What's behind Acoustic Peaks in the Cosmic Microwave Background Anisotropies

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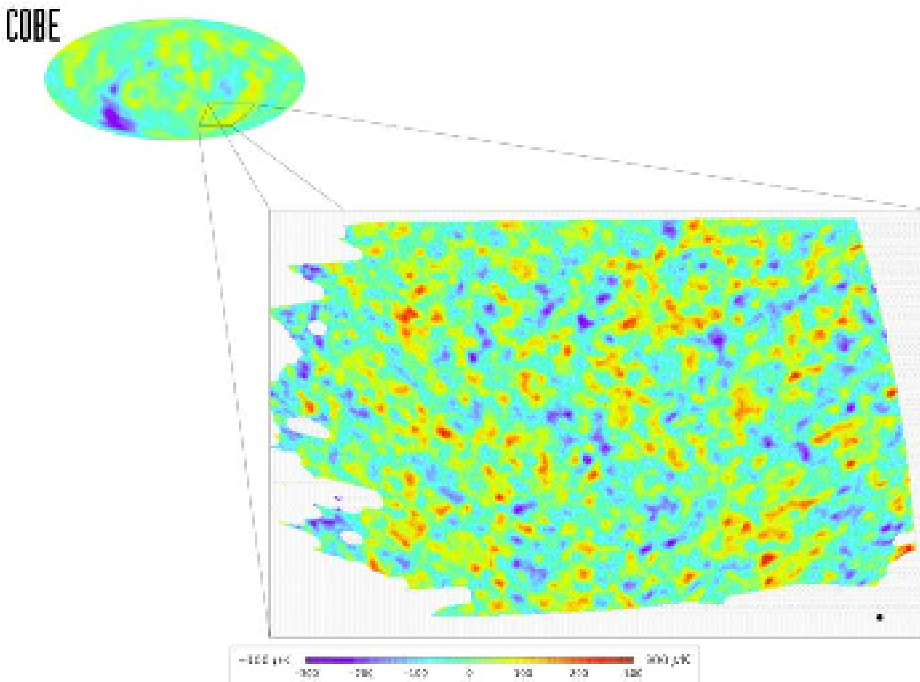
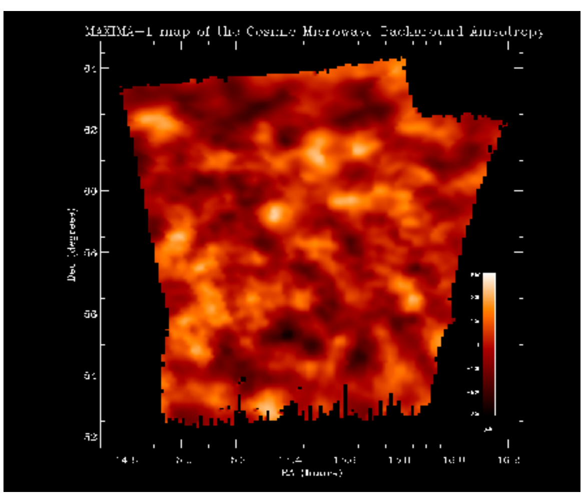
- A look to the data:
  maps, CMB snapshot, power spectra, sky coverage, error bars, cosmic variance, ...

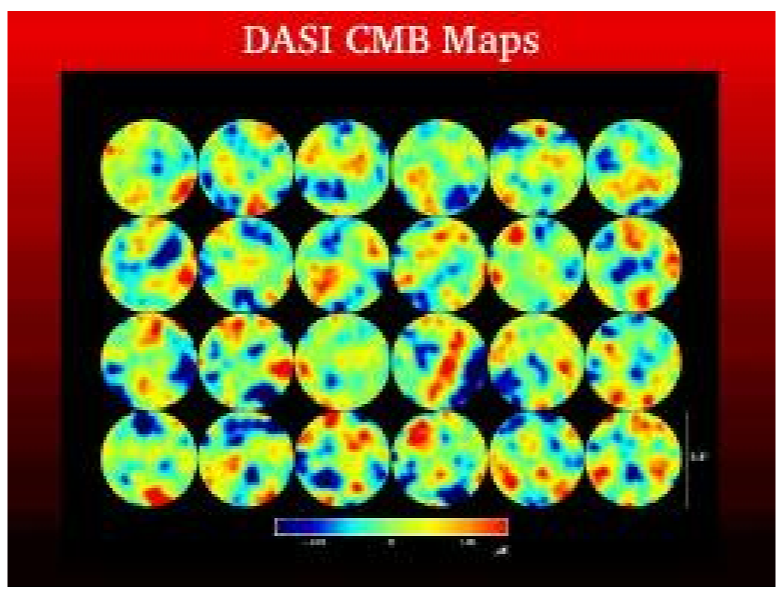
- Putting together the pieces of the puzzle:
  the cosmological system, perturbations, initial conditions, inflationary phaenomenology, ...

- Looking at the holes:
  cosmological parameters, dark things, matter, energy, unobserved CMB signals, ...

The sub-degree CMB anisotropies







CMB Anisotropies

The Cosmic Snapshot At $1+z \simeq 1100$, $T \simeq 3000K$

$$\frac{\delta T}{T}(here, now, \hat{n}) = \int_{\infty}^{0} \frac{\delta T}{T}(r(z), z, \hat{n}) P(z) dz$$

$P(z)$ is the last scattering probability, which is fixed by the cosmological recombination hystory (Jones & Wise, A& A 149, 144, 1984). It is a narrow peak with mean and dispersion

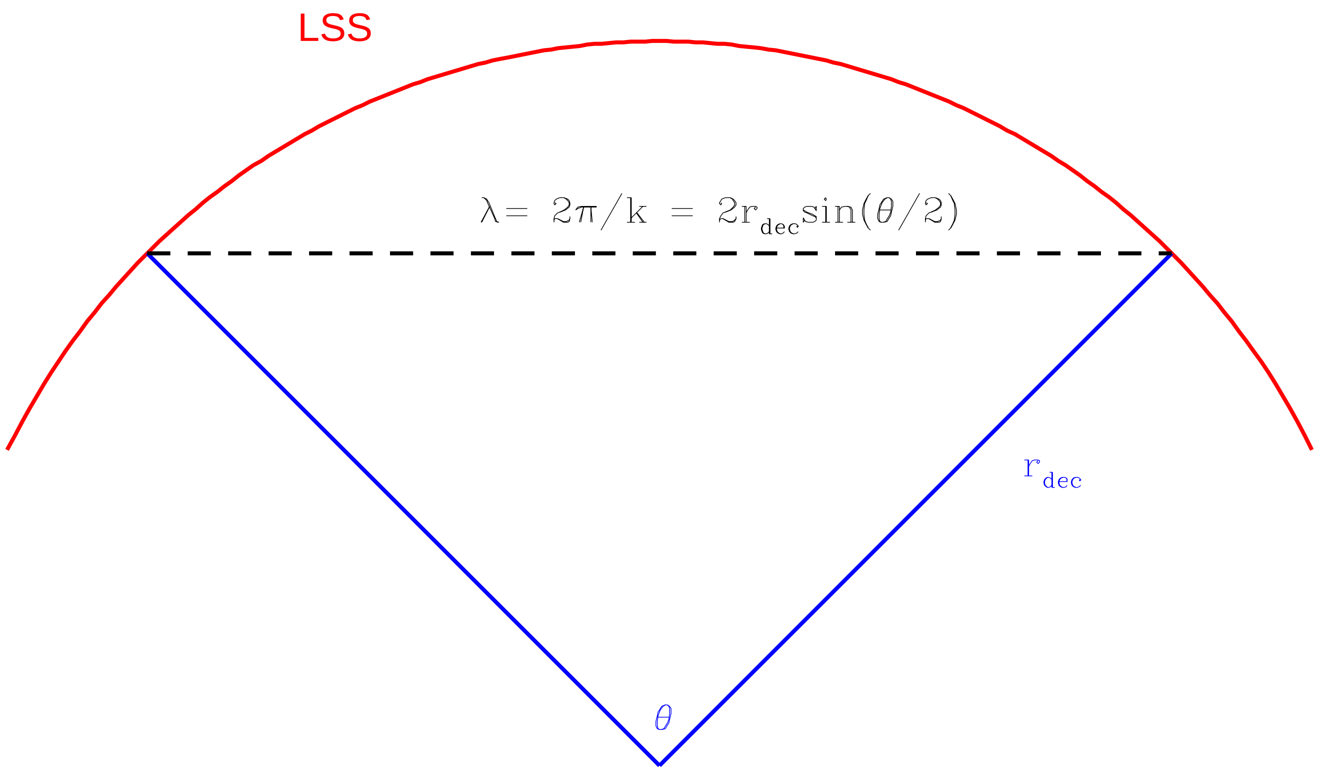
$$1 + z_{dec} \simeq 1100 \ , \ \Delta z \simeq 100$$

corresponding to comoving distances

$$r_{dec} \simeq 6000 h^{-1} \ \text{Mpc} \ , \ \Delta r_{dec} \simeq 10 h^{-1} \ \text{Mpc}$$

CMB Anisotropies

Probing Angles, Probing Scales



Horizon at decoupling: $100h^{-1}$ Mpc, $\theta \simeq 1^\circ$

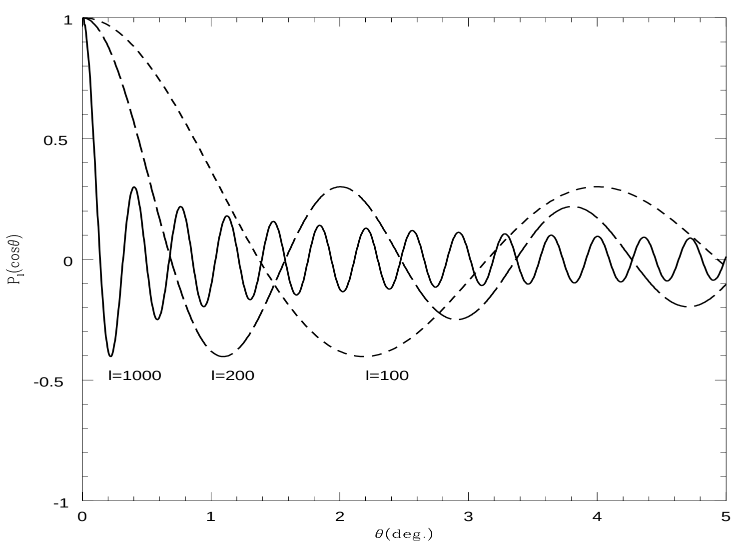
CMB angular Power Spectra

$$\frac{\delta T}{T} \equiv \Theta(\hat{n}) = \sum_{lm} a_{lm} Y_l^m(\hat{n}) \quad , \; C_l = \frac{1}{2l+1} \sum_m |a_{lm}|^2$$

$$< a_{lm} a^*_{l'm'} >= C_l \delta_{ll'} \delta_{mm'}$$

$$\Downarrow$$

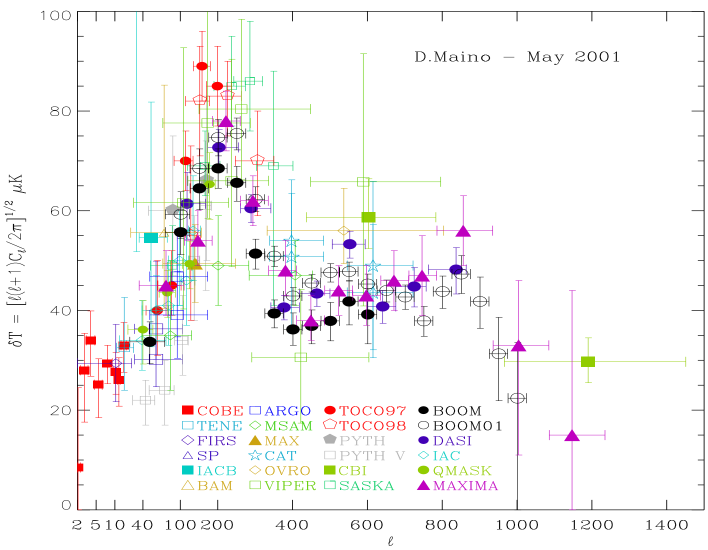
$$< \Theta(\hat{n}) \Theta(\hat{n'}) >_{\hat{n}\cdot\hat{n'}=\cos\theta} = \sum_l \frac{2l+1}{4\pi} C_l P_l(\cos\theta)$$

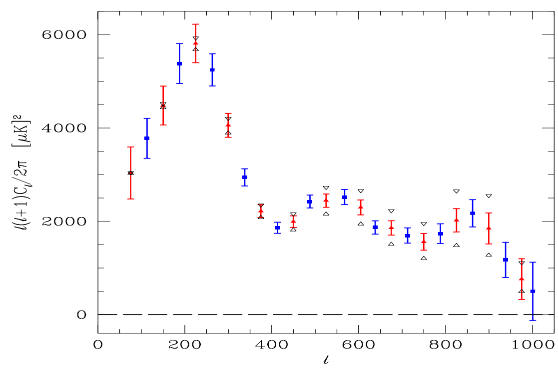


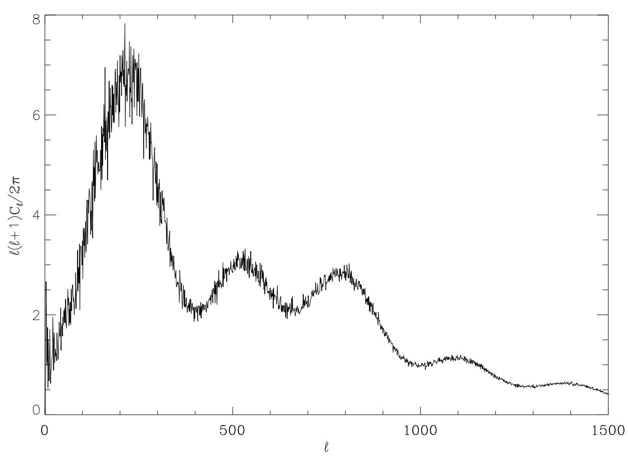
Power on scale

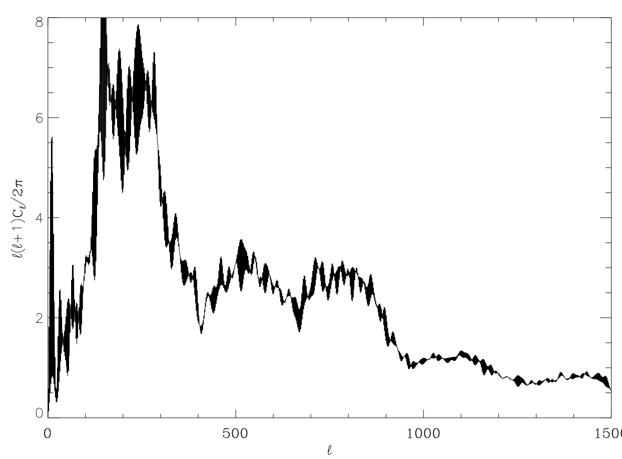
$$\theta \simeq \frac{200}{l} \text{ deg.}$$

Error bars and Cosmic Variance









CMB Polarisation Anisotropies

$$\vec{E} = (E_x \cos(\omega t + \phi_x), E_y \cos(\omega t + \phi_y))$$

$$Q =< E_x^2 > - < E_y^2 > \ , \ U =< 2E_xE_y \cos(\phi_x - \phi_y) >$$

$$I =< E_x^2 > + < E_y^2 > \ , \ V =< 2E_xE_y \sin(\phi_x - \phi_y) >$$

focus on Q and U, construct the polarisation tensor

$$\mathcal{P}(\hat{n}) = \begin{pmatrix} Q(\hat{n}) & -U(\hat{n}) \\ -U(\hat{n}) & -Q(\hat{n}) \end{pmatrix}$$

and expand in tensor spherical harmonics

$$\mathcal{P}(\hat{n}) = \sum_{l=2}^{\infty} \sum_{m=-l}^{l} \left[a_{lm}^E \mathcal{Y}_{lm}^E(\hat{n}) + a_{lm}^B \mathcal{Y}_{lm}^B(\hat{n}) \right]$$

CMB Polarisation Anisotropies

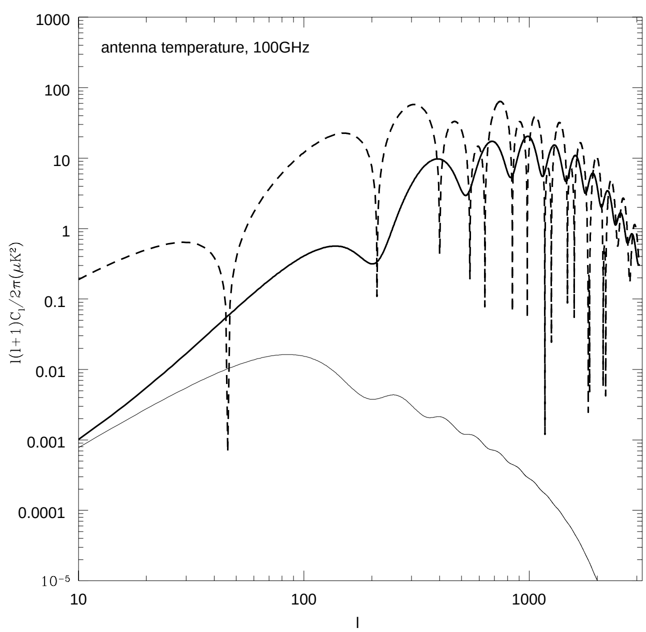
Coding Scalars, Vectors and Tensors in Cosmological Perturbations

$$\mathcal{P}(\hat{n}) = \sum_{l=2}^{\infty} \sum_{m=-l}^{l} \left[a_{lm}^{E} \mathcal{Y}_{lm}^{E}(\hat{n}) + a_{lm}^{B} \mathcal{Y}_{lm}^{B}(\hat{n}) \right]$$

$\mathcal{Y}_{lm}^{E}$ and $\mathcal{Y}_{lm}^{B}$ can be suitably chosen so that polarisation codifies cosmological perturbation according to the following scheme:

scalars + vectors + tensors $\rightarrow E$ modes

vectors + tensors $\rightarrow B$ modes



Cosmological Perturbation Theory

Useful links:

Early:

Lifshitz E.M., J.Phys. (USSR) 10, 116, 1946
Hawking S.W., Astrophys.J.145, 544 1966
Peebles P. J. E. & Yu J. T., Ap.J. 162, 815, 1970
Bardeen J.M., Phys.Rev.D22, 1882, 1980

General:

Kodama I. & Sasaki M.,
Progr. of Theor.Phys.Supp. 78, 1984
Hwang J.C., Astrophys.J. 375, 443 1991

CMB oriented:

Ma C.P. & Bertschinger E., Ap.J. 455, 7, 1995

Hu W., Seljak U., White M., Zaldarriaga M.,
Phys.Rev.D56, 596, astro-ph/9709066, 1997

Cosmological Perturbation Theory

Background Dynamics

$$ds^2 = a(\eta)^2 \left(-d\eta^2 + \frac{dr^2}{1 - Kr^2} + r^2 d\Omega^2 \right)$$

$$G_{\mu\nu} = 8\pi G T_{\mu\nu} \ , \ T^{\nu}_{\mu\,;\nu} = 0$$

$$\Downarrow$$

$$\frac{1}{a^2}\left(\frac{\dot{a}}{a}\right)^2 = \frac{1}{a^2}\mathcal{H}^2 = \frac{8\pi G}{3}\sum_x \rho_x + \frac{K}{a^2} \ ,$$

$$\frac{p_x}{\rho_x} = w_x \ , \ \rho_x \propto \frac{1}{a^{3(1+w_x)}}$$

Cosmological Perturbation Theory

Perturbation Dynamics

$$g_{\mu\nu}(\vec{x},\eta) = a(\eta)^2(\gamma_{\mu\nu} + h_{\mu\nu}(\vec{x},\eta)) \ , \ h_{\mu\nu} \ll 1$$

$$\delta G_{\mu\nu} = 8\pi G \delta T_{\mu\nu} \ , \ \delta T^{\nu}_{\mu\, ;\nu} = 0$$

Perturbations are classified into scalar, vector and tensor under spatial rotations.

Linearity makes convenient working in the Fourier space:

$$\delta(\vec{k},\eta) = \int d^3x \delta(\vec{x},\eta) e^{i\vec{k}\cdot\vec{x}}$$

and introduces a gauge freedom which reduces the number of independent components in $h_{\mu\nu}$.

Cosmological Perturbation Theory

Scalar Perturbations

Fourier space, Newtonian gauge

Metric:

The independent perturbed quantities are

$$\Psi = \frac{h_{00}}{2} \ , \ \Phi = \frac{h_{11}}{2} = \frac{h_{22}}{2} = \frac{h_{33}}{2} \ ,$$

Non-relativistic species (dark matter, baryons):

density and velocity perturbations:

$$\delta_x \equiv \frac{\delta \rho_x}{\rho_x} \ , \ v_x$$

Relativistic species (photons, massless neutrinos):

thermodynamical temperature fluctuations are expanded in angular eigenfunctions to describe the depenence on $\cos\theta_{\hat{k}} = \hat{n} \cdot \hat{k}$:

$$\frac{\delta T}{T}(\vec{k}, \eta, \hat{n}) \equiv \Theta(\vec{k}, \eta, \hat{n}) = \sum_l \Theta_l(\vec{k}, \eta) P_l(\hat{n} \cdot \hat{k})$$

zeroth, first and second order coefficients are density, velocity and anisotropic stress perturbations:

$$4\Theta_0 = \delta_x \ , \ \Theta_1 = v_x \ , \ 12\Theta_2 = 5\pi_x \ , ...$$

Quadrupole and higher order moments for electromagnetic scattering excite photon polarization:

$$E \equiv E_2, E_3, ...$$

Cosmological Perturbation Theory

Scalar Evolution Equations

Fourier space, Newtonian gauge

Metric:

$$\left(\frac{k}{a}\right)^2 \Phi = 4\pi G \sum_{non-rel.} \left[\rho_x \delta_x + \left(\frac{3}{k}\right) \mathcal{H} \rho_x v_x\right] +$$

$$+4\pi G \sum_{rel.} \left[\rho_x \delta_x + \left(\frac{4}{k}\right) \mathcal{H} \rho_x v_x\right]$$

$$-k^2(\Psi + \Phi) = \frac{8\pi G}{3} (\rho_\gamma \pi_\gamma + \rho_\nu \pi_\nu)$$

Cosmological Perturbation Theory

Scalar Evolution Equations

Fourier space, Newtonian gauge

Non-Relativistic species:

$$\dot{\delta}_x = -kv_x - 3\dot{\Phi} \ , \ \dot{v}_x = -\mathcal{H}v_x + k\Psi \ ,$$

Thomson scattering affects baryons evolution:

$$\dot{v}_b = -\mathcal{H}v_b + k\Psi + (4\rho_\gamma/3\rho_b)\dot{\tau}(v_\gamma - v_b) \ ,$$

where $\dot{\tau} = ax_i n_e \sigma_T$ is the differential optical depth: σ_T is the Thomson scattering cross section, $n_e(\eta)$ the electron number density and $x_i(\eta)$ the ionisation fraction.

Cosmological Perturbation Theory

Scalar Evolution Equations

Fourier space, Newtonian gauge

Relativistic species:

Photons temperature and polarization fluctuations:

$$\dot{\Theta}_0 = -\frac{k}{3}\Theta_1 - \dot{\Phi} \ , \ \dot{\Theta}_1 = k\Theta_0 - \frac{2}{5}k\Theta_2 + \dot{\tau}(v_b - \Theta_1) + k\Psi \ ,$$

$$\dot{\Theta}_2 = \frac{2}{3}k\Theta_1 - \frac{3}{7}k\Theta_3 - \dot{\tau}\left(\frac{9}{10}\Theta_2 - \frac{\sqrt{6}}{10}E_2\right)$$

$$\dot{E}_2 = -\frac{\sqrt{5}}{7}kE_3 - \dot{\tau}\left(\frac{1}{10}\Theta_2 + \frac{2}{5}E_2\right) \ ,$$

and for multipoles $l \geq 3$:

$$\dot{\Theta}_l = k\left[\frac{l}{2l-1}\Theta_{l-1} - \frac{l+1}{2l+3}\Theta_{l+1}\right] - \dot{\tau}\Theta_l$$

$$\dot{E}_l = k\left[\frac{\sqrt{l^2-4}}{2l-1}E_{l-1} - \frac{\sqrt{(l+1)^2-4}}{2l+3}E_{l+1}\right] - \dot{\tau}E_l \ .$$

Massless neutrinos can be treated as photons without polarization and Thomson scattering terms.

Cosmological Perturbation Theory

Photon-Baryon Tight Coupling

Fourier space, Newtonian gauge

$\rho_\gamma \gg \rho_b, 1/\dot{\tau} \to 0$ regime:

Equations for photons and baryons imply $v_\gamma = v_b, \dot{\delta}_b = (3/4)\dot{\delta}_\gamma$ and allow to write an equation for photons and gravity only (Hu & Sugiyama, ApJ 444, 489, 1995):

$$\ddot{\Theta}_0 + \mathcal{H}\frac{3\rho_b}{4\rho_\gamma + \rho_b}\dot{\Theta}_0 + k^2\frac{1}{3}\frac{4\rho_\gamma}{4\rho_\gamma + 3\rho_b}\Theta_0 = F \ ,$$

$$F = -\ddot{\Phi} - \mathcal{H}\frac{3\rho_b}{4\rho_\gamma + 3\rho_b}\dot{\Phi} - k^2\frac{1}{3}\Psi$$

which in the limit $\rho_b/\rho_\gamma = 0, \ddot{\Phi} = 0$ becomes

$$\ddot{\Theta}_0 + \frac{k^2}{3}\Theta_0 = -\frac{k^2}{3}\Psi$$

$$\Theta_0(\eta) = [\Theta_0(0) + \Psi]\cos\frac{k\eta}{\sqrt{3}} + \frac{\sqrt{3}}{k}\dot{\Theta}_0(0)\sin\frac{k\eta}{\sqrt{3}} - \Psi$$

Initial Cosmological Perturbations

General Scheme And Classes

(i) Choose initial $\delta_x(\vec{x})$, $v_x(\vec{x})$, ...

(ii) Compute initial Fourier modes.

(iii) Integrate equations.

(iv) Transform back to the real space and obtain the cosmological perturbations at any time.

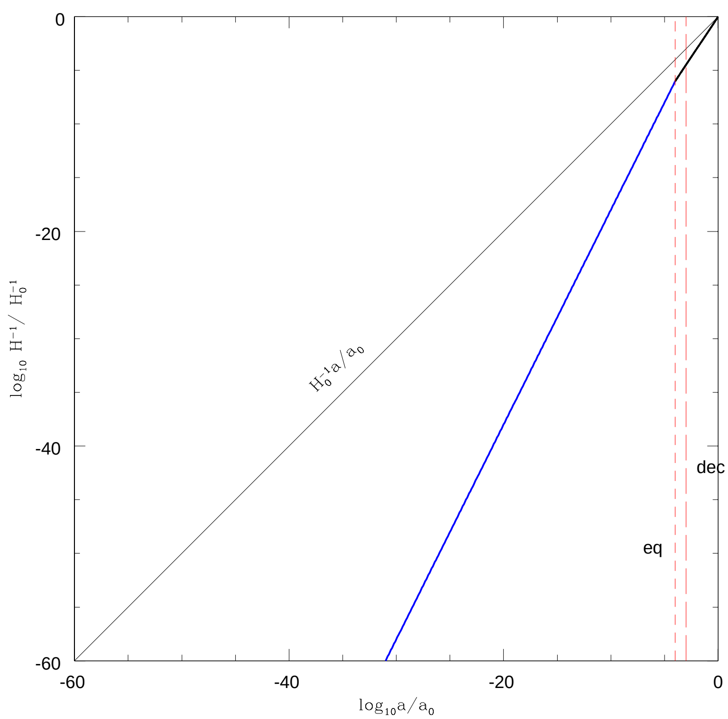
$$\delta(\vec{k}) = |\delta(\vec{k})| e^{i\phi_{\vec{k}}}$$

Gaussian Perturbations:
$|\delta(\vec{k})|$ Gaussian distributed, $\phi_{\vec{k}}$ random between 0 and 2π.

non-Gaussian Perturbations:
$|\delta(\vec{k})|$ arbitrarily distributed, $\phi_{\vec{k}}$ ordered (symmetric seeds [Baccigalupi PRD59, 123004, 1999], topological defects, ...).

Initial Cosmological Perturbations

Horizon Entry



$$\ddot{\Theta}_0 + \frac{k^2}{3}\Theta_0 = -\frac{k^2}{3}\Psi$$

Adiabatic: no entropy perturbations between different species ⇒ curvature $\propto \Psi$ perturbed: $\Psi \propto \Theta_0 \neq 0$, $\dot{\Theta}_0 = 0$

Isocurvature: non-zero entropy perturbations between different species keeping curvature unperturbed ⇒ mixture motion between different species: $\Psi \propto \Theta_0 = 0$, $\dot{\Theta}_0 \neq 0$

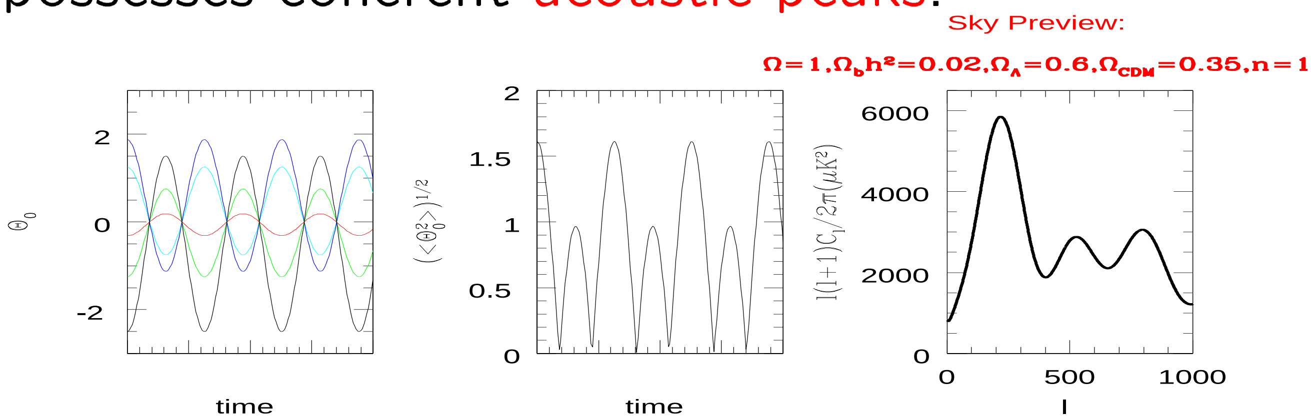
Initial Cosmological Perturbations

Adiabatic Spectra

$$\Theta_0 \propto \Psi \neq 0 \ , \ \dot{\Theta}_0 = 0$$

$$\Theta_0(\eta) = [\Theta_0(0) + \Psi] \cos \frac{k\eta}{\sqrt{3}} - \Psi$$

Oscillations for different wavevectors with the same $|\vec{k}|$ have same zeros; quadratic power possesses coherent acoustic peaks:



Peaks locations: $k\eta = n\pi\sqrt{3}$

Harrison Zel-Dovich:

Perturbations on all super-horizon scales, Gaussianity, same power (scale-invariance):

$$< k^3 \Psi(k)^2 > \propto k^{n-1} \ , \ n = 1$$

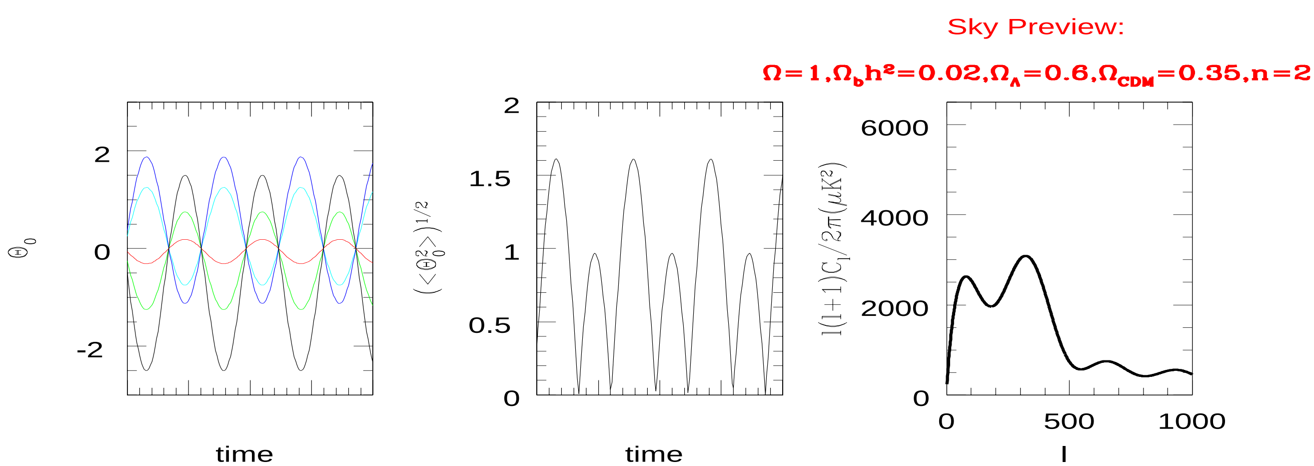
Initial Cosmological Perturbations

Isocurvature Spectra

$$\Theta_0 = \Psi = 0 \; , \; \dot{\Theta}_0 \neq 0$$

$$\Theta_0(\eta) = \frac{\sqrt{3}}{k} \dot{\Theta}_0(0) \sin \frac{k\eta}{\sqrt{3}}$$

Oscillations for different wavevectors with the same $|\vec{k}|$ have same zeros; quadratic power possesses coherent acoustic peaks:



Peaks locations: $k\eta = (n + 1/2)\pi\sqrt{3}$

Initial Cosmological Perturbations

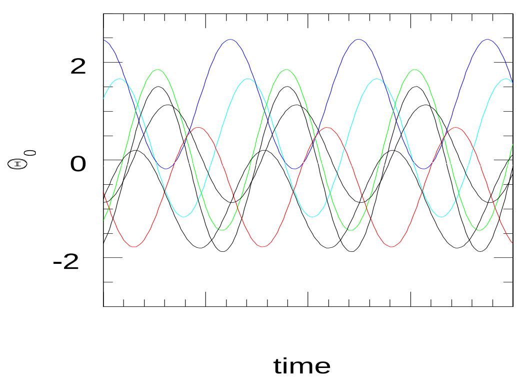
Non-Gaussian Spectra

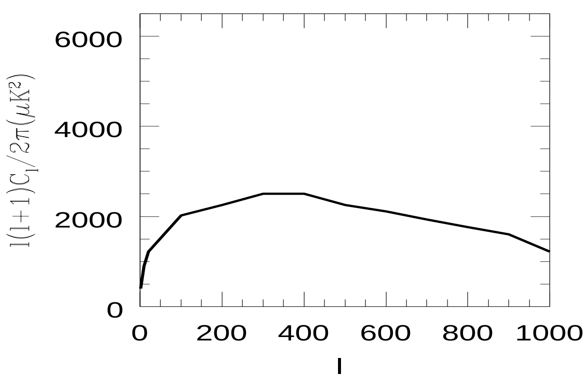
$$\Theta_0 \neq 0 \ , \ \dot{\Theta}_0 \neq 0$$

$$\Theta_0(\eta) = [\Theta_0(0) + \Psi] \cos \frac{k\eta}{\sqrt{3}} + \frac{\sqrt{3}}{k} \dot{\Theta}_0(0) \sin \frac{k\eta}{\sqrt{3}} - \Psi$$

Different directions reflect the non-Gaussian seed and have generally different amplitude and velocity. Acoustic oscillations do not have the same zeros; no peaks:

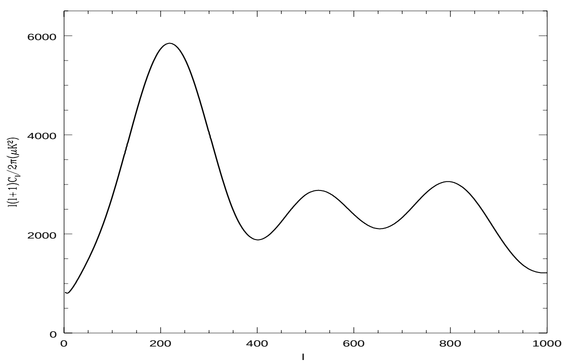
Sky Preview:
Albrecht, Battye, Robinson PRD, 023508, 1999

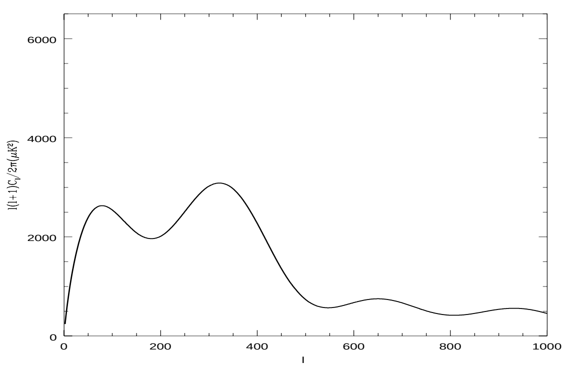


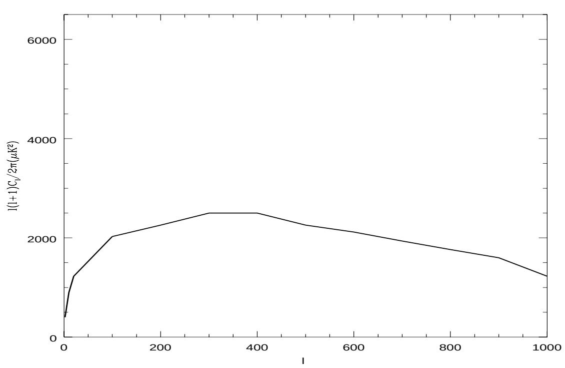


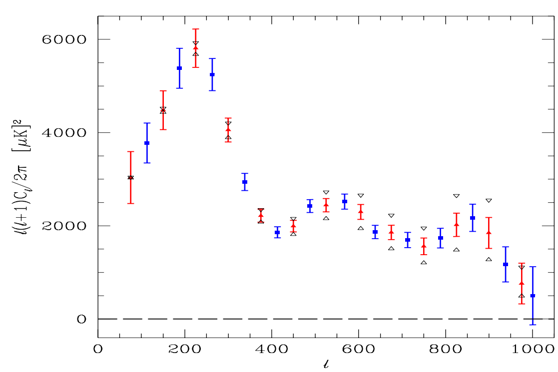
CMB Anisotropies

Which Is The Right One?









Inflation

Basic Concept

$\phi(t), V(\phi) > 0, d\phi/dt \simeq 0 \Rightarrow V(\phi) \simeq constant,$ (slow rolling)

$$H^2 = \left(\frac{1}{a^2}\frac{da}{dt}\right)^2 = \frac{8\pi G}{3}\left[\frac{1}{2}\left(\frac{d\phi}{dt}\right)^2 + V\right] + \frac{K}{a^2}$$

$$\downarrow$$

$$H^2 = \frac{8\pi G}{3}V$$

$$a(t) \propto e^{Ht} = e^{\sqrt{8\pi GV/3}}$$

One Scale Process

Inflation in Particle Physics, Still An Open Issue:

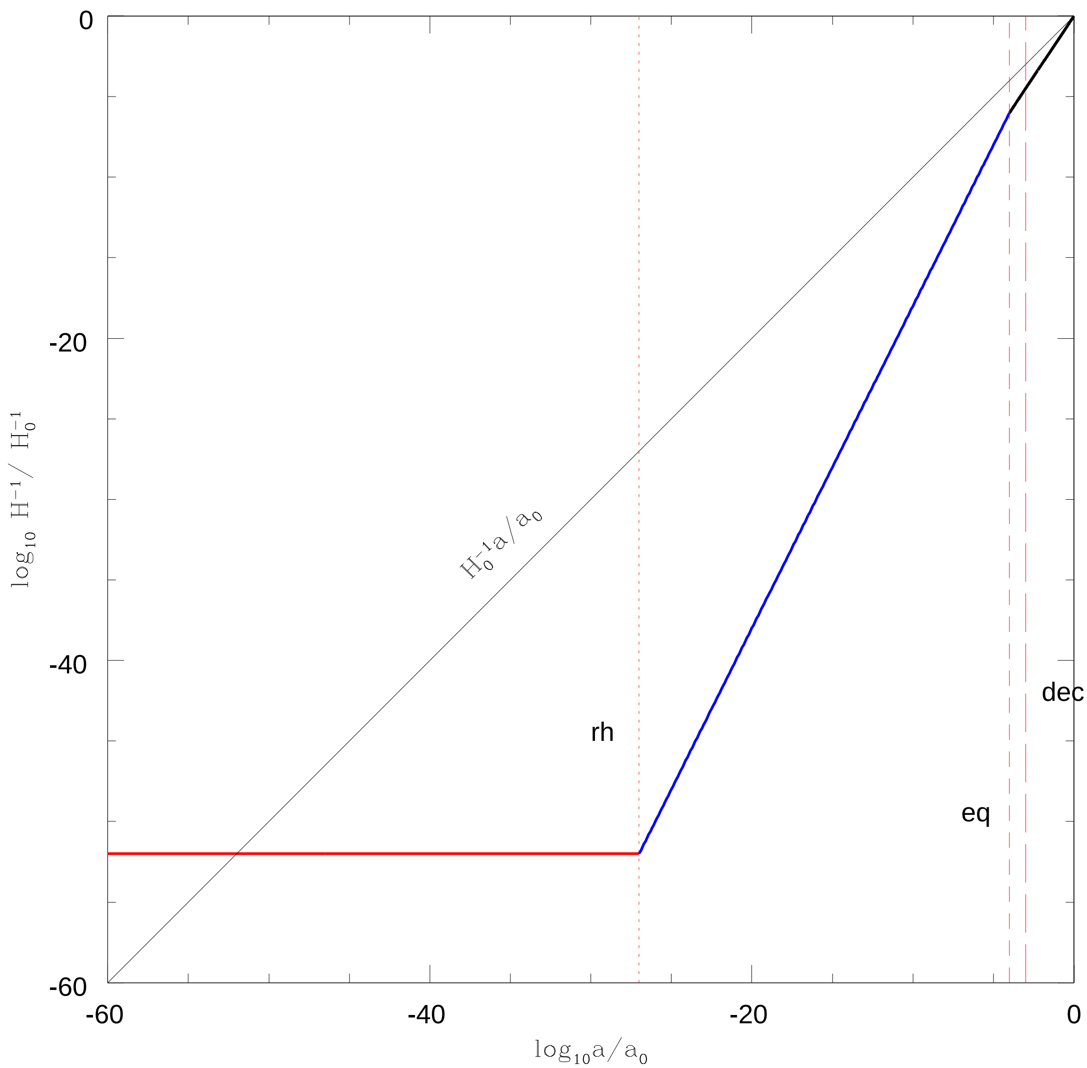
Lidsey, Wands, Copeland, PR337, 343, 2000

Lyth, Riotto, PR314, 1, 1999

...

Inflationary Dynamics

Scales And Horizon during Inflation And After



Inflationary Perturbations

The Beginning

Split inflation in background + perturbations:

$$\phi(\vec{x},t) = \phi(t) + \delta\phi(\vec{x},t)$$

Walk in a mined field without worrying too much, quantizing in a curved background ignoring for example the Fock space broken invariance due to Bogoliubov transformations:

$$\delta\phi(\vec{x},t) \rightarrow \delta\phi_{\vec{k}} = w_{\vec{k}}(t) a_{\vec{k}} e^{i\vec{k}\cdot\vec{x}} + w^*_{\vec{k}}(t) a^+_{\vec{k}} e^{-i\vec{k}\cdot\vec{x}}$$

Solve Perturbed Klein Gordon equation in the slow rolling regime: $(V'' \ll 8\pi G V \simeq H^2)$

$$\ddot{w}_{\vec{k}} + 3H\dot{w}_{\vec{k}} + \left(\frac{k}{a}\right)^2 w_{\vec{k}} = 0$$

Realize that, miraculously, flatness is recovered inside the horizon, $k/aH \gg 1$:

$$\ddot{w}_{\vec{k}} + \left(\frac{k}{a}\right)^2 w_{\vec{k}} = 0$$

so we can define an initial unambiguous vacuum state $|0>$.

Inflationary Perturbations

The Primordial White Noise

The solution to $\ddot{w}_{\vec{k}} + 3H\dot{w}_{\vec{k}} + (k/a)^2 w_{\vec{k}} = 0$ is:

$$w_{\vec{k}}(t) = \frac{H}{\sqrt{2k^3}} \left(i + \frac{k}{aH} \right) \exp \left(i \frac{k}{aH} \right)$$

At early times

$\delta t \ll H^{-1}$, $k/aH \simeq k/aH|_0 - (k/a)\delta t$:

$$w_{\vec{k}}(t) = \frac{1}{\sqrt{E_{\vec{k}}}} \exp(-iE_{\vec{k}}\delta t) \ , \ E_{\vec{k}} = k/a$$

After the horizon crossing $k/aH \ll 1$:

$$w_{\vec{k}} = \frac{iH}{\sqrt{2k^3}} \ \text{frozen}$$

$$\Downarrow$$

Fluctuation power after horizon crossing:

$$P_\phi(k) = \frac{k^3}{2\pi^2} < |w_{\vec{k}}|^2 >= \left(\frac{H}{2\pi} \right)^2$$

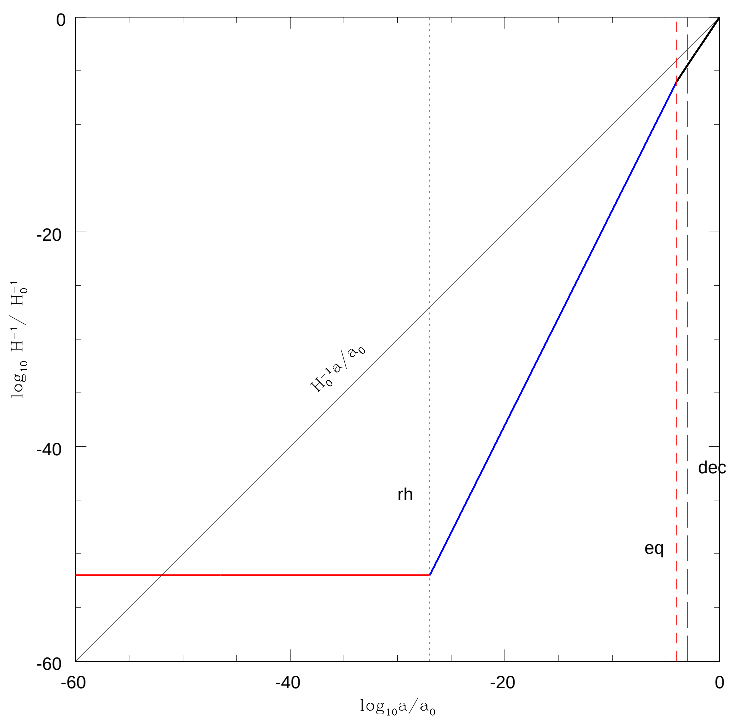
One Scale Process $\Rightarrow$ Scale Invariance
Quantum Fluctuations $\Rightarrow$ Gaussianity

Inflationary Perturbations

Horizon Exit

Translate inflation fluctuations into gravitational potential units linking the horizon H to the potential V during slow-rolling. Vacuum energy decays adiabatically recovering thermal equilibrium between different species. Observations at horizon re-entry constrain inflation:

$$\text{COBE} \Rightarrow 10^{-10} = P_{\Theta} \simeq P_{\Psi} \simeq 10 \left(\frac{GV}{V'^{2/3}} \right)^3$$



Cosmological Parameters

1. Ω_K: < 0 increases the distance to the last scattering surface, shifting acoustic peaks towards higher (smaller) multipoles.

2. Ω_Λ: $\uparrow$ decreases the distance to the last scattering surface, shifting acoustic peaks towards smaller multipoles; enhances acoustic oscillations increasing the radiation/mass ratio.

3. Ω_{CDM}: $\uparrow$ reduces acoustic oscillations because increasing the radiation/mass ratio.

4. Ω_b: $\uparrow$ enhances compression over rarefaction peaks because increasing the effective mass of the oscillator.

5. $h = H_0/(100\ \mathrm{km/sec/Mpc})$: $\uparrow$ as Ω_b due to the combination $\Omega_b h^2$.

6. $\Omega_\nu = 1 - \Omega_K - \Omega_{CMB} - \Omega_b$: $\uparrow$ subtracts power to the CDM, enhances acoustic oscillations and increases the diffusion damping scale.

7. N_ν: $\uparrow$ enhances acoustic oscillations increasing the radiation/mass ratio. Hints from lepton asymmetry (Lesgourgues & Liddle, astro-ph/0106361).

8. τ_c: $\uparrow$ reduces CMB power on angular scales smaller than reionization horizon.

Cosmological Parameters

9. Scalar Adiabatic Amplitude: sets super-horizon perturbation power.

10. Scalar Adiabatic Spectral Index n_s: $\neq 1$ breaks scale invariance. It is an expected feature due to the change of H during inflation.

11. Scalar Isocurvature Amplitude: sets the amplitude of isocurvature components. It can be generated in multiple field inflation (Bartolo, Matarrese, Riotto astro-ph/0106022).

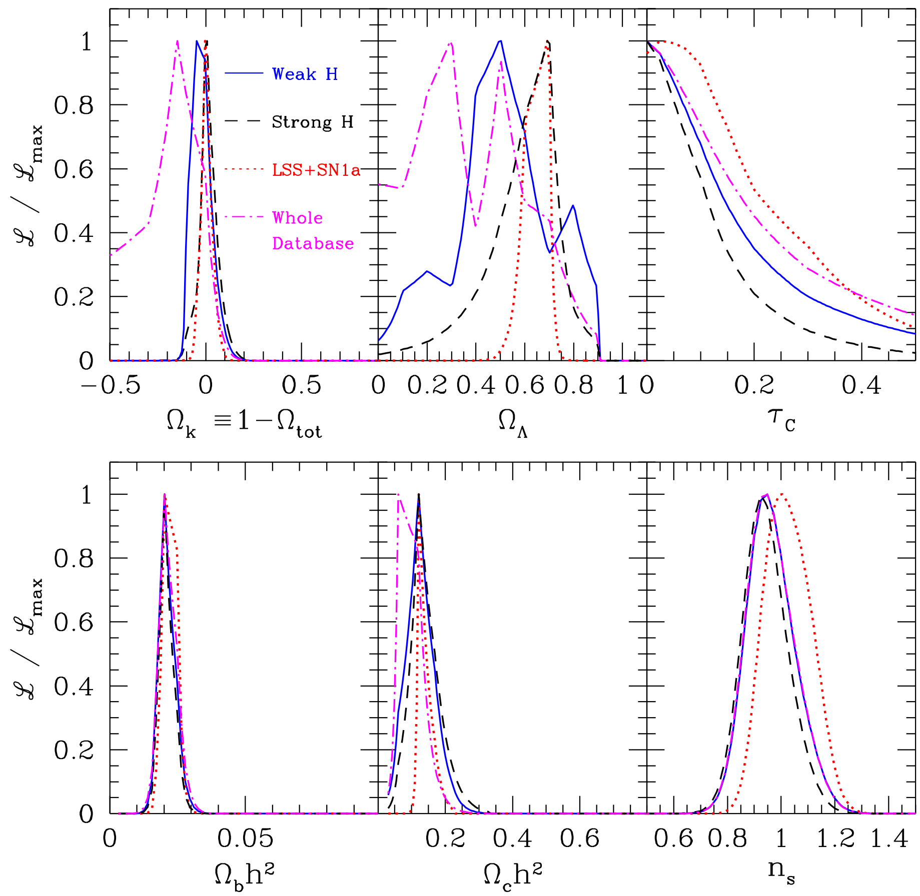
12. Scalar Isocurvature Spectral Index: sets scale dependence of the isocurvature component.

13. Tensor Amplitude. $\uparrow$ enhances super-horizon CMB power, without altering sub-degree CMB peaks. Deviations from slow rolling during inflation can account for tensor perturbations.

14. Tensor Spectral Index n_t: sets scale dependence of the cosmological gravitational wave spectrum.

Cosmological Parameters

From BOOMERanG 98, Netterfield et al. astro-ph/0104460



Weak H: $0.45 < h < 0.90$,

Strong H: $h = 0.71 \pm 0.08$,

LSS+SNIa: cluster abundance+SN data,

The other 8 parameters are supposed to be fixed.

Cosmological Parameters

Dark Things

15. Dark Energy, $\Omega_\Lambda \to \Omega_Q, w_Q$: $w_Q > -1$ reduces the distance to the last scatteting; also alters gravitational potential dynamics at low redshifts.

16. Dark Matter: interactions, species, ...

Fundamental Constant Variations

17. Gravitational constant variation: changes the distance to the last scatteting; also alters gravitational potential dynamics at low redshifts (Baccigalupi, Matarrese, Perrotta PRDD62, 123510, 2000).

18. Fine Structure Constant variation: Murphy et al. claim $\Delta\alpha/\alpha = (-0.72 \pm 0.18)10^{-5}$ by distant quasars absorbtion lines, astro-ph/0012419; reduces the distance to the last scatteting by delaying recombination.

Conclusion

- The observed sub-degree CMB acoustic oscillations strongly support the existence of super-horizon curvature perturbations in linear regime at decoupling.

- This is consistent with the idea that Big Bang Cosmology experienced a phase of vacuum energy dominated, super-horizon expansion much before decoupling, $T \gg 3000K$.

- Within this scenario, CMB indicates a baryon amount which is consistent with the measure from Big Bang Nucleosynthesis, requiring that the dominant component of the energy density today interact only gravitationally with known particles.

The Satellites