$$H = J\sum_i \left[\left(\vec{S}_i \cdot \vec{S}_{i+1}\right)_\Delta + j\left(\vec{S}_i \cdot \vec{S}_{i+2}\right)_{\Delta'} + D\left(S_i^z\right)^2 \right]$$

D=0 (and Δ=Δ′) phase diagram

[Hikihara et al., J. Phys. Soc. Jpn. **69**, 259 (2000)]



Chirality

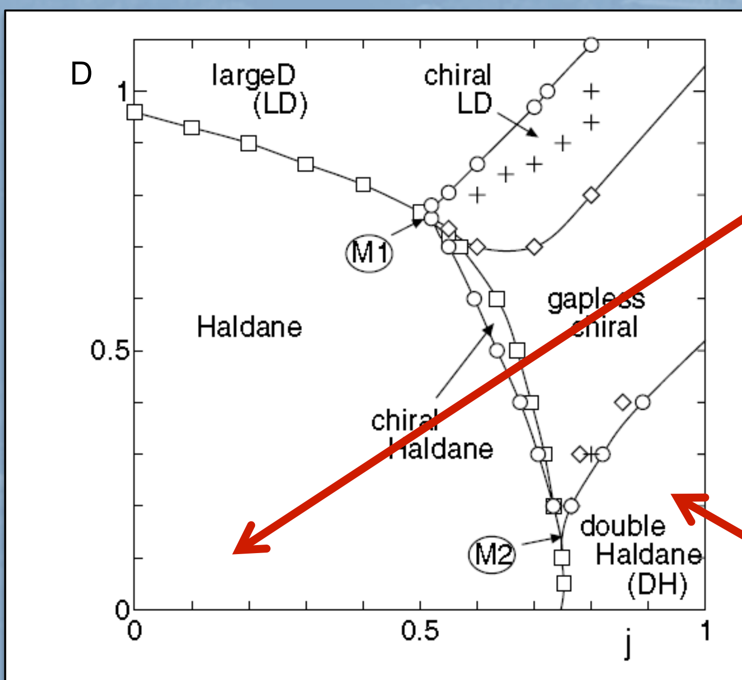
$$\vec{\kappa}_i \equiv \left\langle \left(\vec{S}_i \times \vec{S}_{i+1}\right) \right\rangle \neq 0$$

Frustrated AF spin-1 chains

$$H = J\sum_i \left[\left(\vec{S}_i \cdot \vec{S}_{i+1}\right)_\Delta + j\left(\vec{S}_i \cdot \vec{S}_{i+2}\right)_{\Delta'} + D\left(S_i^z\right)^2 \right]$$

D>0 (and Δ=Δ′=1) phase diagram

[Hikihara, J. Phys. Soc. Jpn. **71**, 319 (2002)]



Haldane phase is expected (even for Δ=0) for sufficiently low D.

For Δ=0 gapless chiral here

Bosons in zig-zag optical lattices

$$H=\sum_i\left[|t|b_i^+b_{i+1}+|t'|b_i^+b_{i+2}+H.c.\right]+\frac{U_0}{2}\sum_i n_i\left(n_i-1\right)$$

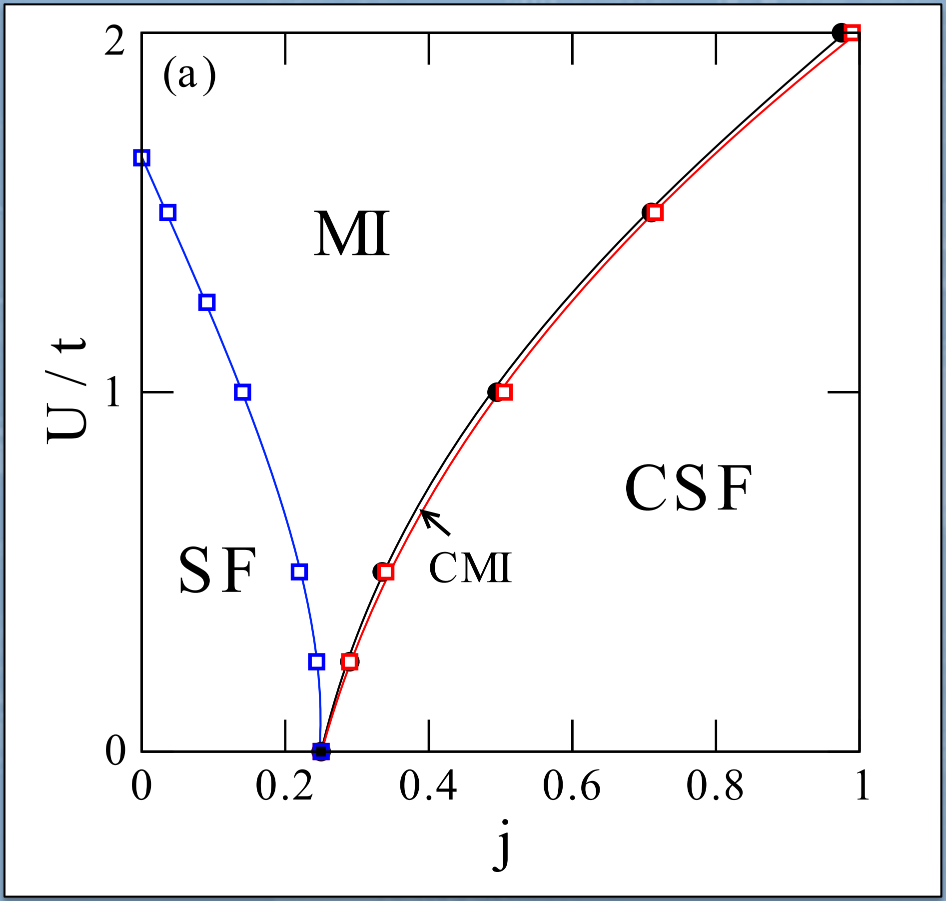
In absence of dipolar interactions (U_1=U'_1=0, and hence Δ=0)

Haldane phase is expected (even for Δ=0) for sufficiently low D. → Haldane-insulator expected (even for non-polar gases) for sufficiently low U_0.

However, for unconstrained bosons this is not what occurs, since for low U_0 the boson ↔spin mapping fails

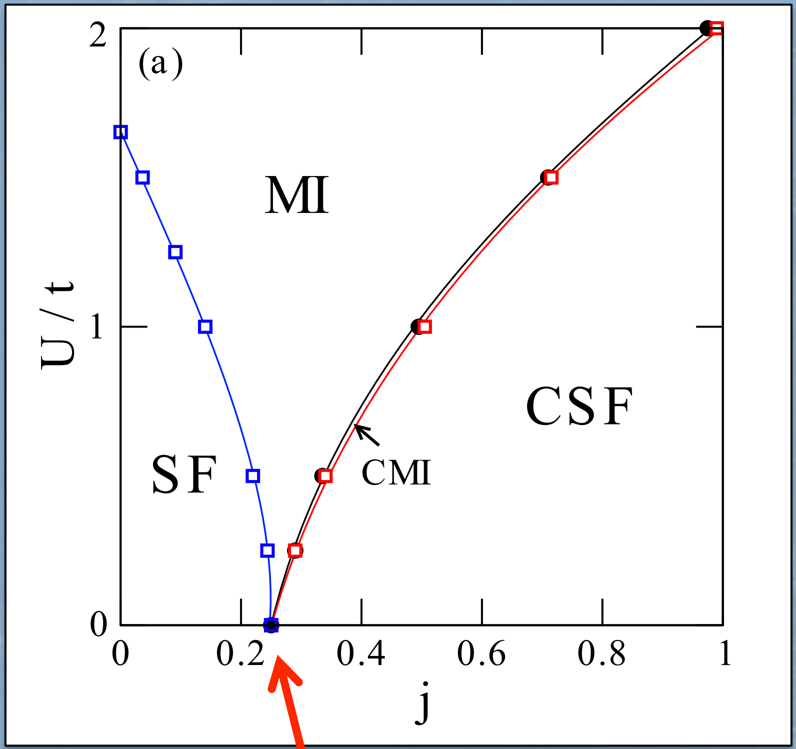
Bosons in zig-zag optical lattices (unconstrained bosons)

$$H=\sum_i\left[|t|b_i^+b_{i+1}+|t'|b_i^+b_{i+2}+H.c.\right]+\frac{U_0}{2}\sum_i n_i\left(n_i-1\right)$$



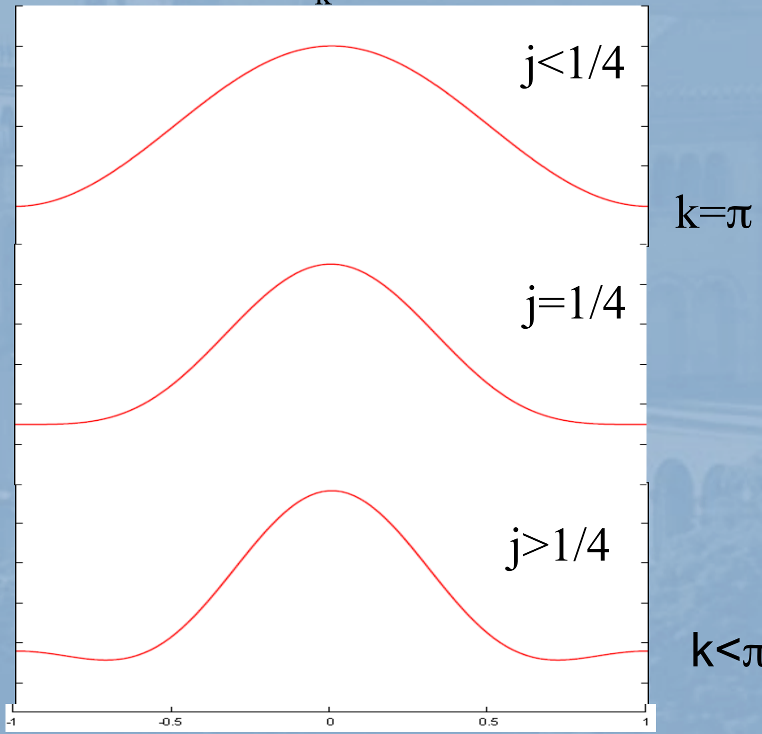
Bosons in zig-zag optical lattices (unconstrained bosons)

$$H = \sum_i \left[|t| b_i^+ b_{i+1} + |t'| b_i^+ b_{i+2} + H.c. \right] + \frac{U_0}{2} \sum_i n_i (n_i - 1)$$



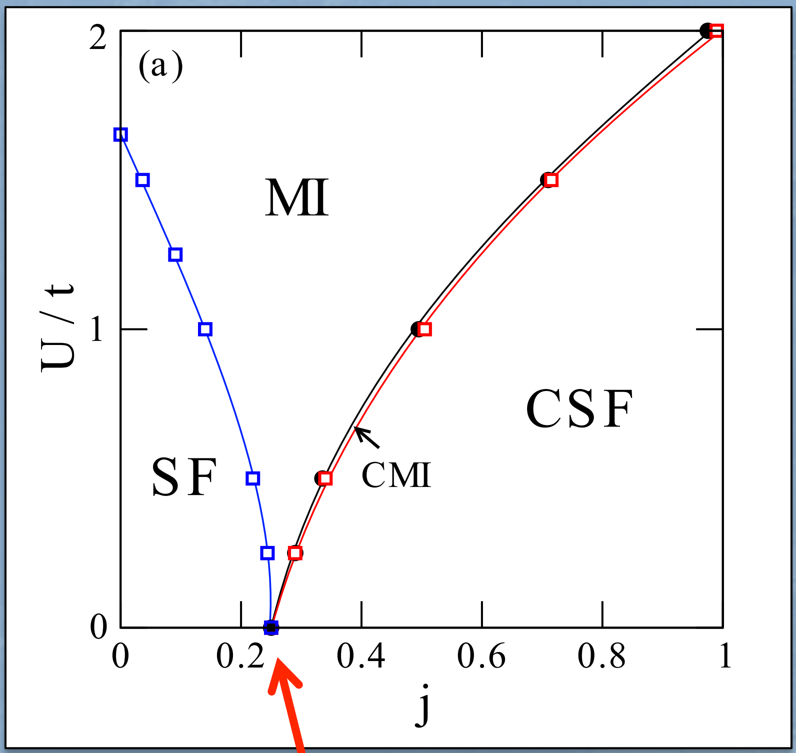
Lifshitz-point (j=1/4)

For $U_0=0$: $H = 2|t| \sum_k \left[\cos k + j \cos 2k \right] b_k^+ b_k$



Bosons in zig-zag optical lattices (unconstrained bosons)

$$H=\sum_i\left[|t|b_i^+b_{i+1}+|t'|b_i^+b_{i+2}+H.c.\right]+\frac{U_0}{2}\sum_i n_i(n_i-1)$$



Lifshitz-point (j=1/4)

For U_0=0: $H=2|t|\sum_k\left[\cos k+j\cos 2k\right]b_k^+b_k$

Chirality $\kappa_i^z\equiv\frac{i}{2}\left\langle b_{j+1}^+b_j-b_j^+b_{j+1}\right\rangle$

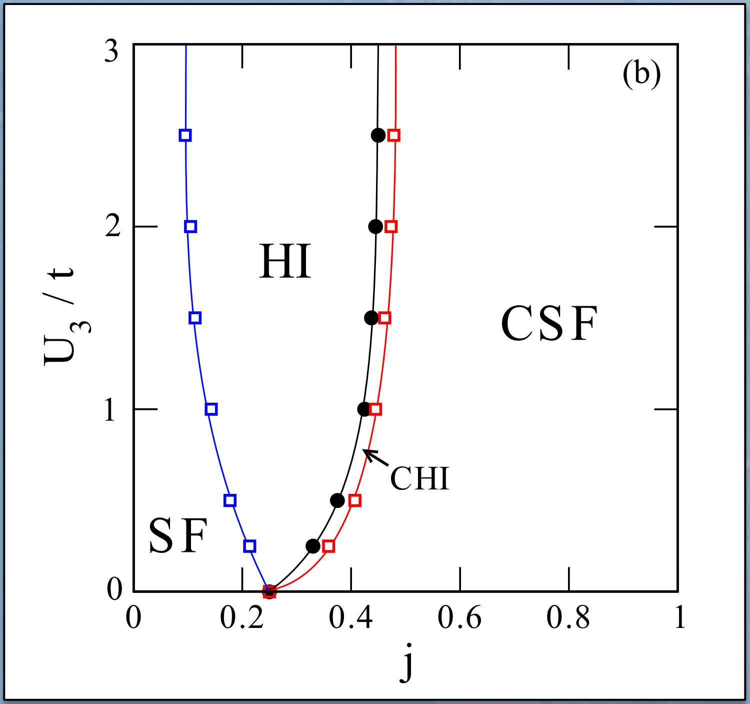
$$\kappa\equiv\sum_i\kappa_i^z=\sum_k\sin k\left\langle b_k^+b_k\right\rangle$$

When U_0 grows a Mott-Insulator (=large-D for spin-1 chains) opens starting at the Lifshitz point

Bosons in zig-zag optical lattices (constrained bosons)

$$H = \sum_i \left[|t| b_i^+ b_{i+1} + |t'| b_i^+ b_{i+2} + H.c. \right] + \frac{U_0}{2} \sum_i n_i (n_i - 1) \qquad \left(b_i^+ \right)^3 = 0$$

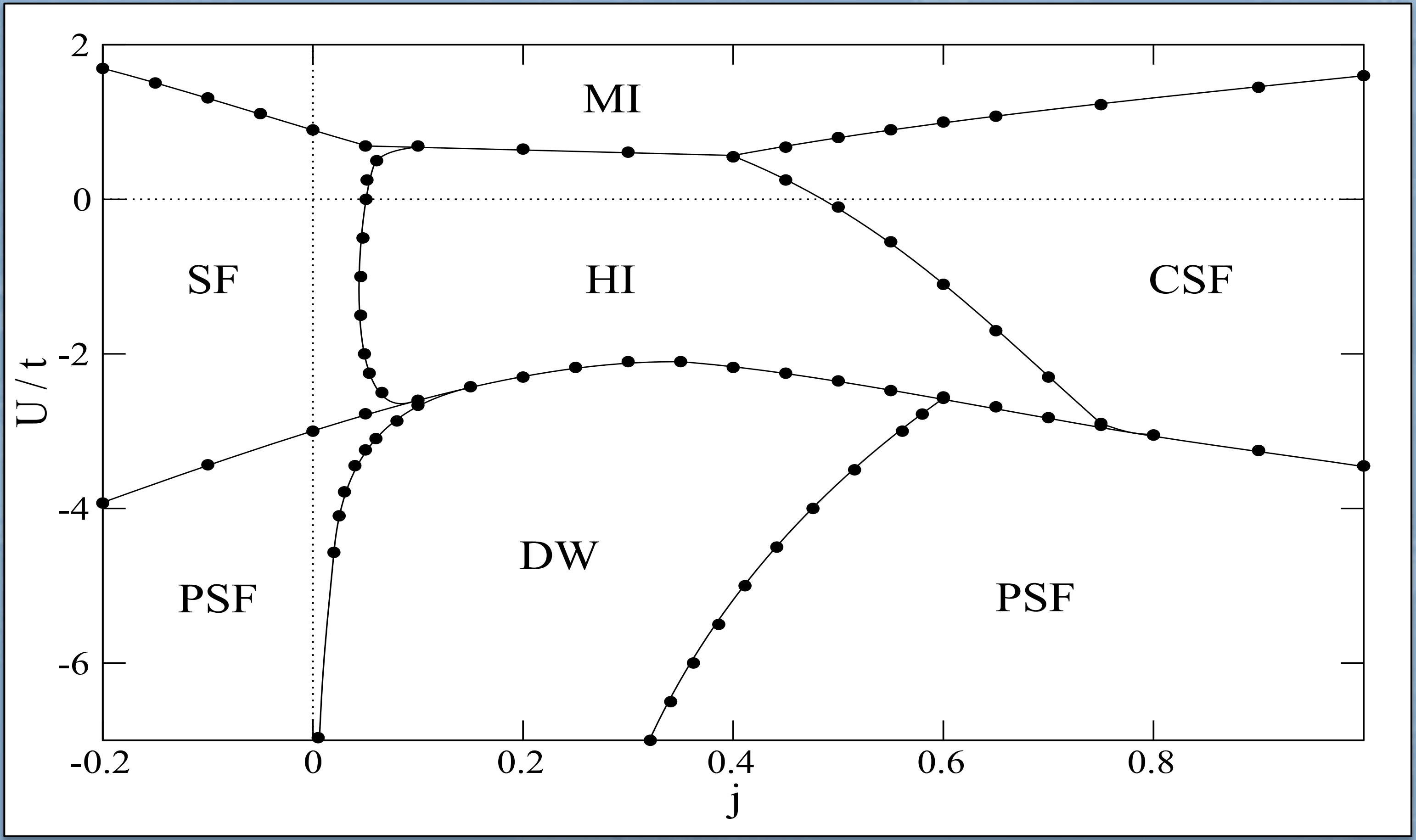
$$H = \sum_i \left[|t| b_i^+ b_{i+1} + |t'| b_i^+ b_{i+2} + H.c. \right] + U_3 \sum_i \left(b_i^+ \right)^3 \left(b_i \right)^3$$



A Haldane-insulator (HI) phase opens for finite U_3 at the Lifshitz point

Bosons in zig-zag optical lattices (constrained bosons)

$$H = \sum_i \left[|t| b_i^+ b_{i+1} + |t'| b_i^+ b_{i+2} + H.c. \right] + \frac{U_0}{2} \sum_i n_i (n_i - 1) \qquad (b_i^+)^3 = 0$$



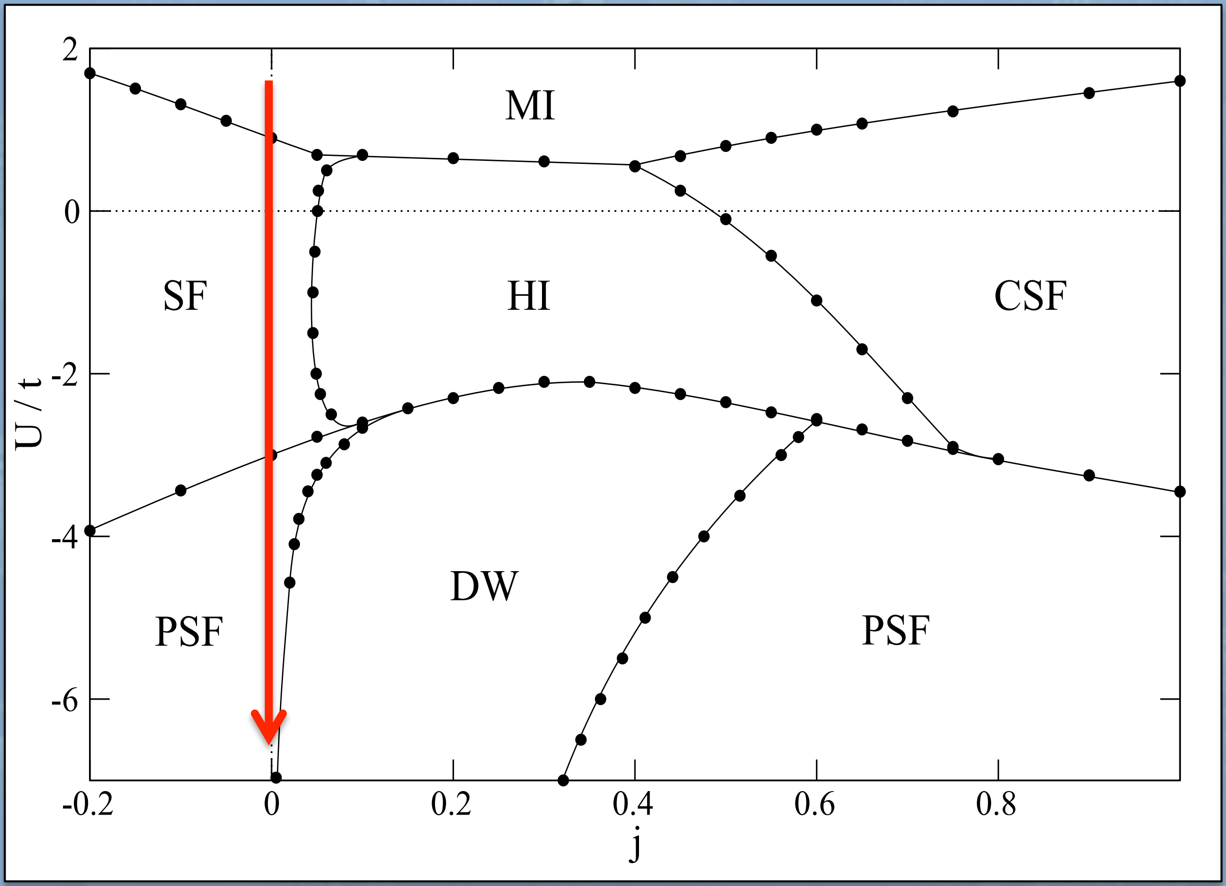
Bosons in zig-zag optical lattices (constrained bosons)

$$H=\sum_i\left[|t|b_i^+b_{i+1}+|t'|b_i^+b_{i+2}+H.c.\right]+\frac{U_0}{2}\sum_i n_i(n_i-1)\qquad \left(b_i^+\right)^3=0$$

One may study large U<0 (D<0), which leads to pair-superfluidity

$$G_{ij}^{(1)}\equiv\left\langle b_i^+b_j\right\rangle$$

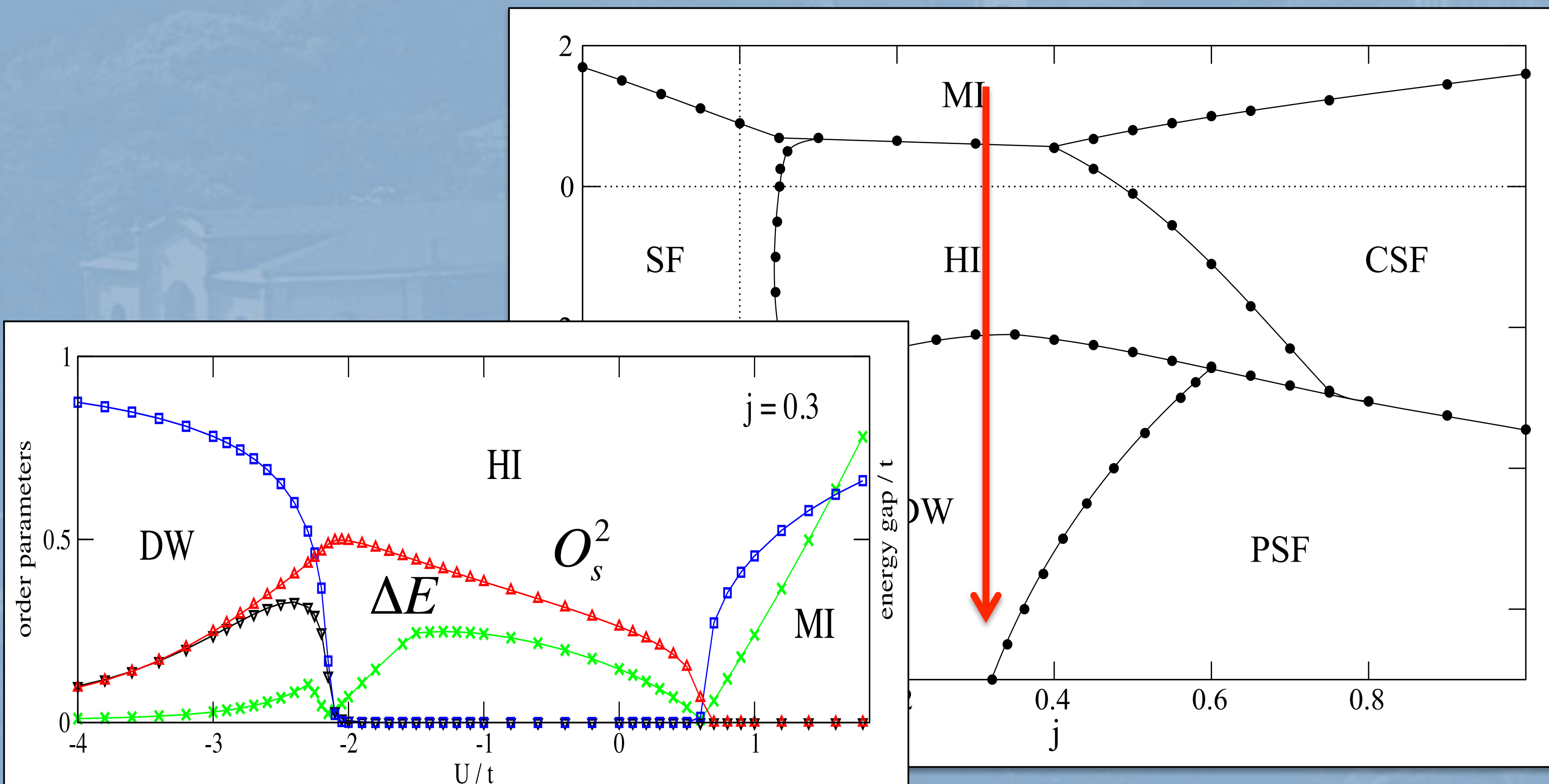
$$G_{ij}^{(2)}\equiv\left\langle\left(b_i^+\right)^2 b_j^2\right\rangle$$



[Daley et al., PRL **102**, 040402 (2009);
Bonnes and Wessel, PRL **106**, 185302 (2011)]

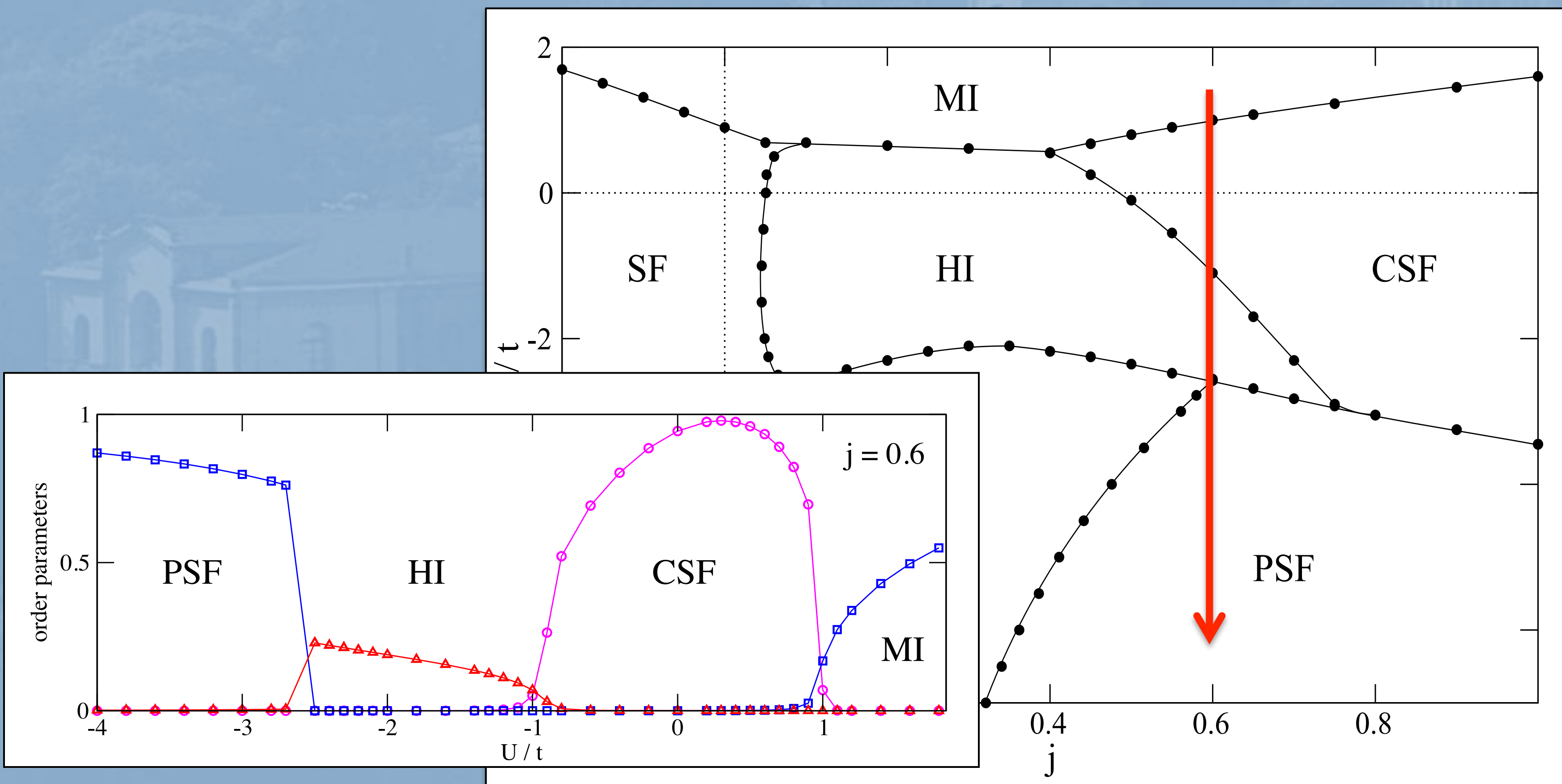
Bosons in zig-zag optical lattices (constrained bosons)

$$H = \sum_i \left[|t| b_i^+ b_{i+1} + |t'| b_i^+ b_{i+2} + H.c. \right] + \frac{U_0}{2} \sum_i n_i (n_i - 1) \qquad \left(b_i^+\right)^3 = 0$$



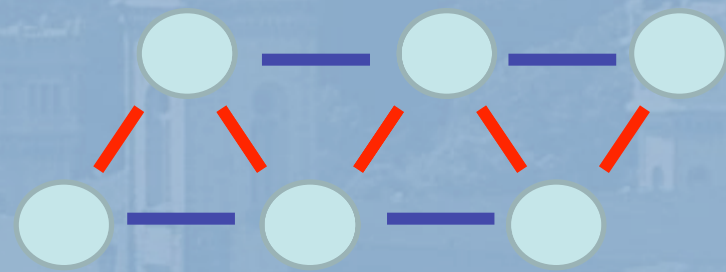
Bosons in zig-zag optical lattices (constrained bosons)

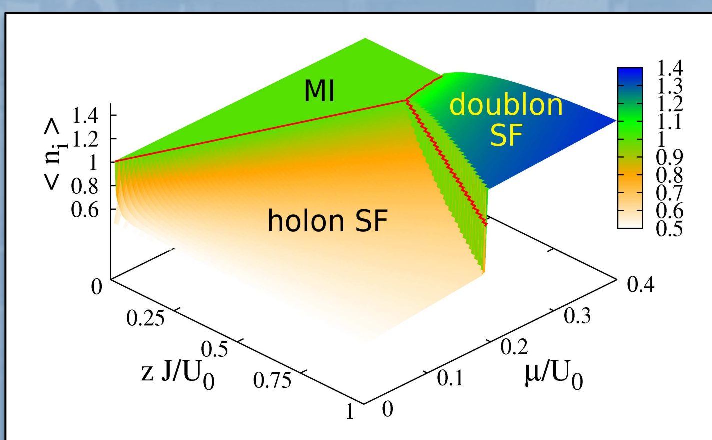
$$H = \sum_i \left[|t| b_i^+ b_{i+1} + |t'| b_i^+ b_{i+2} + H.c. \right] + \frac{U_0}{2} \sum_i n_i \left(n_i - 1\right) \qquad \left(b_i^+\right)^3 = 0$$



This talk

Ultra-cold bosons
in zig-zag optical lattices





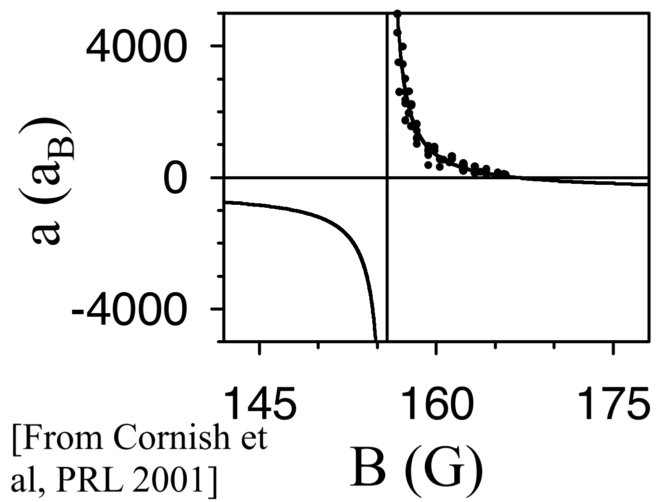
Atoms with periodically modulated interactions

[Rapp et al., arXiv:1207.0641]

Lattice gases with periodically modulated interactions

Periodically modulated magnetic field B(t)=B(t+2π/ω) in the vicinity of a Feshbach resonance

$$a(t) = a_{bg}\left(1 + \frac{\Delta B}{B(t) - B_r}\right) = a_0 + \sum_{l>0} a_l \cos(l\omega t)$$



[From Cornish et al, PRL 2001]

$$H(t) = -J\sum_{\langle ij\rangle} b_i^\dagger b_j + \frac{U(t)}{2}\sum_i \hat{n}_i(\hat{n}_i - 1) - \sum_i \mu\,\hat{n}_i$$

$$U(t) = U_0 + \sum_{l>0} U_l \cos(l\omega t) = U_0 + \tilde{U}(t)$$

Floquet analysis (similar as for shaken lattices...)

Floquet basis

$$|\{n_j\},m\rangle = e^{im\omega t}e^{-i\frac{V(t)}{2}\sum_j \hat{n}_j(\hat{n}_j-1)}|\{n_j\}\rangle$$

$$V(t) \quad = \quad \int^t \tilde{U}(t')dt'/\hbar$$

$$\langle\langle\{n'_j\},m'|\ldots|\{n_j\},m\rangle\rangle \quad = \quad \frac{1}{T}\int_0^T \langle\{n'_j\},m'|\ldots|\{n_j\},m\rangle$$

Matrix elements in the Floquet basis

$$\langle\langle\{n'_j\},m'|[H(t)-i\hbar\partial_t]|\{n_j\},m\rangle\rangle$$

$$= \delta_{m,m'}\ \langle\{n'_j\}|H_m|\{n_j\}\rangle$$

$$-J\sum_{\langle i,j\rangle}\langle\{n'_j\}|b_i^\dagger F_{m'-m}(\hat{n}_i-\hat{n}_j)b_j|\{n_j\}\rangle$$

$$H_m \quad = \quad m\hbar\omega + \frac{U_0}{2}\sum_j \hat{n}_j\,(\hat{n}_j-1) - \sum_j \mu\hat{n}_j$$

$$F_m(x) \quad = \quad \frac{1}{T}\int_0^T dt e^{-imt}e^{iV(t)x}$$

Effective Hamiltonian

$$\hbar\omega \gg J, U_0$$

$$H_{\text{eff}} = -J\sum_{\langle ij\rangle} b_i^\dagger F_0(\hat{n}_i - \hat{n}_j) b_j + \frac{U_0}{2}\sum_i \hat{n}_i(\hat{n}_i - 1) - \mu\sum_i \hat{n}_i$$

Periodically modulated interactions result in a nonlinear hopping that depends on the difference of occupations in nearest neighbors

[This result was first derived for 2-well BECs by Gong et al., PRL **103**, 133002 (2009)]

$$\tilde{U}(t) = U_1 \cos\omega t$$

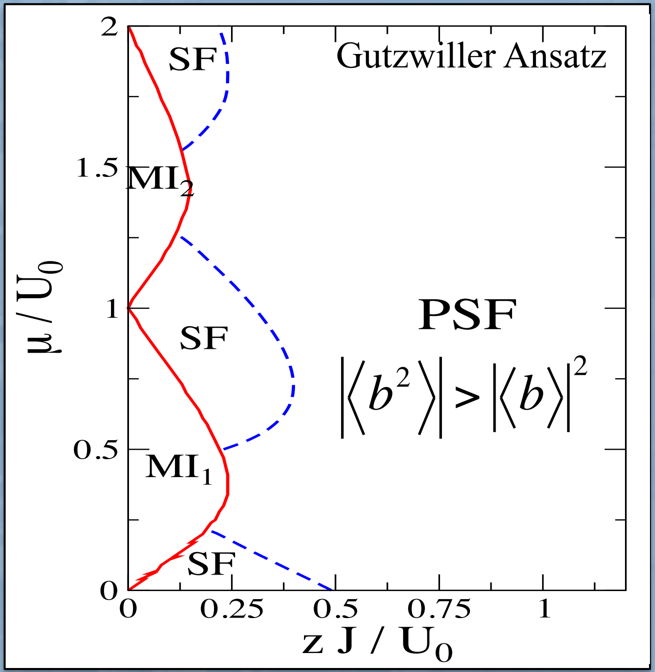
$$F_0(\hat{n}_i - \hat{n}_j) = J_0(\Omega(\hat{n}_i - \hat{n}_j))$$

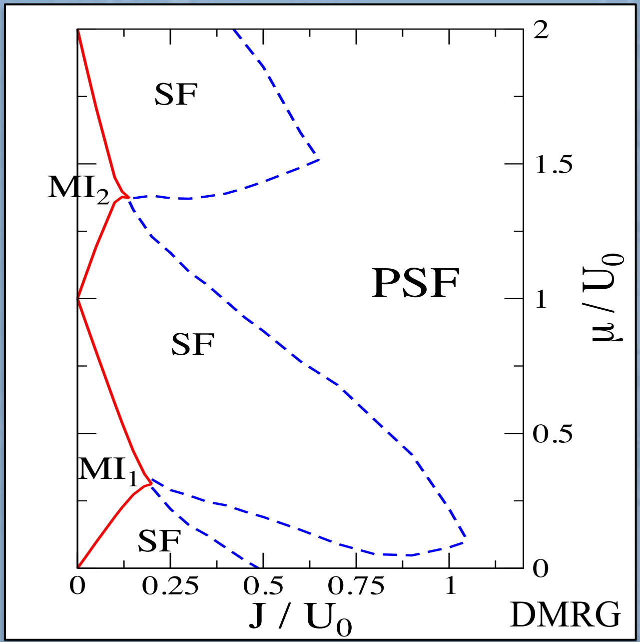
$$\Omega = U_1 / \hbar\omega$$

Pair-superfluidity

If $J_0(\Omega)<0$ the system develops pair superfluidity (PSF)

[This property is shared with other systems with nonlinear hopping, Schmidt et al., PRB **74**, 174508 (2006); T. Sowiński et al., PRL **108**, 115301] (2012)]





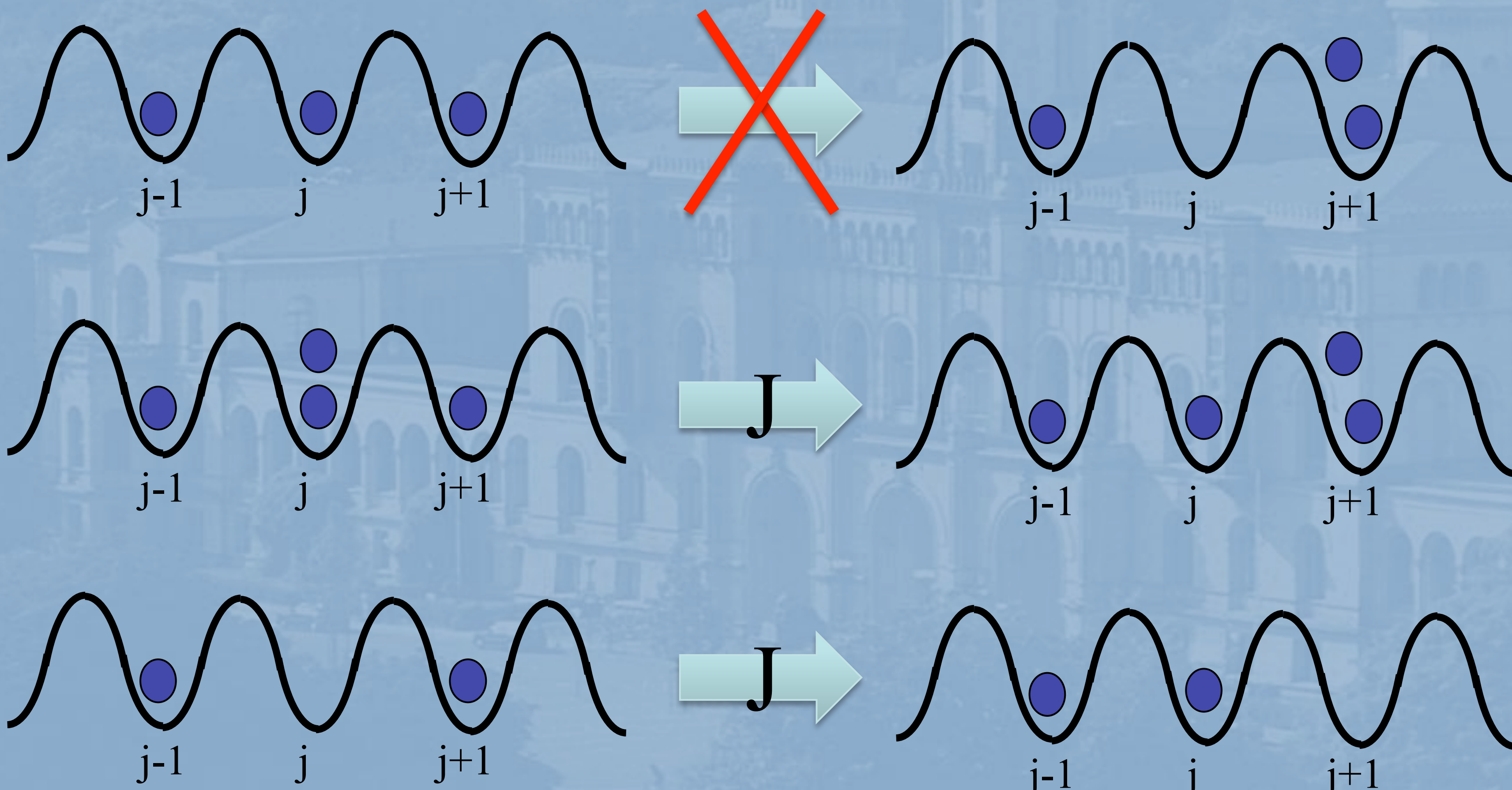
PSF exists also in 1D systems

$G_2(i,j) \equiv \langle (b_i^\dagger)^2 b_j^2\rangle$ presents a slowlier power-law decay than $G_1(i,j) \equiv \langle b_i^\dagger b_j\rangle$

Selective suppression of hopping processes

$$H_{TUN} = -J \sum_{<i,j>} \hat{b}_i^{+} J_0\left(\Omega\left(\hat{n}_i - \hat{n}_j\right)\right)\hat{b}_j$$

Let $J_0(\Omega)=0$



Defect-free Mott insulators

For J=0 the state $\bigotimes_j |n\rangle_j$ is the ground state for $n-1 < \mu/U_0 < n$

In the usual BHM, for J>0 this is not any more an eigenstate, quantum fluctuations lead to a finite population of defects (extra holes or particles)

Here the defect-free Mott state remains the ground-state in the whole Mott lobe !!

$$\left(\Delta n\right) \equiv \left\langle \hat{n}^2 \right\rangle - \left\langle \hat{n} \right\rangle^2 = 0$$

Holon- and doublon-superfluids

Extra particles/holes cannot be destroyed, forming a 2-component Bose-gas with hard-core constraint

$$
\begin{aligned}
H_h &= -Jn \sum_{<i,j>} h_i^\dagger h_j + (\mu - U_0(n-1)) \sum_i h_i^\dagger h_i, \\
H_p &= -J(n+1) \sum_{<i,j>} p_i^\dagger p_j + (U_0 n - \mu) \sum_i p_i^\dagger p_i,
\end{aligned}
$$

$$
p_i^\dagger p_i + h_i^\dagger h_i = 0 \text{ or } 1
$$

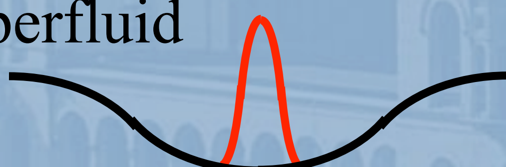
Holon- and doublon-superfluids

$$E_p(\mathbf{q}) = U_0 n - \mu + (n+1)\epsilon^0_{\mathbf{q}}$$

$$E_h(\mathbf{q}) = \mu - U_0(n-1) + n\epsilon^0_{\mathbf{q}}$$

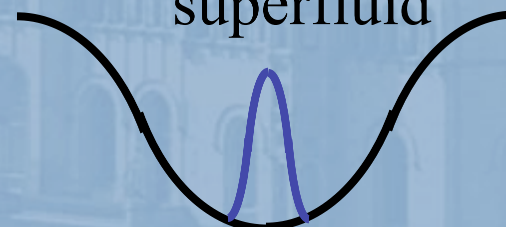
$$\boxed{\mu_c \equiv U_0(n-1/2) - Jz}$$

Pure holon superfluid



$$E_h(\mathbf{q}) = \mu - U_0(n-1) + n\epsilon^0_{\mathbf{q}}$$

Pure doublon superfluid

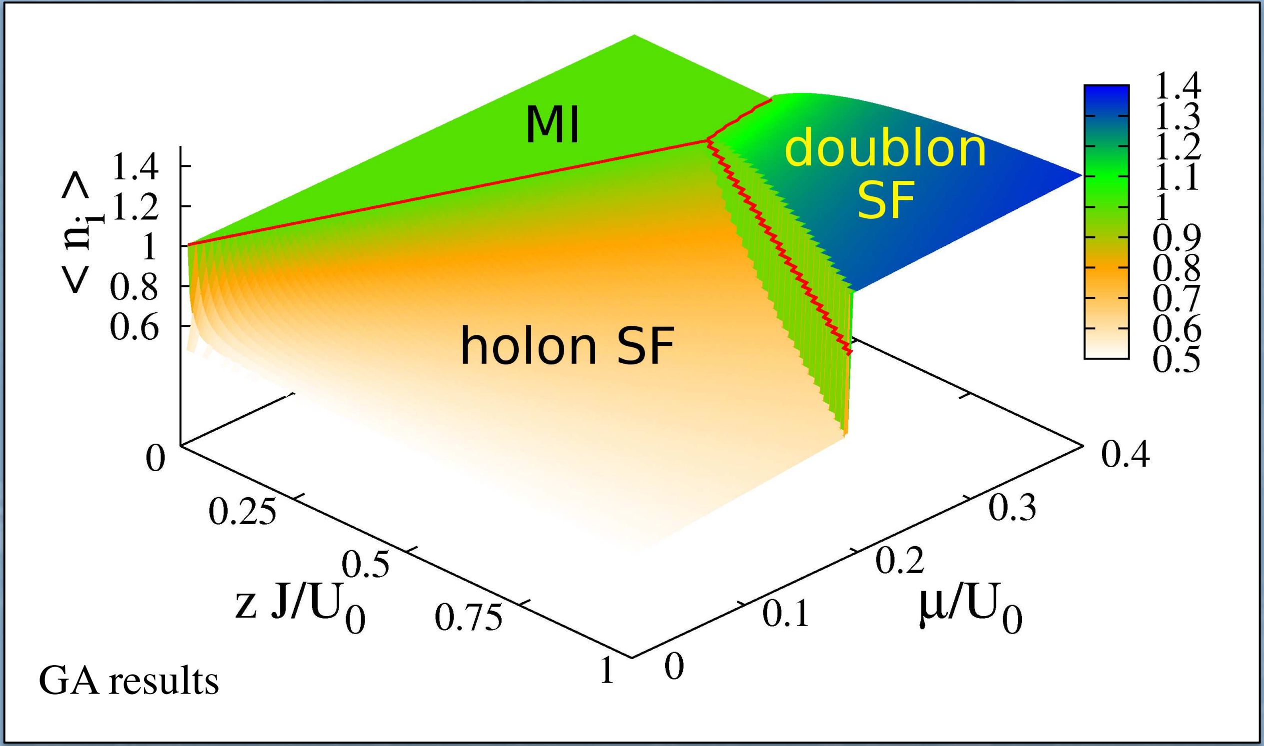


$$E_p(\mathbf{q}) = U_0 n - \mu + (n+1)\epsilon^0_{\mathbf{q}}$$

$$\epsilon^0_{\mathbf{q}} = -2J\sum_{j=x,y,z}\cos(q_j d)$$

Transition from holon-SF to doublon-SF

At $\mu=\mu_c$ one expects a large density jump (diverging compressibility) marking the boundary between the holon and doublon superfluids



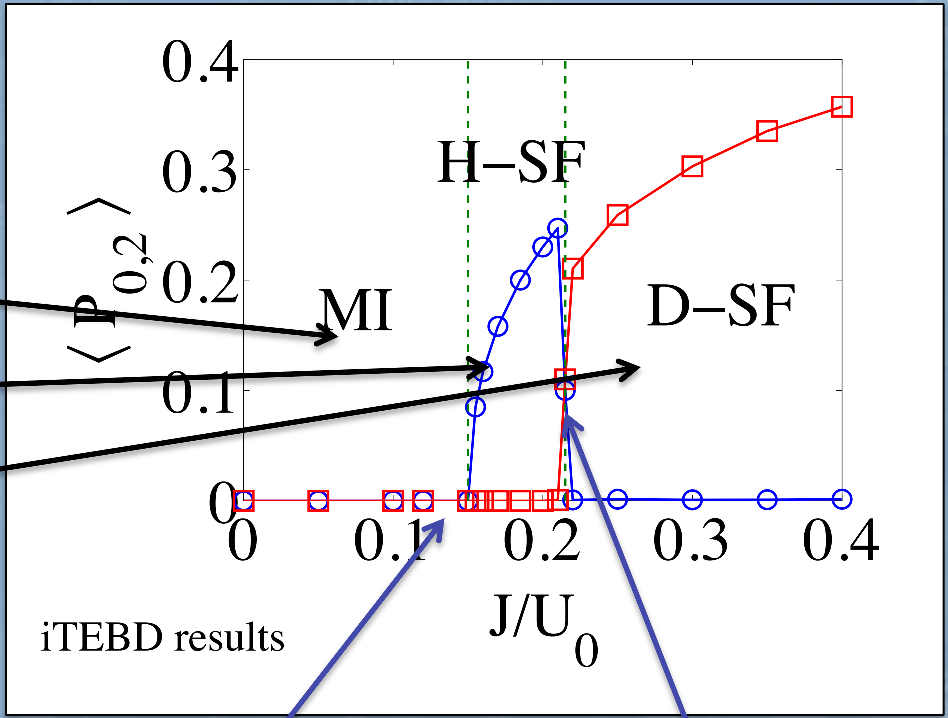
Lattice gases with periodically modulated interactions

Similar results in 1D

Defect-free MI

Holon-SF

Doublon-SF



C-IC transition

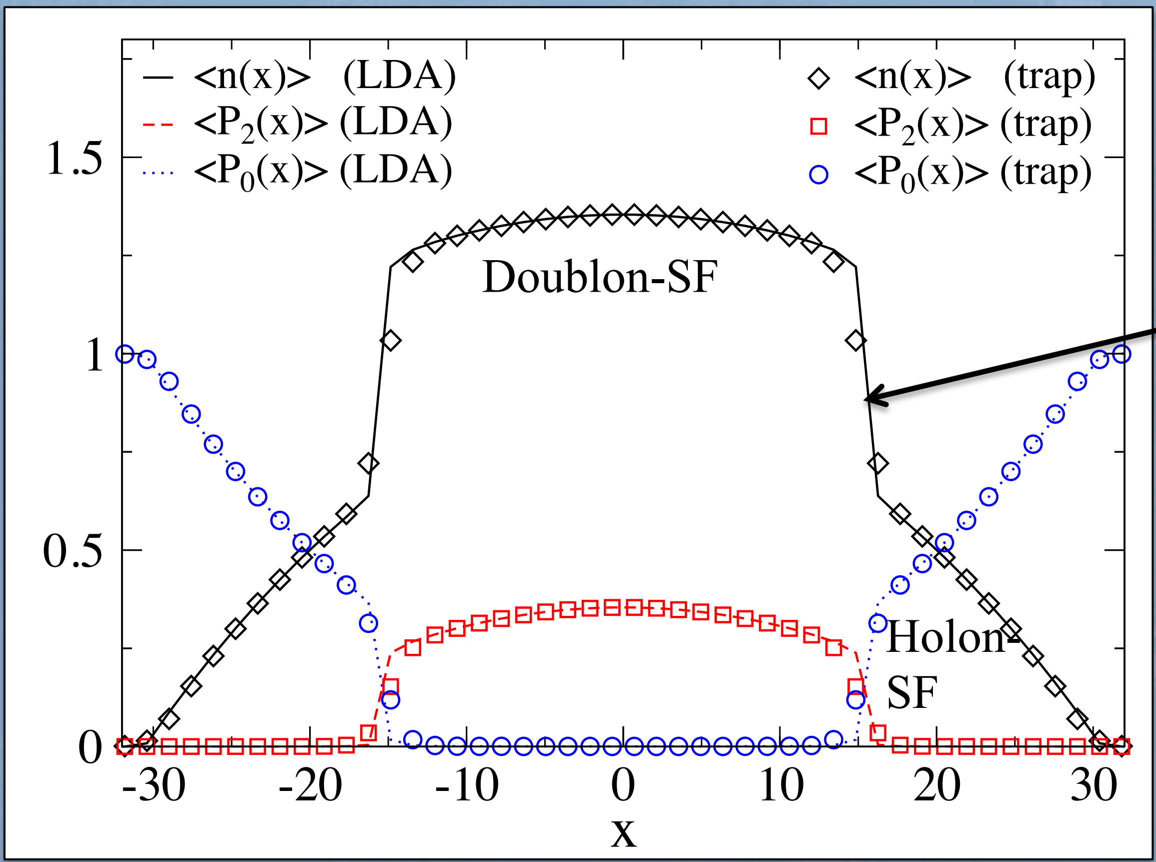
Diverging compressibility

Translating between Floquet and laboratory basis

$$|\{n_j\}, m\rangle = e^{im\omega t} e^{-i\frac{V(t)}{2}\sum_j \hat{n}_j(\hat{n}_j - 1)} |\{n_j\}\rangle$$

- Density measurements are equal in both basis
- Other measurements (in particular TOF) must be „translated“ since the mapping is non-linear, but for holon and doublon-SF phases it is equal in both basis

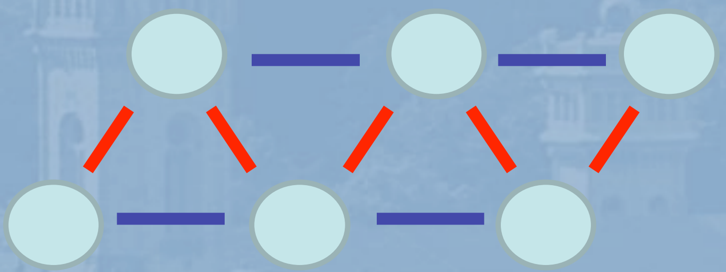
Density profiles in harmonically trapped lattice gases



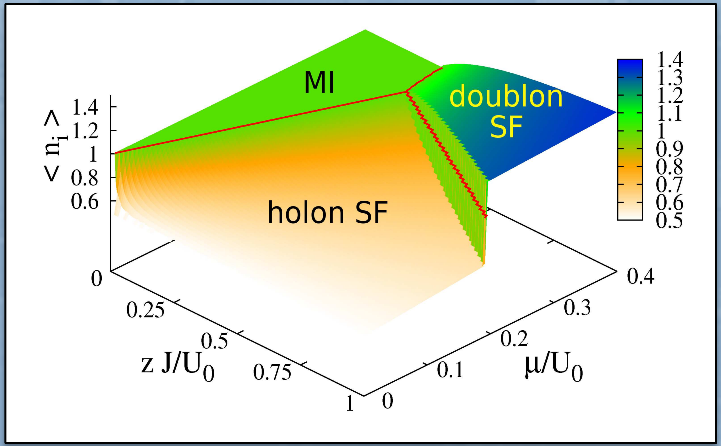
The large compressibility at the holon-SF to doublon-SF transition translates into an abrupt density jump

Summary

Ultra-cold bosons in zig-zag optical lattices



- Mott-insulators for vanishing interactions
- Haldane-insulator without polar interactions



Atoms with periodically modulated interactions

- Pair-superfluidity
- Defect-free Mott states
- Holon- and doublon-SF
- Abrupt density jumps

People

T. Vekua

S. Greschner

A. Rapp

X. Deng

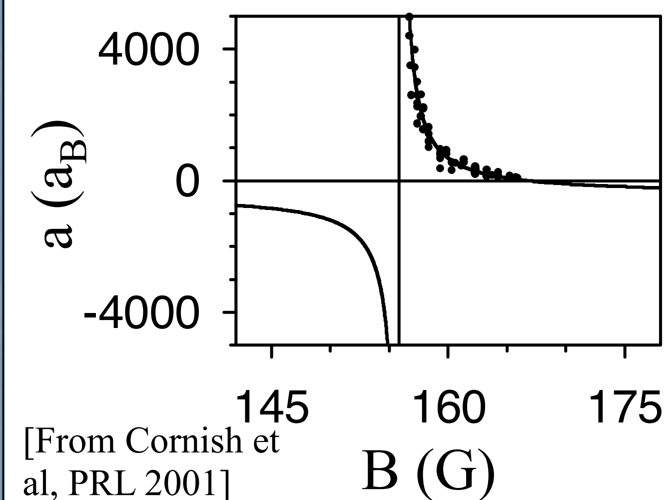
Experimental parameters

E.g. 85Rb

$$a_{bg} \approx -400 a_B$$

$$B_r = 155.2G$$

$$\Delta B = -11.6G$$



[From Cornish et al, PRL 2001]

$$B(t)/G \cong 167.6 + 5.6\cos(\omega t) \quad \Rightarrow \quad a(t)/a_B \cong 20 + 200\cos(\omega t)$$

$$s \equiv \frac{V_0}{E_R} \approx 17 \quad \Rightarrow \quad \begin{array}{l} J << U_0 \\ \Omega = 2.4 \\ \omega \approx 900Hz >> \dfrac{U_0}{\hbar} = 217Hz \end{array}$$