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Traveling Dark Solitons in the Superfluid Fermi Gases across the BEC-BCS crossover

Joachim Brand

Massey University
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Quantum gases at Massey

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Soliton dynamics

Low dimensional BEC Quantum fluctuations and strong correlations

Superfluid Fermi gases: BEC-BCS crossover

Macroscopic quantum superpositions in strongly-interacting systems

Josephson Junction analogs, Vortex tunneling

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Polar Fermionic Molecules

PHYSICAL REVIEW A **82**, 063624 (2010)

Anisotropic superfluidity in the two-species polar Fermi gas

Renyuan Liao and Joachim Brand
Institute for Advanced Study and Centre for Theoretical Chemistry and Physics, Massey University, Auckland 0632, New Zealand
(Received 3 August 2010; revised manuscript received 20 October 2010; published 20 December 2010)

We study the superfluid pairing in a two-species gas of heteronuclear fermionic molecules with equal density. The interplay of the isotropic s-wave interaction and anisotropic long-range dipolar interaction reveals rich physics. We find that the single-particle momentum distribution has a characteristic ellipsoidal shape that can be reasonably represented by a deformation parameter α defined similarly to the normal phase. Interesting momentum-dependent features of the order parameter are identified. We calculate the critical temperatures of both the singlet and triplet superfluids, suggesting a possible pairing symmetry transition by tuning the s-wave or dipolar interaction strength.

Anisotropic momentum distribution

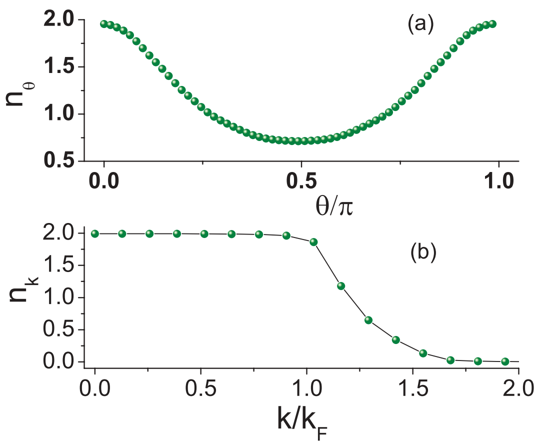


FIG. 2. (Color online) (a) Momentum-integrated angular number distribution. (b) Angle-averaged momentum number distribution $n_k(k) = \int n(\mathbf{k}) d\Omega/4\pi$ at $1/k_F a = -1$ and $C_{dd} = 1$ for the singlet superfluid.

Competition between singlet and triplet superfluid

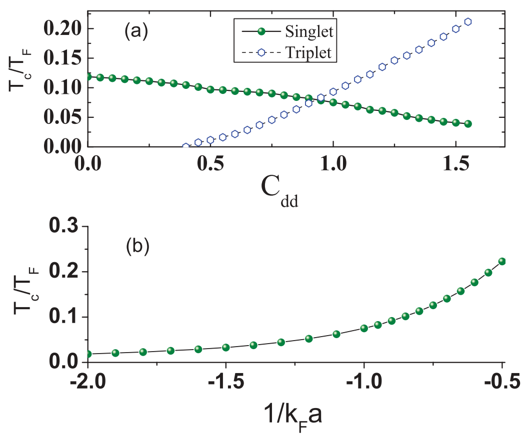


FIG. 5. (Color online) (a) Critical temperature as a function of dipole-dipole interaction strength C_{dd} for singlet superfluid at $1/k_F a = -1$ and for triplet superfluid. (b) Critical temperature as a function of $1/k_F a$ at $C_{dd} = 1$ for singlet superfluid.

This talk: Dark solitons in BEC-BCS crossover

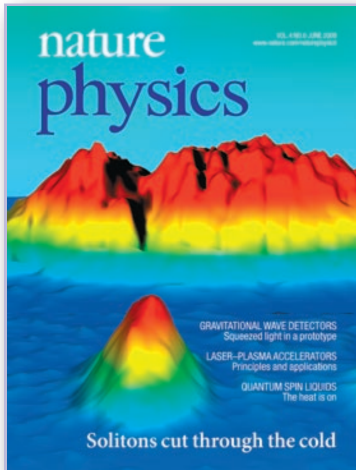
- Introduction
  Dispersion relations are relevant for understanding dynamics

- Analytic disperion relations at unitarity
  Dark soliton dispersion relations from universal scaling and general assumptions

- Crossover mean field theory
  numerical soliton profiles and dispersion relation on homogeneous background

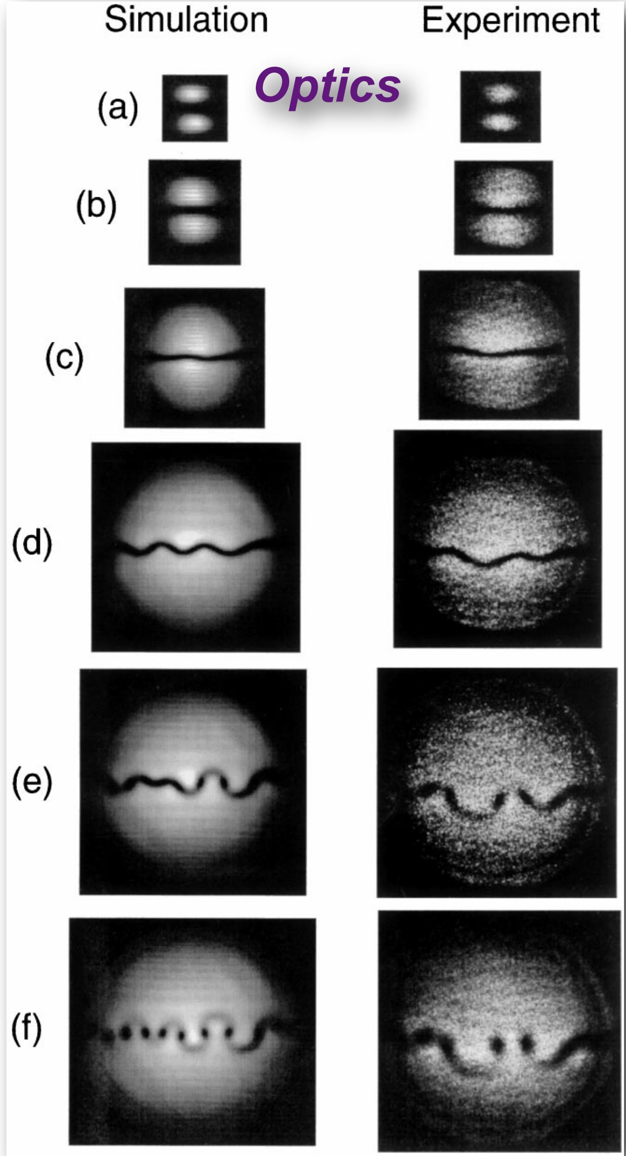
Solitons

credit: Alex Kasman



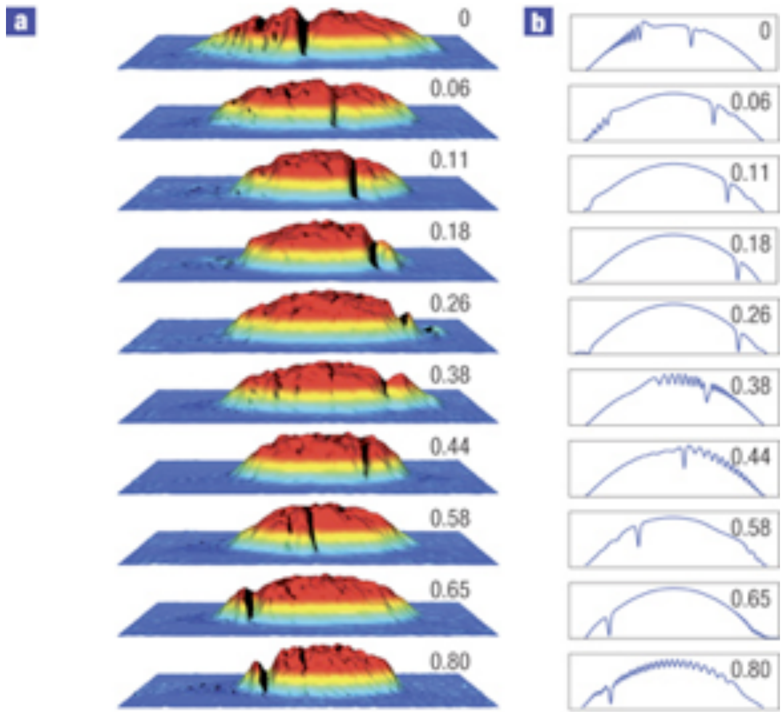
Sengstock group (2008)

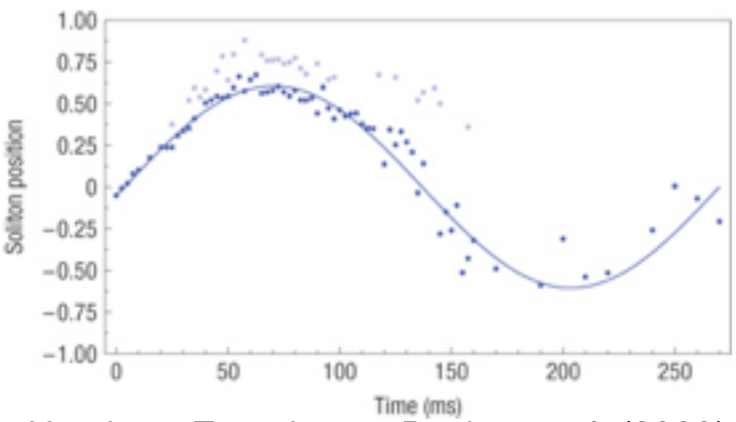
BEC



Tikhonenko et al. (1996)

Dark solitons in a trapped BEC





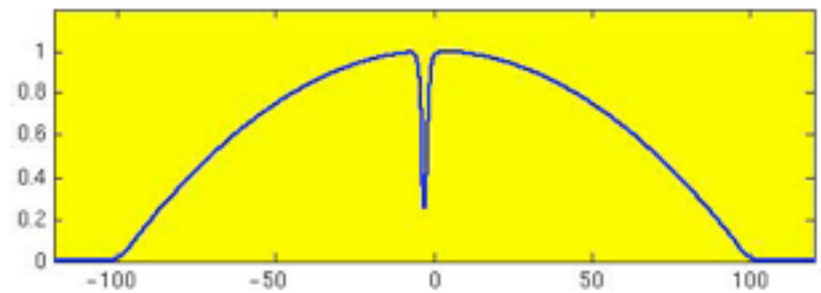
Hamburg Experiment: Becker et al. (2008)

Solitons in trapped BEC oscillate more slowly than COM

$$\omega = \frac{\omega_{\mathrm{trap}}}{\sqrt{2}}$$

Theory:
•Busch, Anglin PRL (2000)
•Konotop, Pitaevskii, PRL (2004)

Experiment:
•Becker et al. Nat. Phys. (2008)
•Weller et al. PRL (2008)



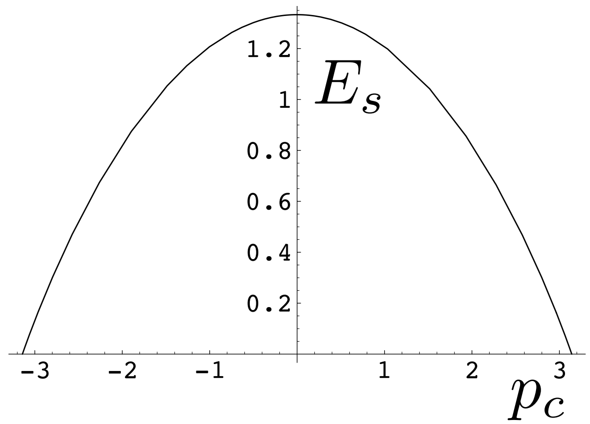
Movie credits: Nick Parker, Univ. Leeds

Landau quasiparticle dynamics

Konotop, Pitaevskii, PRL (2004)

- in *homogeneous* BEC: one parameter family of dark soliton solutions

$$E_s(v_s, \mu)$$



- in *trapped* BECs:
  – soliton moves on a slowly varying background, **locally** conserving energy

$$\frac{dE_s(v_s, \mu(z))}{dt} = 0 \longrightarrow \text{equation of motion}$$

  – BEC solitons **also *locally conserve particle number***

$$N_s \equiv \int (n_s - n_0) d^3 r = -\frac{\partial E_s}{\partial \mu} \qquad N_s = f(E_s(v_s, \mu))$$

Speed limits in the Fermi gas

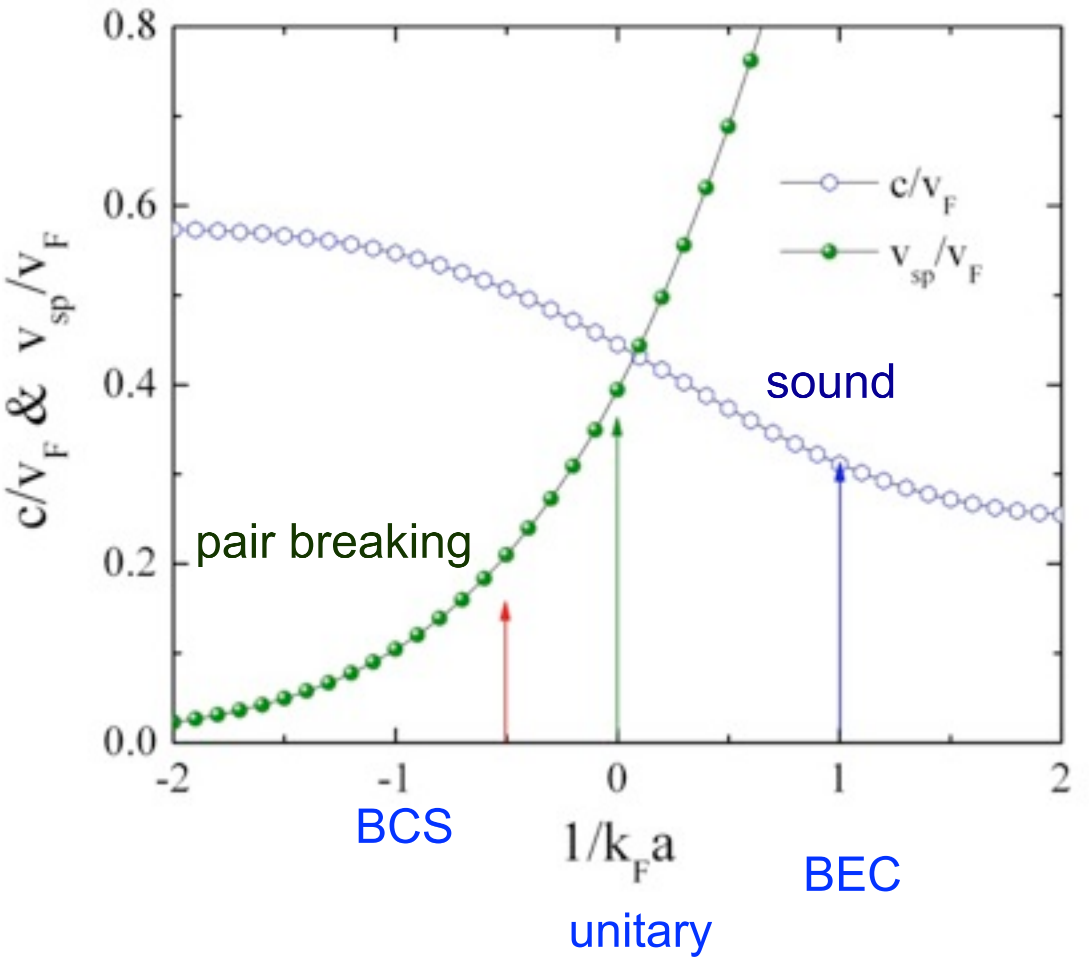
The soliton velocity is limited by

- **Sound speed** *c* (from compressibility)

$$mc^2 = n\frac{\partial \mu}{\partial n}$$

- Velocity of BCS **pair breaking** v_{sp}

$$mv_{sp}^2 = \sqrt{\mu^2 + \Delta_0^2} - \mu$$



Do solitons exist in superfluid Fermi gases and what are their properties?

- In the BEC limit (BEC of composite bosons) we would expect so. Properties predicted by GP theory.
- If we really want to know, have to find them in ***experiment***!
- Approaching from ***theory***, this is a hard problem since we have strongly correlated system:
  - Find (*magic)* solution to numerically simulate the dynamics (or excited states) of strongly correlated many-body problem?
  - Try crossover mean field theory? computationally demanding but doable.
  - Exploit universal scaling of the unitary gas?

Soliton properties at unitarity

For this part, forget mean-field theory!

Scaling arguments: soliton energy for unitary Fermi gas

Grand canonical energy

$$\langle \hat{H} - \mu \hat{N} \rangle = [\varepsilon_h k_F L + \mathcal{E} + \mathcal{O}((k_F L)^{-1})] k_F^2 A E_F$$

Equation of state (homogeneous gas):

$$\mu = (1+\beta) E_F = (1+\beta) \frac{\hbar^2 k_F^2}{2m}$$

Soliton energy:

$$E_s(\mu, v_s) = \mathcal{E} k_F^2 A E_F = \mu^2 B \mathcal{E}(\tilde{v}^2)$$

where $\tilde{v} = \dfrac{v_s m}{\hbar k_F}$ is the dimensionless soliton velocity

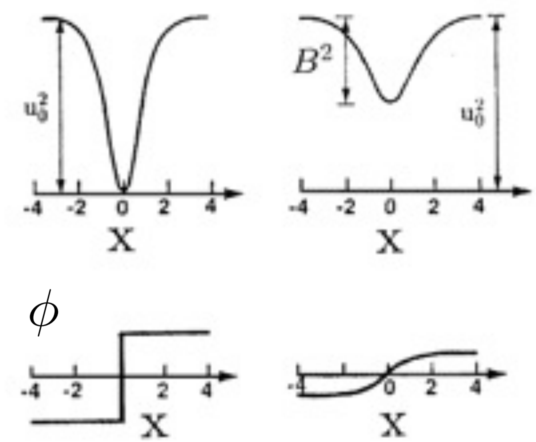
particle number: $N_s = v_s^2 \dfrac{m}{2} (1+\beta) B \mathcal{E}'(\tilde{v}^2) - 2\mu B \mathcal{E}(\tilde{v}^2)$

Three assumptions

(A) *Under adiabatic change of the environment, the soliton can conserve both energy and particle number, simultaneously.*

(B) *Energy and particle number vanish as the soliton velocity approaches the speed of sound c.*

(C) *The superfluid order parameter has a well defined phase step across the soliton that vanishes under the conditions of (B).*



Three assumptions

(A) *Under adiabatic change of the environment, the soliton can conserve both energy and particle number, simultaneously.*
Consequence:

$$\frac{\partial N_s}{\partial \mu}\frac{\partial E_s}{\partial v_s} = \frac{\partial N_s}{\partial v_s}\frac{\partial E_s}{\partial \mu}$$

can be solved for dimensionless soliton energy:

$$\mathcal{E}(\tilde{v}^2) = \mathring{e}(\mathring{v}^2 - \tilde{v}^2)^2$$

so far two undetermined parameters $\mathring{e}, \mathring{v}$

Three assumptions

(B) *Energy and particle number vanish as the soliton velocity approaches the speed of sound c.*

Consequences:

$$\mathring{v} = \frac{cm}{\hbar k_F} = \sqrt{\frac{1+\beta}{3}}$$

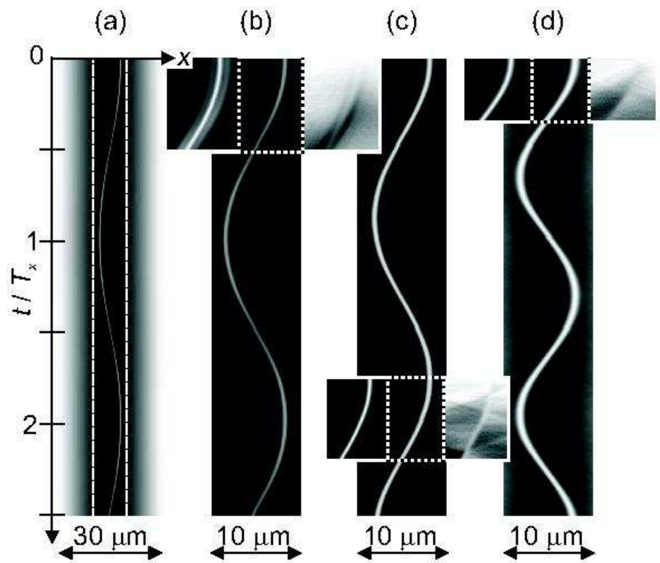
for dynamics of solitons on Thomas-Fermi density profile with $\mu(z) = \mu_0 - \frac{1}{2} m\omega_{\text{trap}}^2 z^2$

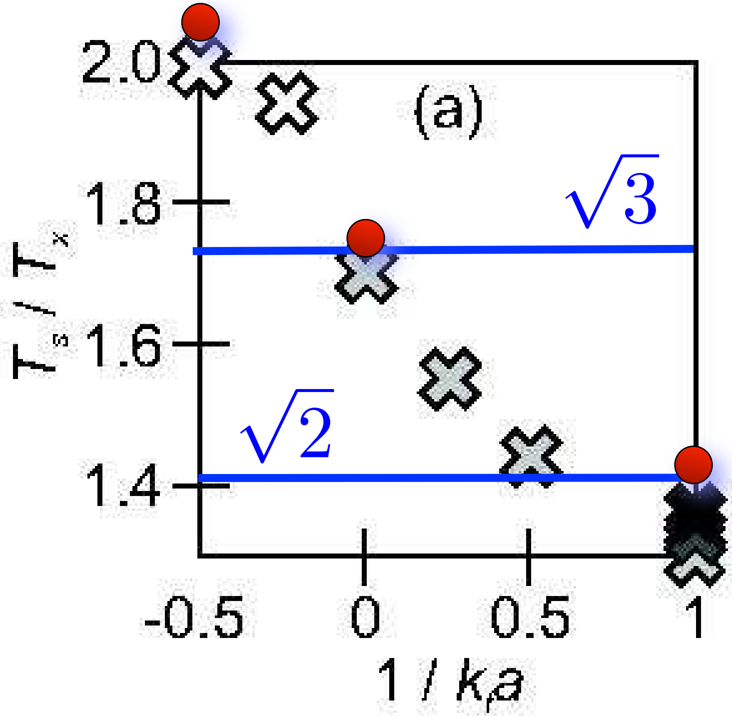
$$\omega^2 z_s^2 + \frac{d^2 z}{dt^2} = 0$$

soliton oscillation frequency: $\boxed{\frac{\omega}{\omega_{\text{trap}}} = \frac{1}{\sqrt{3}}}$

Oscillation period from time-dependent simulations

Time-dependent simulations were performed by the Trento group





- Analytics
- Our numerics:
  - 2.083 at eta = -0.5 (BCS)
  - 1.748 at eta = 0 (unitarity)
  - 1.456 at eta = +1 (BEC)
- Trento data (time dependent)
  Scott et al. PRL (2011)

Scott, Dalfovo, Pitaevskii, Stringari PRL (2011)

Three assumptions

(C) *The superfluid order parameter has a well defined phase step across the soliton that vanishes under the conditions of (B).*

Consequence: determines final parameter

recall: $p_s - p_c = \hbar n_1(\pi - \delta\phi)/2$ Pitaevskii (2010)

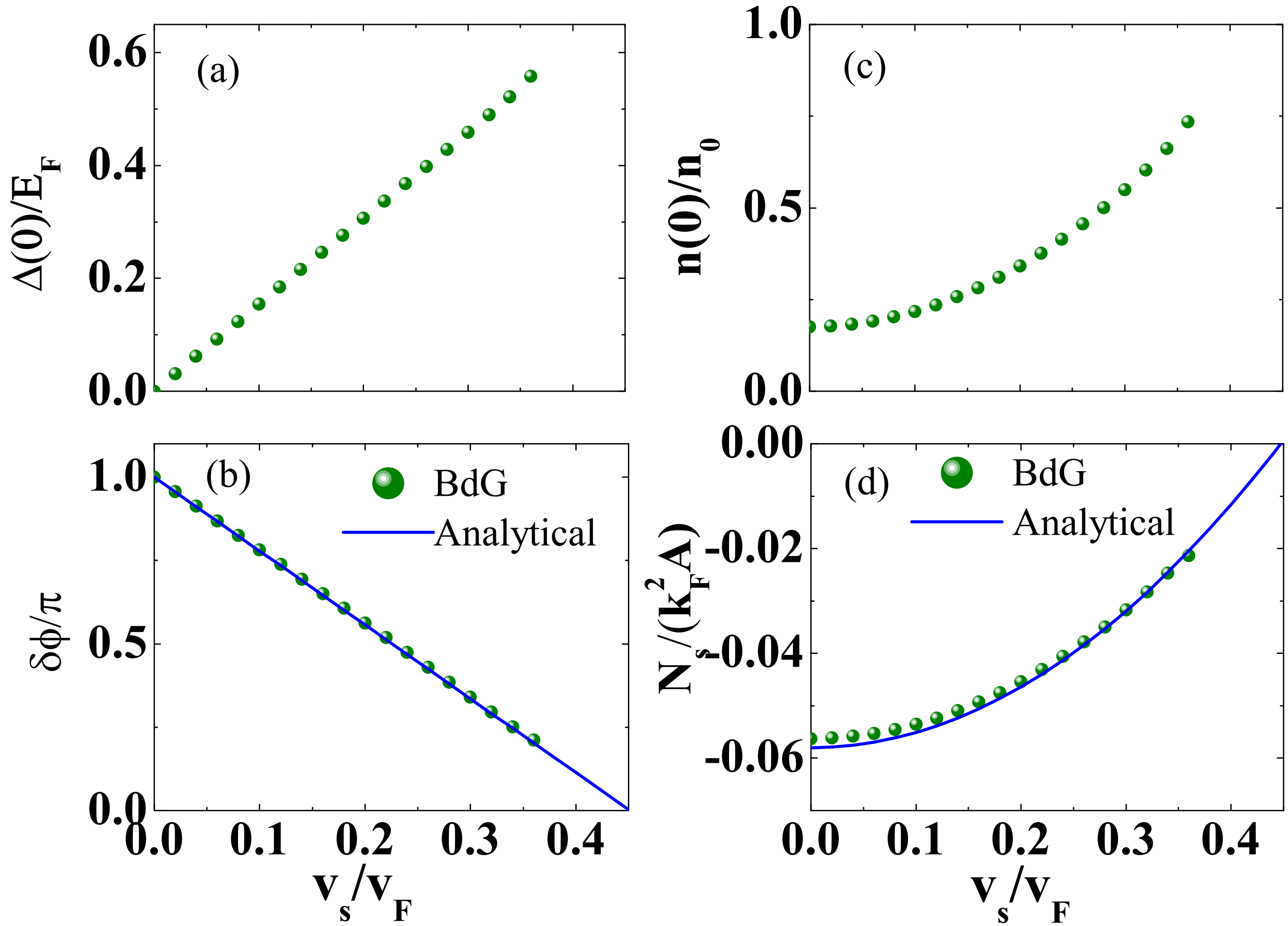
physical momentum: $p_s = mN_s v_s$

canonical momentum: $\dfrac{\partial E_s}{\partial p_c} = v_s$

yields

$$\mathring{e} = \frac{\mathring{v}^{-3}}{8\pi}, \quad \delta\phi = \pi(1 - v_s/c)$$

Solitons at unitarity



Assumptions (A), (B), (C) seem to be fulfilled!

Liao, Brand PRA **83**, 041604(R) (2011)

Crossover mean-field theory

find soliton solutions numerically

Mean-field theory

- BCS crossover theory (Leggett 1980)
  - Extend the use of BCS / BdG theory to the crossover problem (with renormalised coupling constant)
  - Qualitatively correct equation of state
  - Yields GP equation in BEC limit with $a_{GP} = 2a_{BdG}$ (correct: $a_{GP} = 0.6a_{BdG}$) (Pieri, Strinati 2003)

Bogoliubov-de Gennes equation

- self-consistently solve

$$i\hbar\partial_t \begin{pmatrix} u_\nu(\mathbf{r},t) \\ v_\nu(\mathbf{r},t) \end{pmatrix} = \begin{pmatrix} \hat{h} & \Delta(\mathbf{r},t) \\ \Delta^*(\mathbf{r},t) & -\hat{h} \end{pmatrix} \begin{pmatrix} u_\nu(\mathbf{r},t) \\ v_\nu(\mathbf{r},t) \end{pmatrix}$$

$$\hat{h} = \frac{\hbar^2}{2m}\nabla^2 - \mu$$

$$\Delta(z,t) = \Delta(z - v_s t) = \Delta(\xi)$$

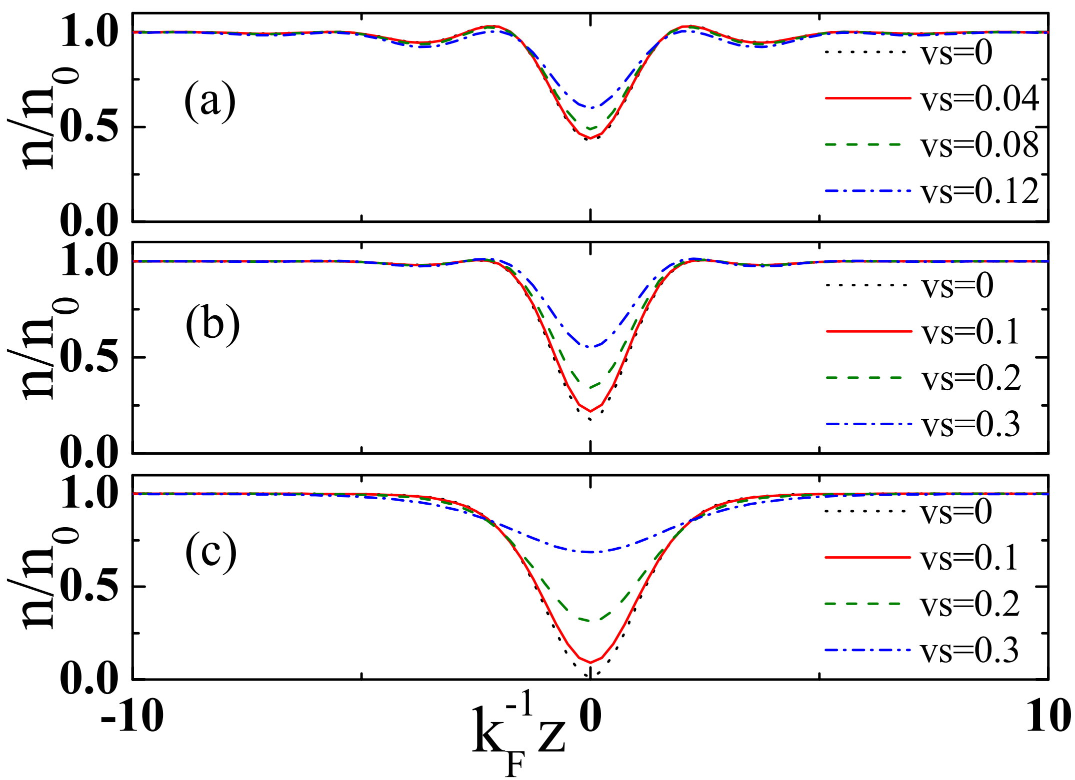
$$\Delta(\xi) = -g\sum_{\mathbf{p},n} u_{\mathbf{p},n}(\xi)v^*_{\mathbf{p},n}(\xi)$$

$$1/g = m/(4\pi\hbar^2 a) - 1/\Omega\sum_\nu 1/2\epsilon_\nu$$

- implemented Broyden's (generalized secant) method

We find dark soliton solutions, so they do exist!

Density profiles



BCS: $1/k_F a = -0.5$

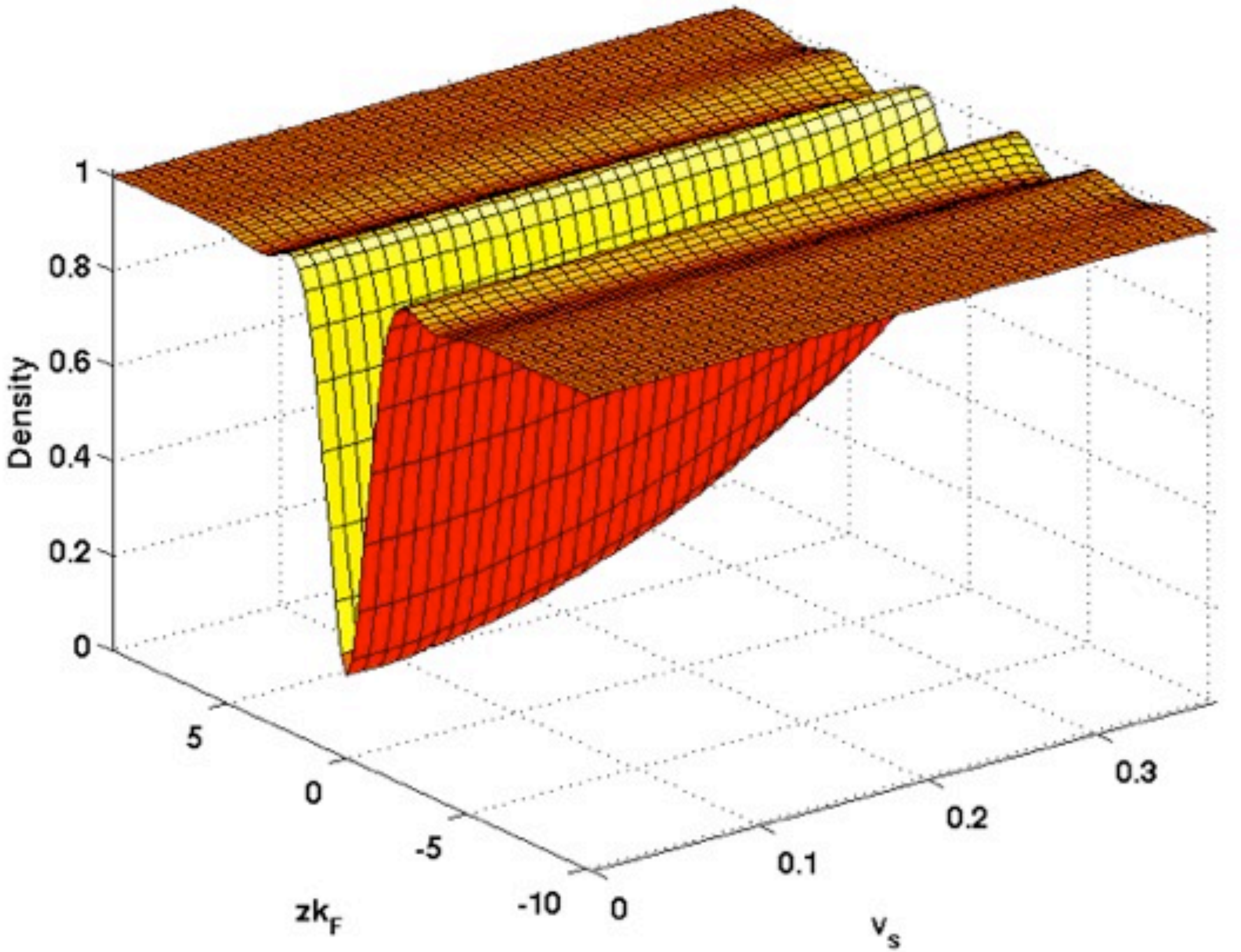
unitarity: $1/k_F a = 0$

BEC: $1/k_F a = 1$

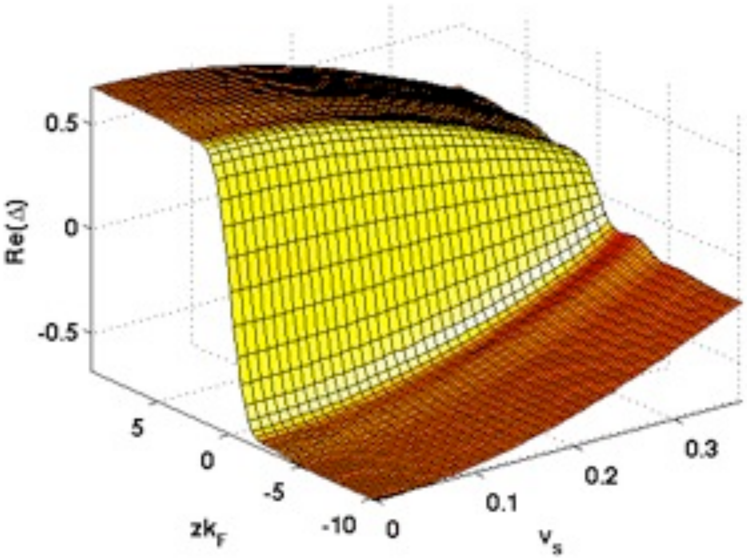
Liao, Brand PRA **83**, 041604(R) (2011)

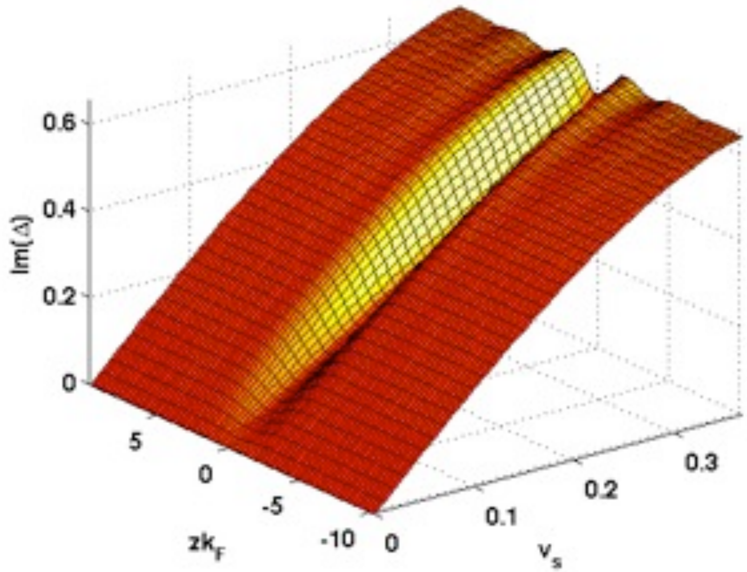
c.f. Spuntarelli, Carr, Pieri, Strinati NJP **13**, 035010 (2011)

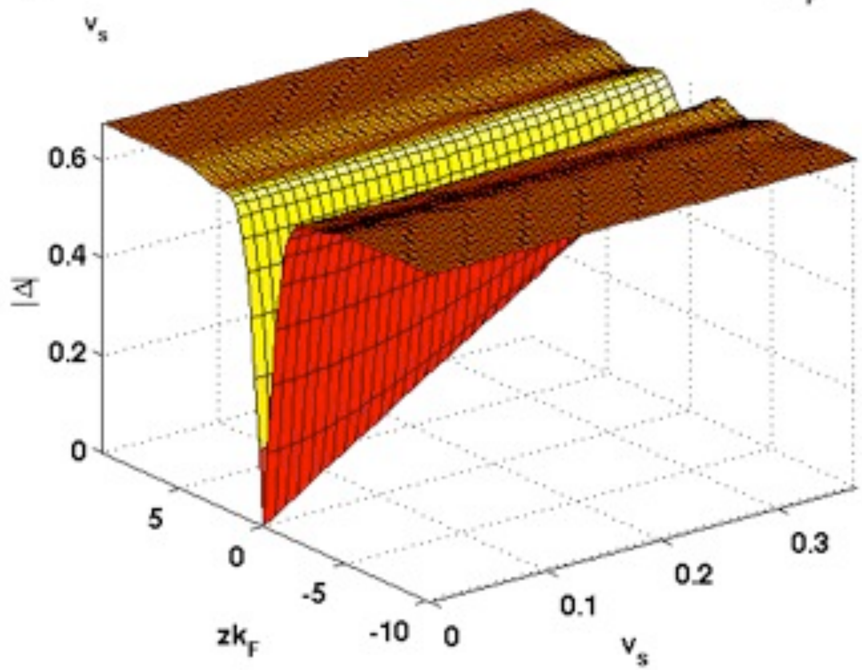
Density at unitarity



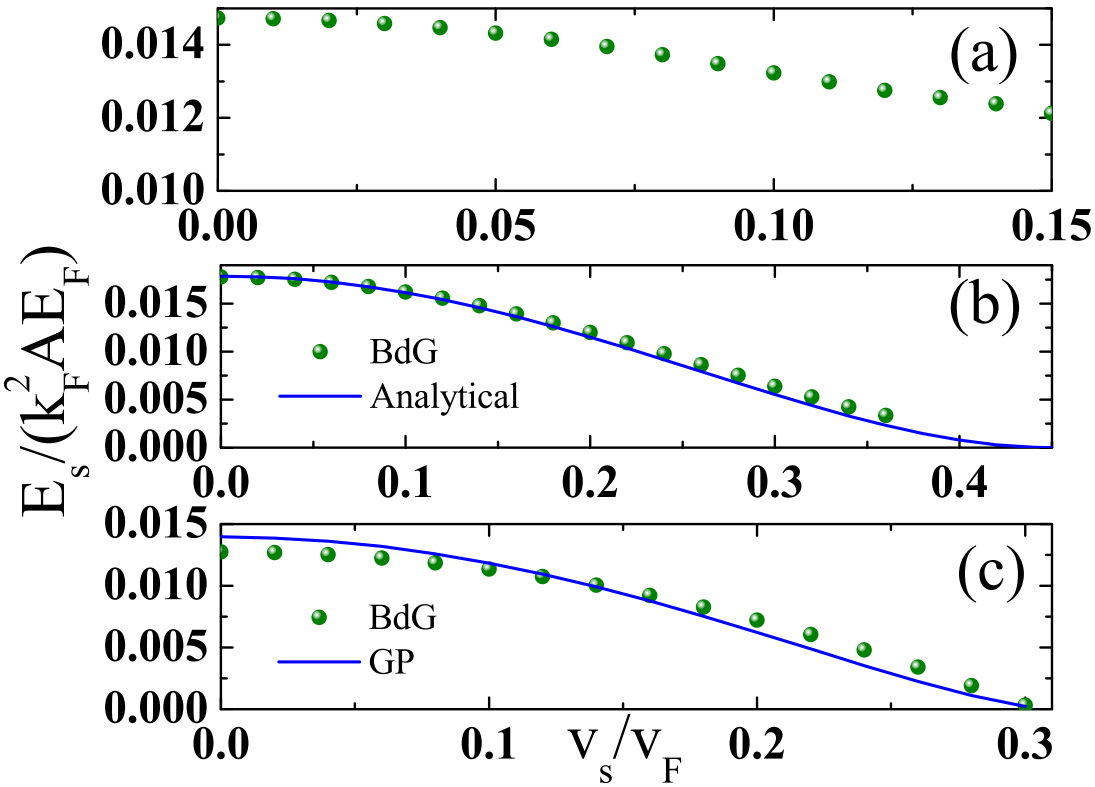
Order parameter at unitarity







Dispersion relations



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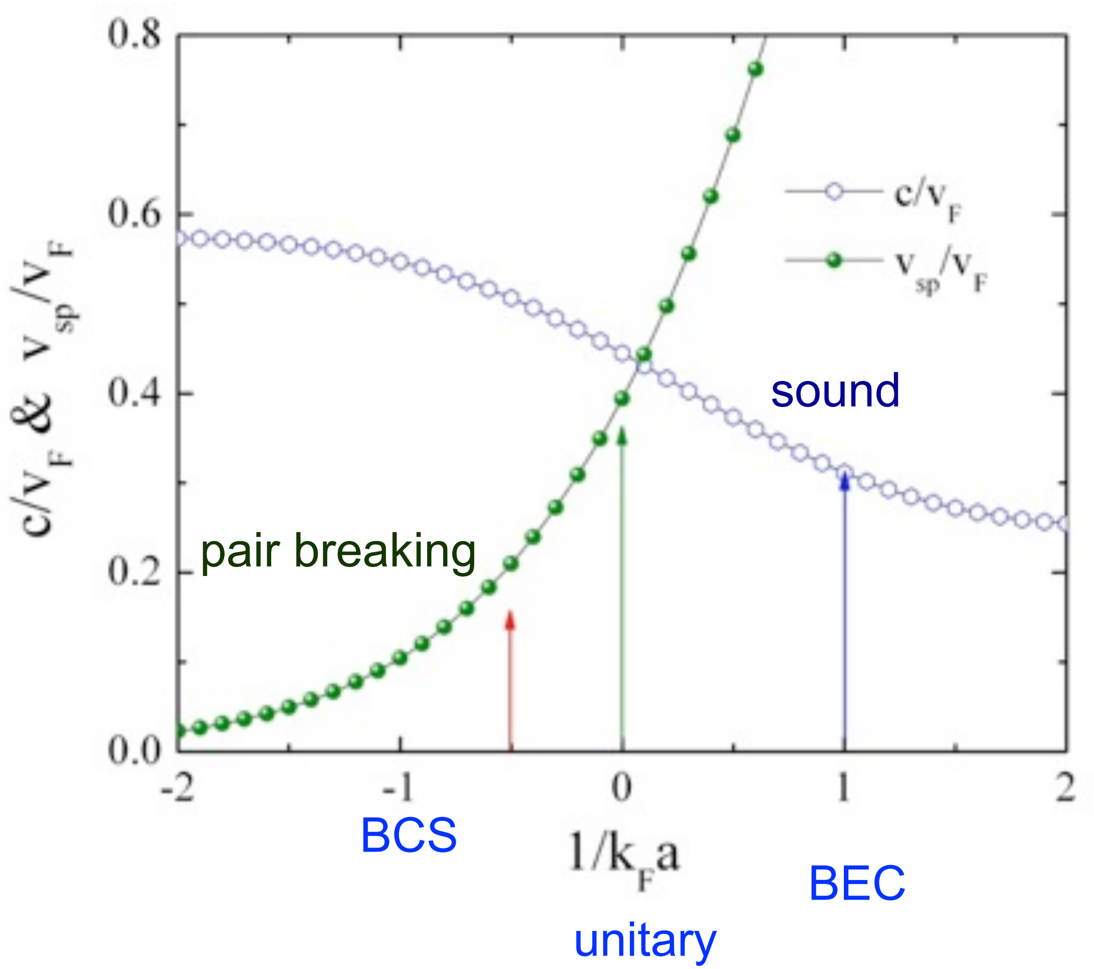
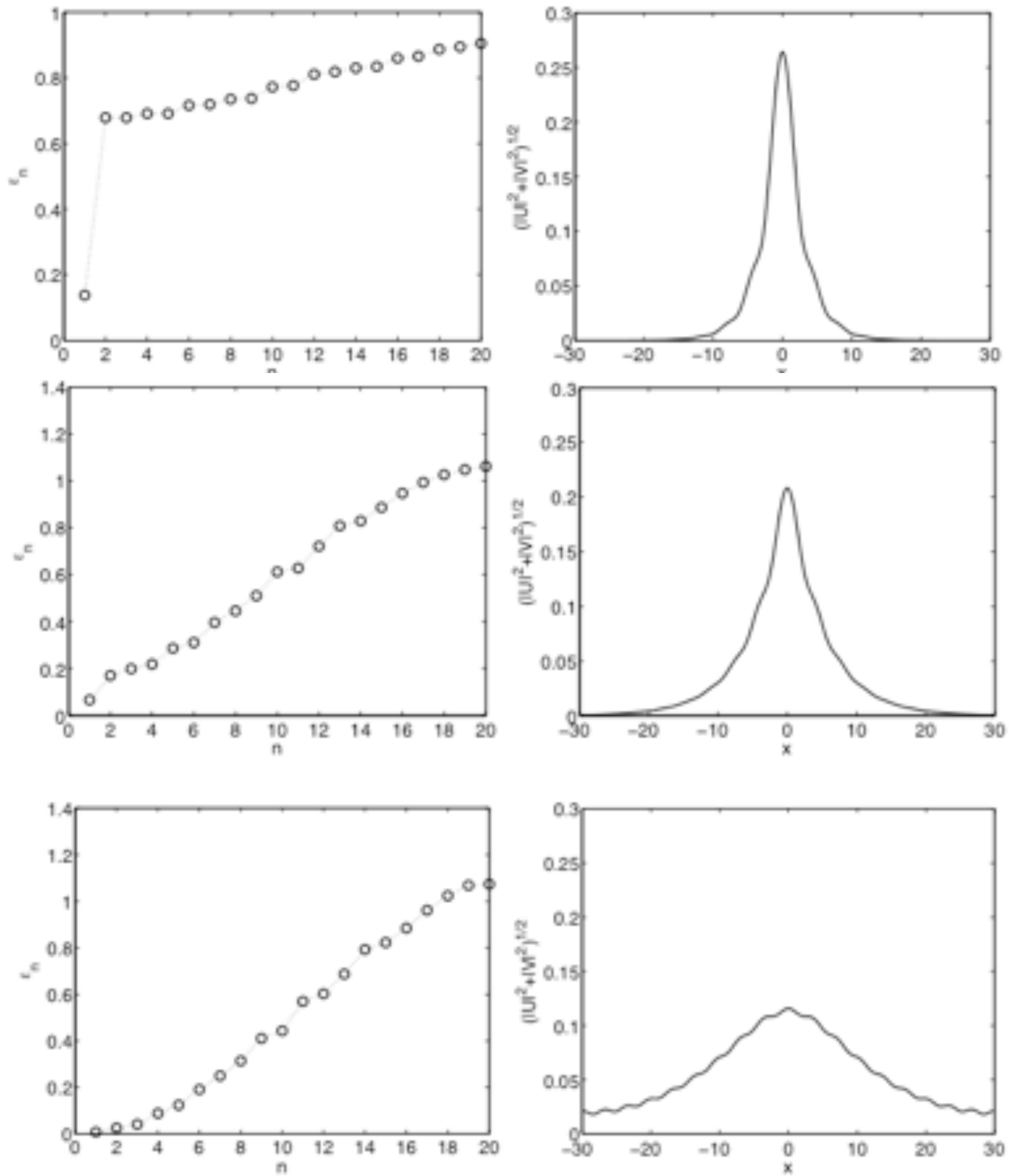


Andreev bound states



$v = 0$

$v = 0.3\ v_F$

$v = 0.38\ v_F$

How to make a soliton in the lab?

1. Phase imprinting
   NIST, Hannover, JILA, Hamburg

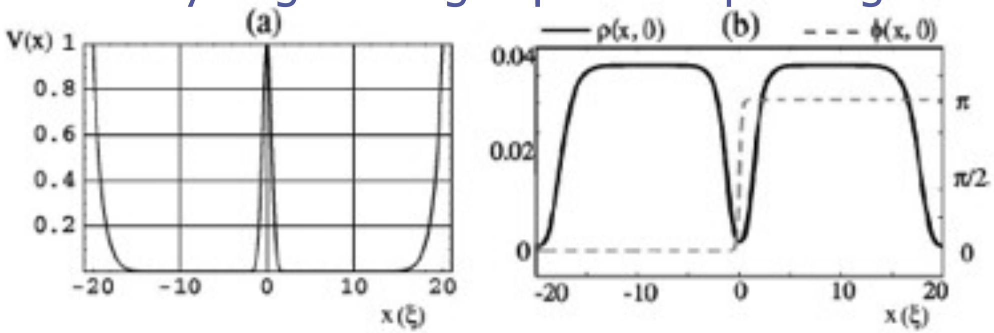
2. Collision of moving BECs
   Heidelberg

3. Cavity collapse
   Harvard

4. Combined density engineering and phase imprinting

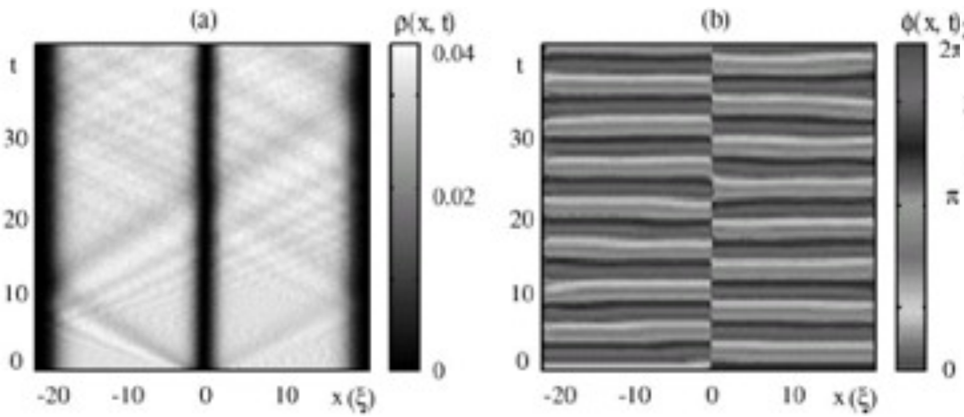
Generation and interaction of solitons in a quasi-1D BEC

Density engineering & phase imprinting



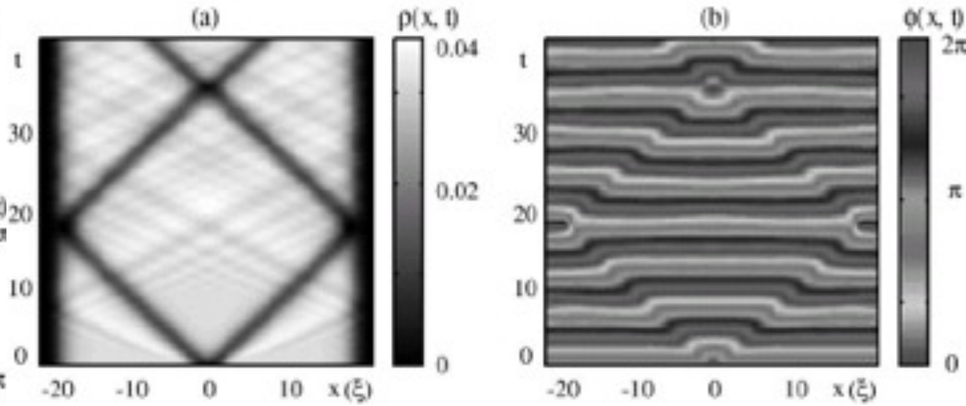
$$\phi(x) = \Delta\phi \tanh(2x/\Delta x_\phi)$$

Phase step of π generates single soliton



Density manipulation on the size scale of the healing length allows the specific engineering of one or several solitons.

Density engineering alone generates 2 or more solitons.



L.D. Carr, J. Brand, S. Burger, A. Sanpera, PRA **63**, 051601(R) (2001)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