An introduction to CMB lensing

PhD course
Linear Cosmological Perturbations and CMB anisotropies

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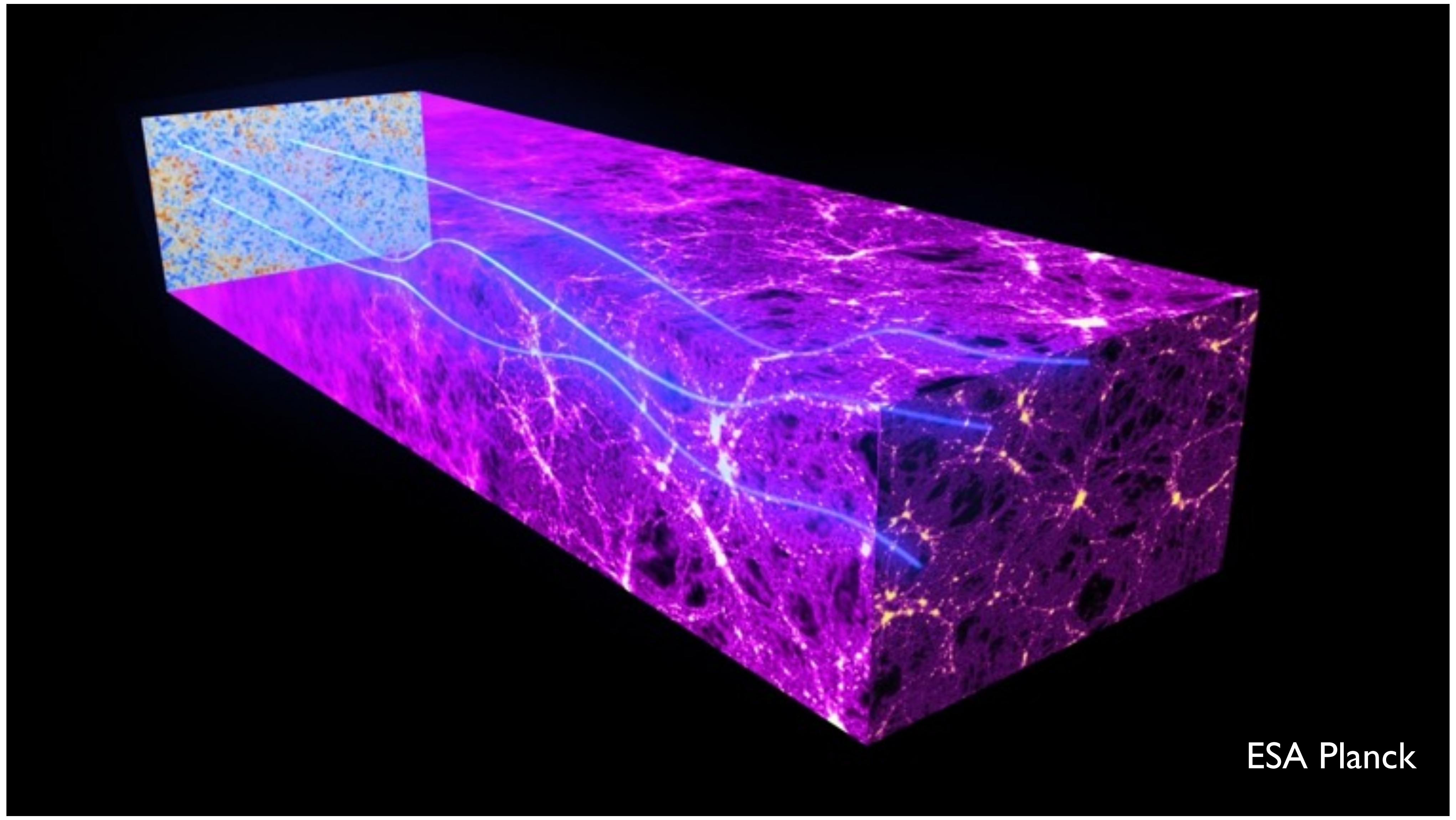
SISSA

CMB weak lensing


ESA Planck

Intervening structures bend the path of CMB photons

➡ observed CMB is a “distorted picture” of the last scattering surface

Review references:

- Lewis and Challinor 2006: comprehensive review
- Hanson, Challinor, Lewis 2010 simpler and concise
- Reasonably recent cosmology books, e.g. Dodelson 2003

Obstacle

➡ statistics is modified

- Power spectrum, smoothed and small scales power
- Non gaussianities

➡ Conversion of primordial E to observed B

…

Probe

➡ Information about structure formation

- Dark energy
- Neutrino mass

…

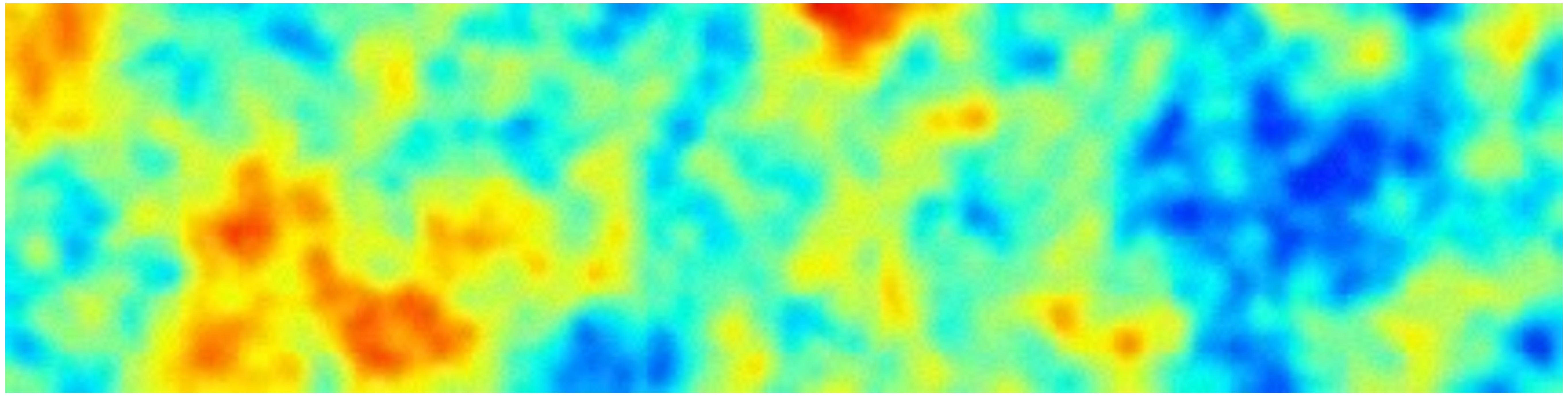
CMB weak lensing: why

- Lensing, in general:
  - **Backlight** with some known property
  - **Intervening** gravitational **field** alter this property
  - Use this change to constrain the intervening field

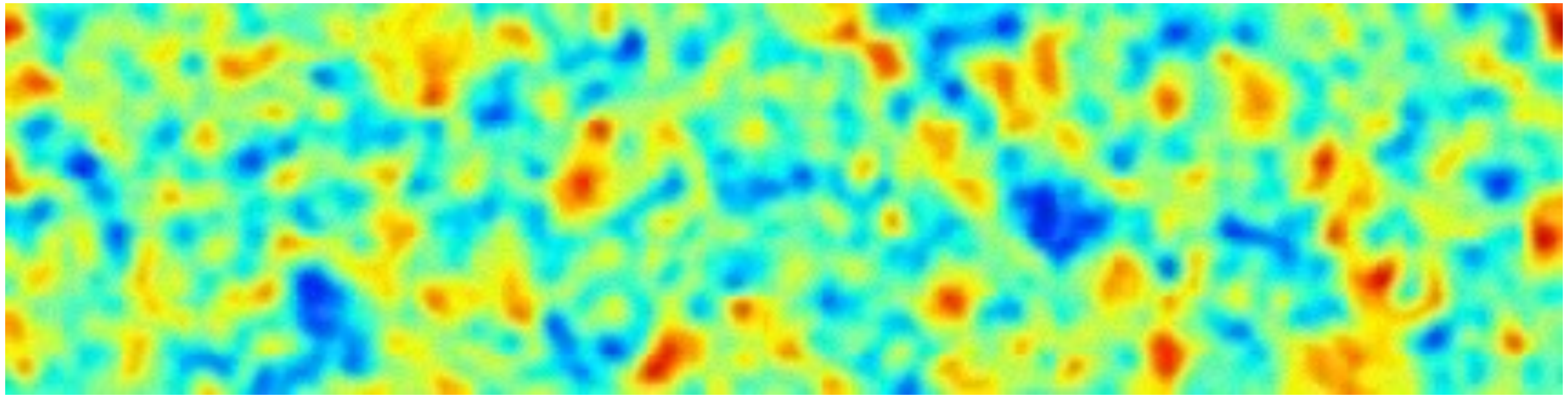
- CMB lensing
  - ✓ The most distant light (z ~ 1100)
  - ✓ Full-sky
  - ✓ CMB is linearly polarized: 3 observables per line of sight
  - ✓ To (very) good approximation Gaussian
    Moreover, two-point correlation well understood

Lensing in action

Lensing OFF



$T(\hat{n})\ (\pm 350\mu K)$



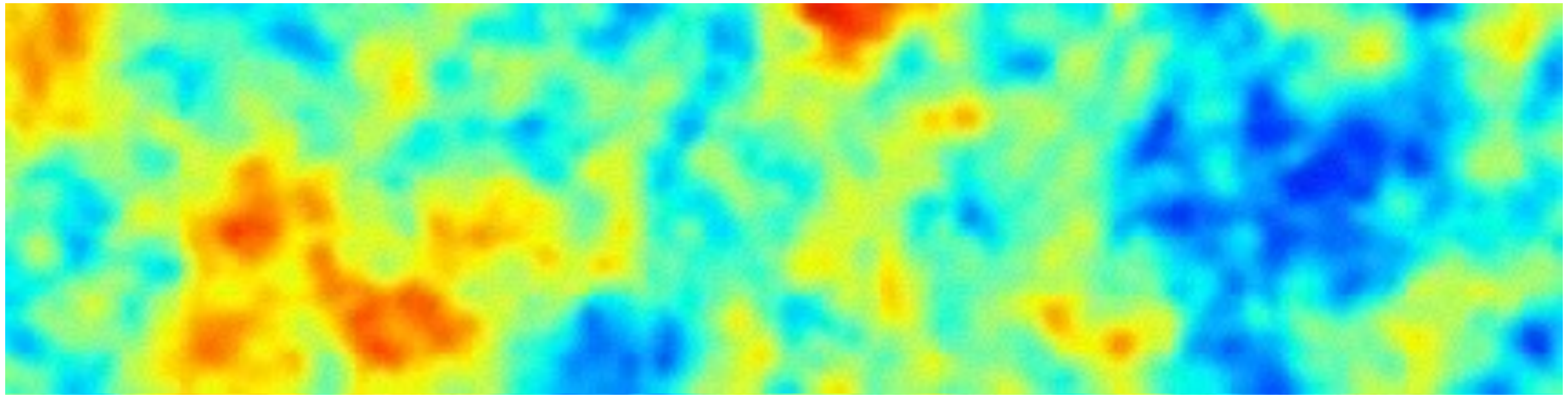
$E(\hat{n})\ (\pm 25\mu K)$

$B(\hat{n})\ (\pm 2.5\mu K)$

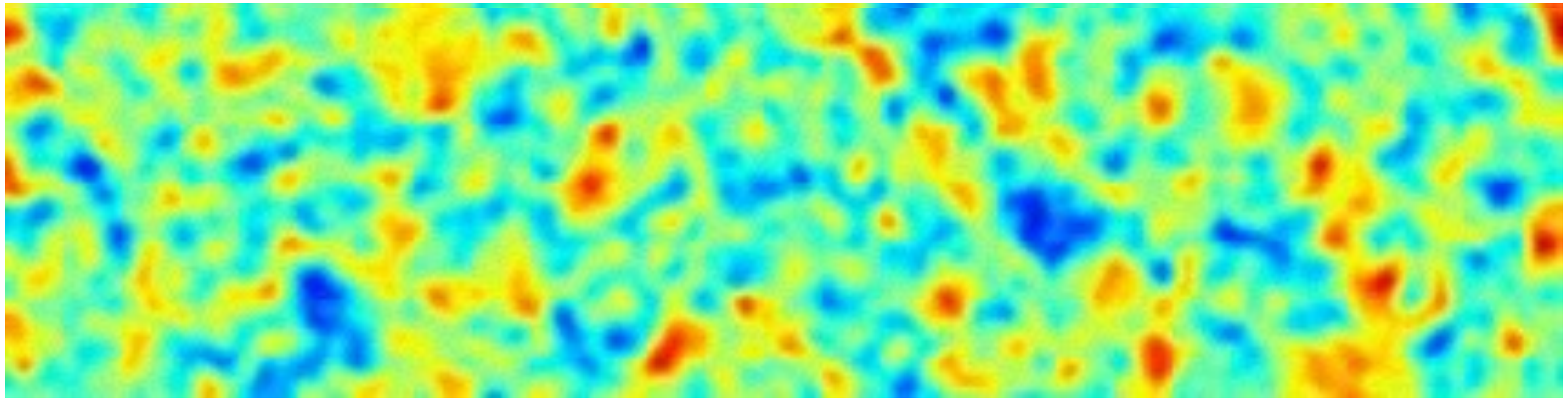
Credit: D. Hanson

Lensing in action

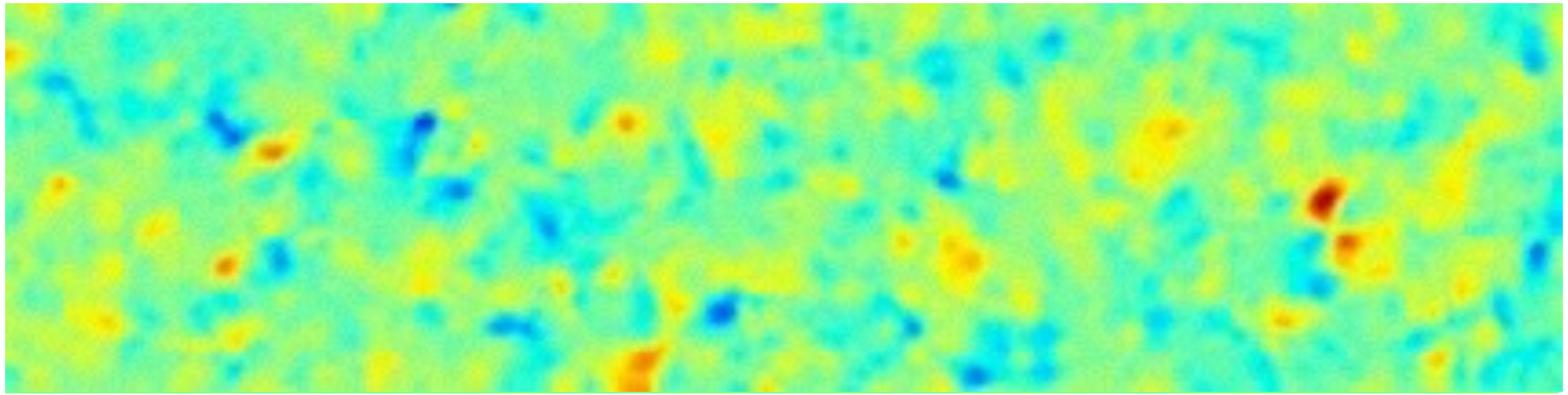
Lensing ON



$\mathrm{T}(\hat{n})\ (\pm 350\mu K)$



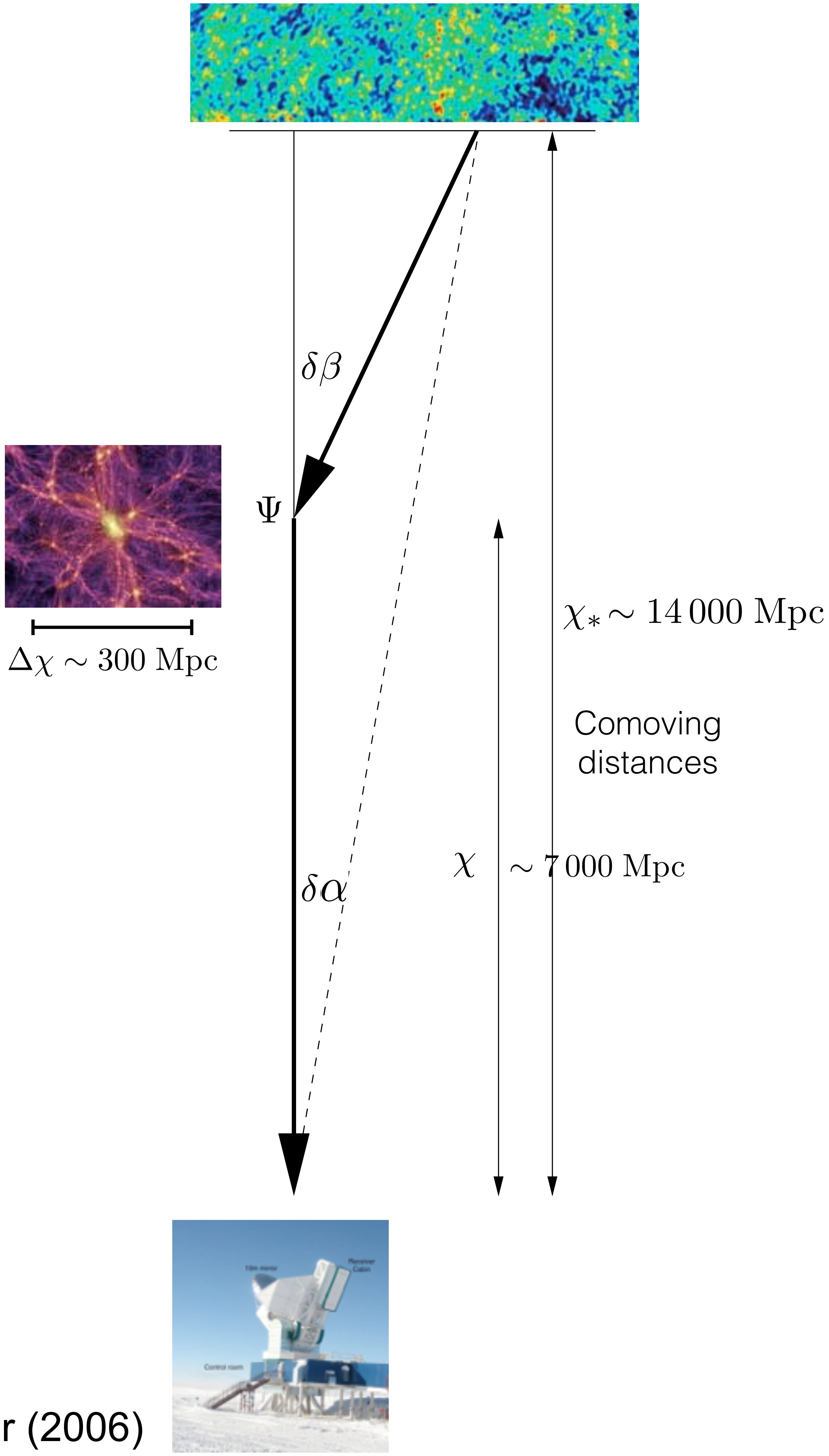
$\mathrm{E}(\hat{n})\ (\pm 25\mu K)$



$\mathrm{B}(\hat{n})\ (\pm 2.5\mu K)$

Credit: D. Hanson

The order of magnitude of the effect



Adapted from
Lewis & Challinor (2006)

- $\Psi \sim 2 \cdot 10^{-5}$ **depth** of the potential

➡ $\delta\beta \sim 2\Psi \sim 10^{-4}$

- $\Delta\chi \sim 300$ Mpc characteristic comoving **size** of matter perturbations

➡ affects the total deflection
  - $14000/300 \sim 50$ incoherent lenses encountered
  - total deflection is $\alpha \sim \sqrt{50}\ \delta\beta \sim 7 \cdot 10^{-4} \sim 2$ arcmin

➡ affects the coherency of the deflection
  - photons within $300/7\,000 \sim 2$ deg see the ~same lenses

CMB lensing: arcminute effect coherent over a degree scale

Displacement field and lensing potential

- To first order

$$\alpha(\mathbf{x}) = \nabla\psi(\mathbf{x})$$

Displacement field

Lensing potential

$$\psi(\mathbf{x}) = -2\int_0^{\chi_*} d\chi \left(\frac{\chi_* - \chi}{\chi_*\chi}\right)\Psi(\chi\mathbf{x};\eta_0 - \chi)$$

Gravitational potential

- Power spectrum of the lensing potential

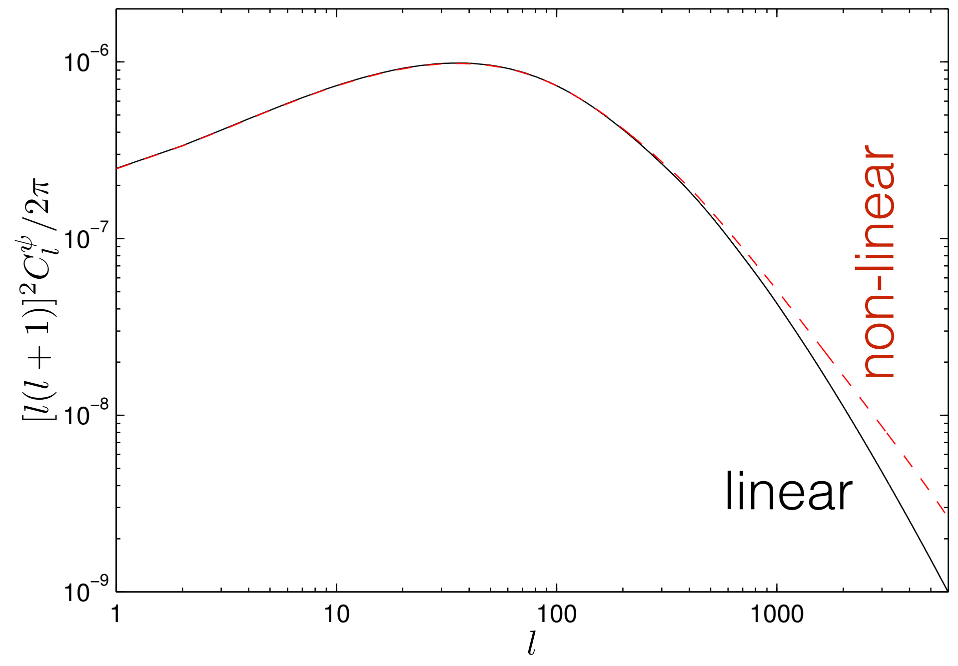
$$C_l^{\psi\psi} = 16\pi\int\frac{dk}{k}\mathscr{P}_{\mathscr{R}}(k)\left[\int_0^{\chi_*} d\chi\, T_\Psi(k;\eta_0-\chi)\, j_l(k\chi)\left(\frac{\chi_*-\chi}{\chi_*\chi}\right)\right]^2$$

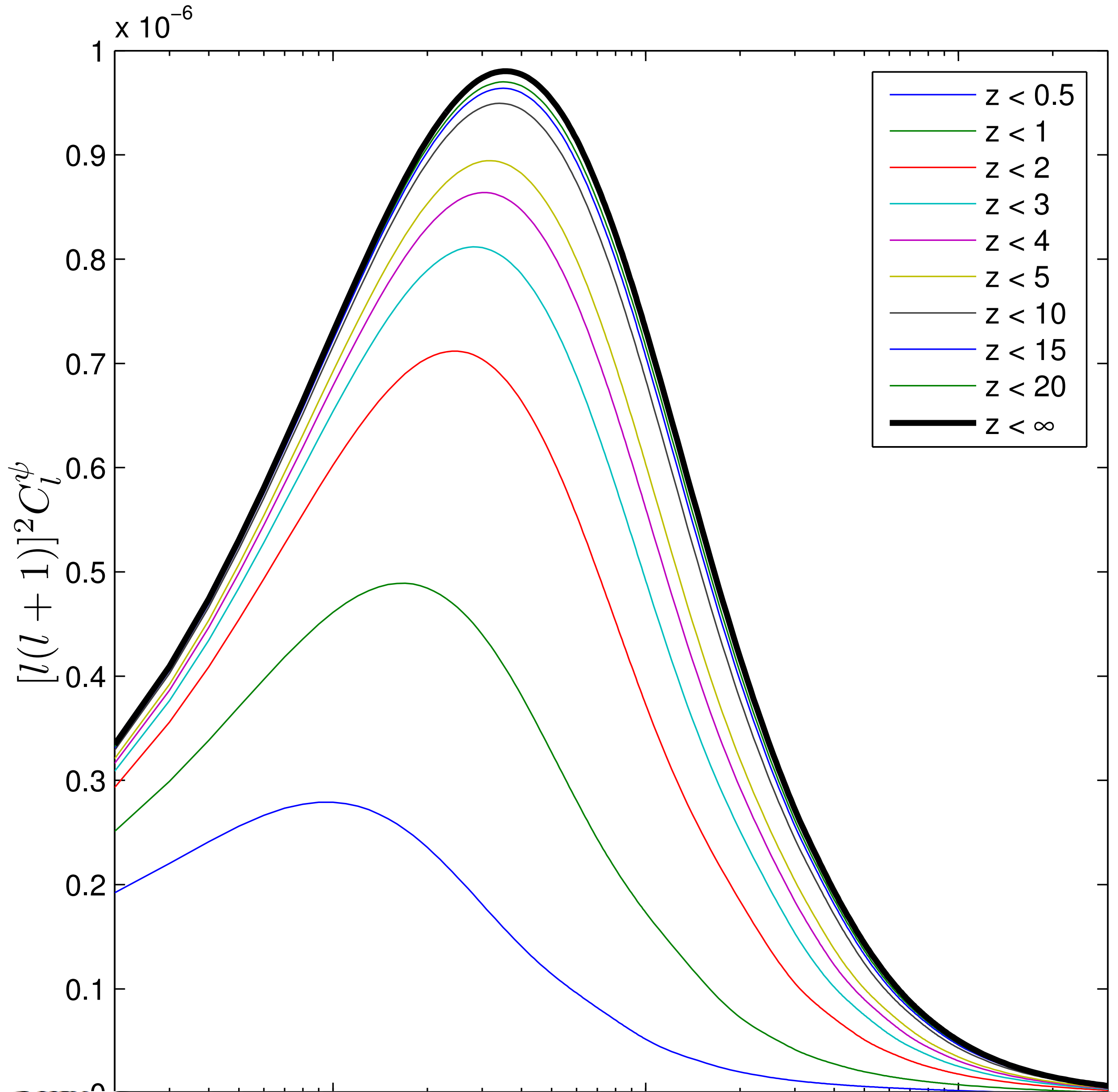
Primordial 3-D power spectrum of density **perturbations**

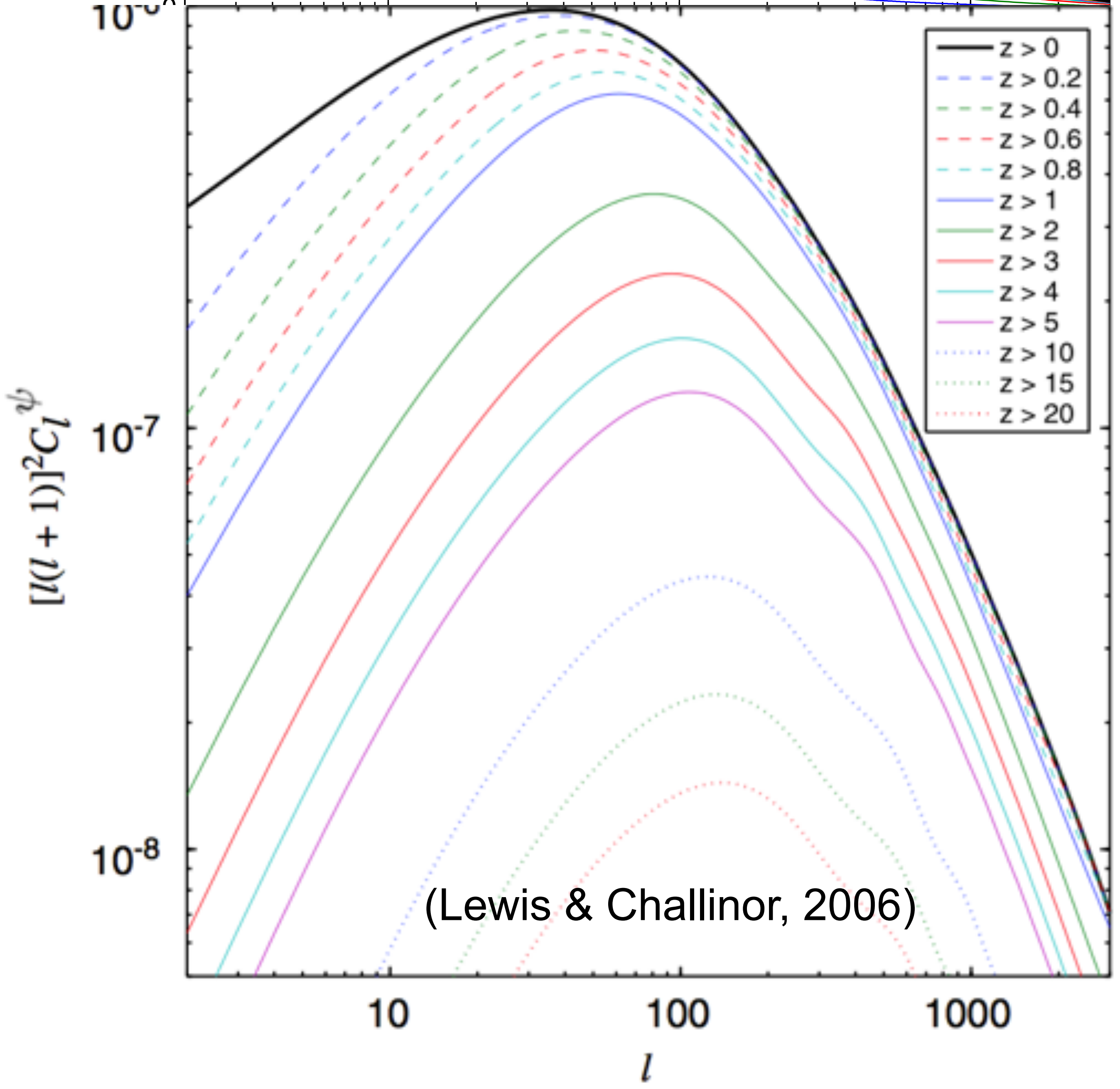
Structure formation:
- Dark energy
- neutrino mass

and actually also
- non-linear evolution

Bessel function







(Lewis & Challinor, 2006)

Effect on CMB power spectrum

$$\text{Observed} \longrightarrow \tilde{T}(\mathbf{x}) = T(\mathbf{x}+\alpha(\mathbf{x})) \longleftarrow \text{Unlensed}$$

$$= T(\mathbf{x}) + \underline{\alpha^a \nabla_a T(\mathbf{x})} + \frac{1}{2}\alpha^a \alpha^b \nabla_a \nabla_b T(\mathbf{x}) + \cdots$$

$$\left[\tilde{Q} \pm i\tilde{U}\right](\mathbf{x}) = [Q \pm iU](\mathbf{x}+\alpha(\mathbf{x}))$$

$$= [Q \pm iU](\mathbf{x}) + \underline{\alpha^a \nabla_a [Q \pm iU](\mathbf{x})} + \frac{1}{2}\alpha^a \alpha^b \nabla_a \nabla_b [Q \pm iU](\mathbf{x}) + \cdots$$

$$\longrightarrow \delta\tilde{T}(\boldsymbol{l}) = \int \frac{d^2 l'}{(2\pi)^2} T(\boldsymbol{l}') \, \mathbf{l}' \cdot (\mathbf{l}-\mathbf{l}') \, \psi(\mathbf{l}-\mathbf{l}')$$

$$\longrightarrow \delta[\tilde{Q} \pm i\tilde{U}] = \int \frac{d^2 l'}{(2\pi)^2} [Q \pm iU](\boldsymbol{l}') \, \mathbf{l}' \cdot (\mathbf{l}-\mathbf{l}') \, \psi(\mathbf{l}-\mathbf{l}')$$

E/B mixing

$$[E(\mathbf{l}) \pm iB(\mathbf{l})] = [Q(\mathbf{l}) \pm iU(\mathbf{l})] e^{\mp 2i\xi_{\mathbf{l}}}$$

$\xi_{\mathbf{l}}$ angle between $\mathbf{l}$ and the *x*-axis

$$\delta\tilde{E}(\boldsymbol{l}) = \int \frac{d^2 l'}{(2\pi)^2} \Big[E(\boldsymbol{l}') \cos 2\varphi_{\boldsymbol{l}'\boldsymbol{l}} \Big] \mathbf{l}' \cdot (\mathbf{l}-\mathbf{l}') \, \psi(\mathbf{l}-\mathbf{l}')$$

$$\delta\tilde{B}(\boldsymbol{l}) = \int \frac{d^2 l'}{(2\pi)^2} \Big[E(\boldsymbol{l}') \sin 2\varphi_{\boldsymbol{l}'\boldsymbol{l}} \Big] \mathbf{l}' \cdot (\mathbf{l}-\mathbf{l}') \, \psi(\mathbf{l}-\mathbf{l}')$$

Suppose no primordial B modes.

Some B-modes are observed.

$$\varphi_{\boldsymbol{l}'\boldsymbol{l}} = \xi_{\mathbf{l}'} - \xi_{\mathbf{l}}$$

Effect on CMB power spectrum

$$C_l^{\tilde T\tilde T} = \left(1 - l^2 R^\psi\right) C_l^{TT} + \int \frac{d^2\mathbf{l}'}{(2\pi)^2} \left[\mathbf{l}' \cdot (\mathbf{l} - \mathbf{l}')\right]^2 C^{\psi\psi}_{|\mathbf{l}-\mathbf{l}'|} C^{TT}_{l'}$$

$$C_l^{\tilde E\tilde E} = \left(1 - l^2 R^\psi\right) C_l^{EE} + \int \frac{d^2\mathbf{l}'}{(2\pi)^2} \left[\mathbf{l}' \cdot (\mathbf{l} - \mathbf{l}')\right]^2 C^{\psi\psi}_{|\mathbf{l}-\mathbf{l}'|} C^{EE}_{l'} \cos^2\left[2\left(\xi_{\mathbf{l}'} - \xi_{\mathbf{l}}\right)\right]$$

$$C_l^{\tilde B\tilde B} = \int \frac{d^2\mathbf{l}'}{(2\pi)^2} \left[\mathbf{l}' \cdot (\mathbf{l} - \mathbf{l}')\right]^2 C^{\psi\psi}_{|\mathbf{l}-\mathbf{l}'|} C^{EE}_{l'} \sin^2\left[2\left(\xi_{\mathbf{l}'} - \xi_{\mathbf{l}}\right)\right],$$

Temperature:

- TT smoothed
- Small scale limit

$$C_l^{\tilde T\tilde T} \approx l^2 C_l^{\psi\psi} \int \frac{dl'}{l'} \frac{(l')^4 C_{l'}^{TT}}{4\pi} \approx l^2 C_l^{\psi\psi} R^T,$$

$$R^\psi \equiv \frac{1}{2}\left\langle |\nabla\psi|^2 \right\rangle = \frac{1}{4\pi} \int \frac{dl}{l} l^4 C_l^{\psi\psi} \sim 3 \times 10^{-7}$$

Polarization:

- EE smoothed
- **Lensing B-modes** are produced from primordial E-modes
- E and B modes, same small scale limit $\frac{1}{2} l^2 C_l^\psi R^E$
- Large scales BB limit is a white spectrum

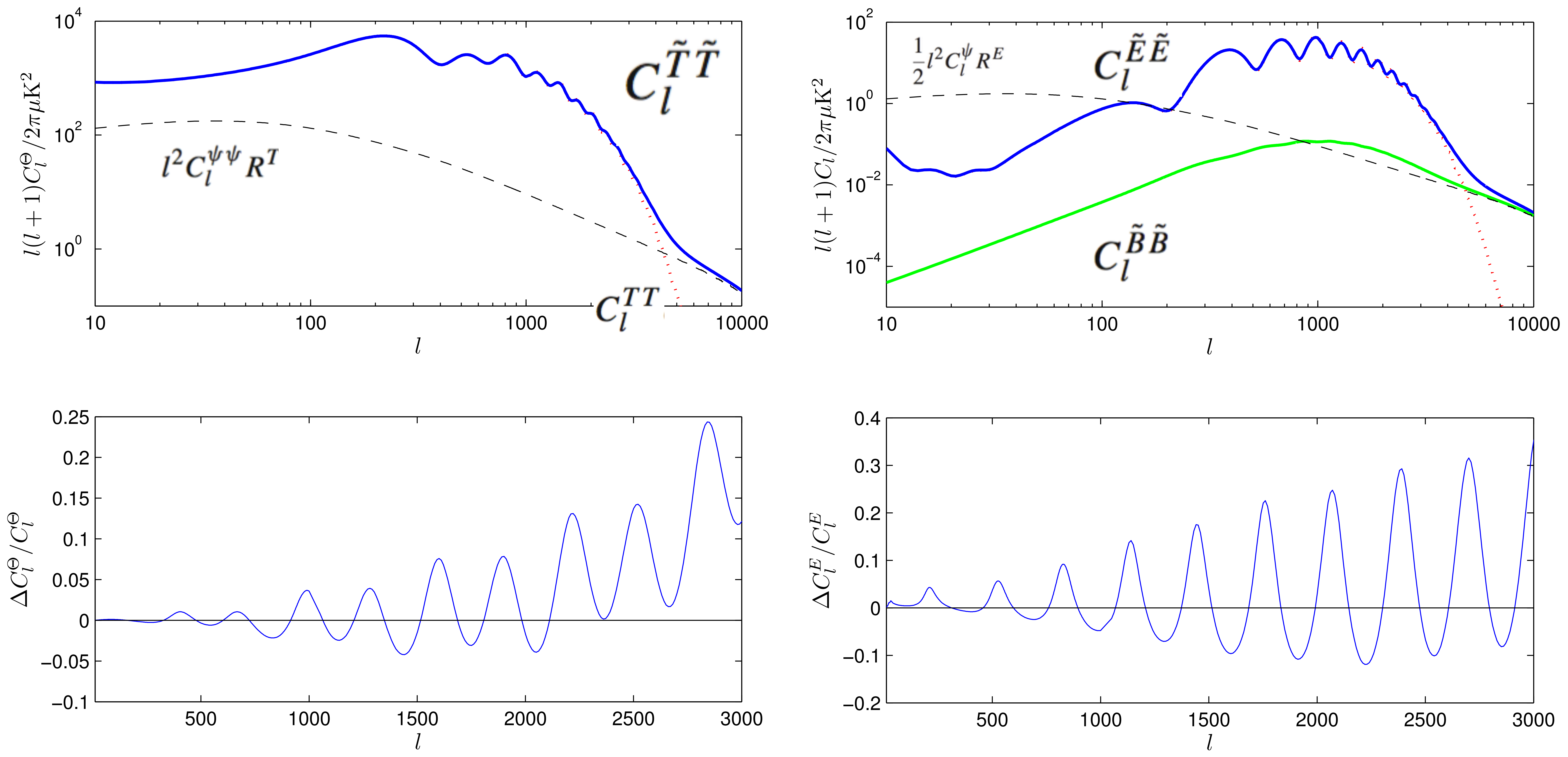
$$C_l^{\tilde B\tilde B} \sim \frac{1}{4\pi} \int \frac{dl'}{l'} l'^4 C_{l'}^{\psi\psi} l'^2 C_{l'}^{EE} \sim 2 \times 10^{-6}\,\mu\mathrm{K}^2$$

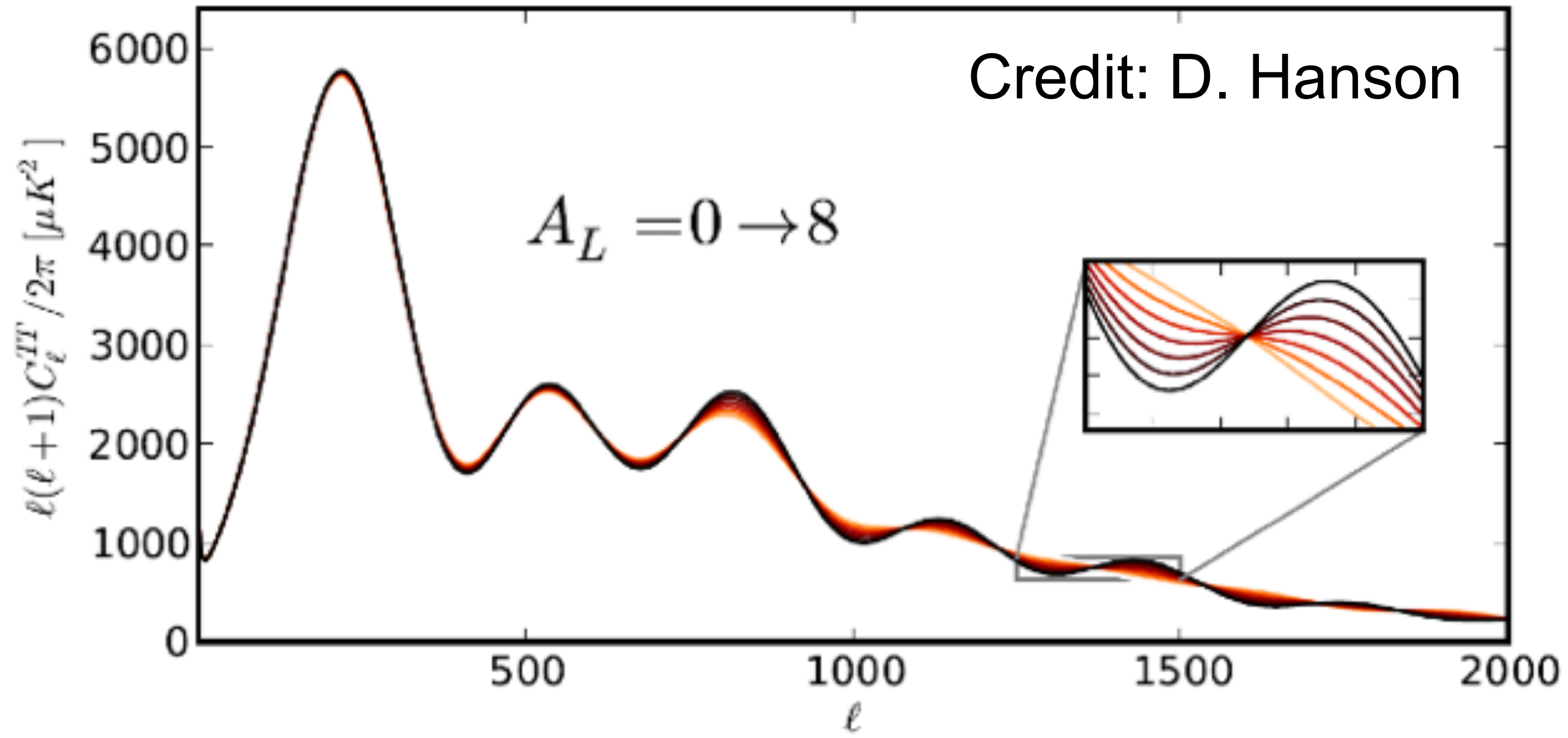
White noise equivalent

$\sim 5\,\mu$K-arcmin

Effect on CMB power spectrum, polarization

Adapted from
Lewis & Challinor (2006)





Credit: D. Hanson

- EE smoothed
  (more than TT because EE peaks sharper)
- Lensing B-modes

Estimating the lensing potential

- Assuming that the primordial CMB was gaussian, for $\mathbf{l} \neq \mathbf{l}'$

$$\langle T(\mathbf{l}) T(\mathbf{l}') \rangle = 0$$

- But with lensing

$$\delta\tilde{T}(\boldsymbol{l}) = \int \frac{d^2 l'}{(2\pi)^2} T(\boldsymbol{l}') \, \mathbf{l}' \cdot (\mathbf{l} - \mathbf{l}') \, \psi(\mathbf{l} - \mathbf{l}')$$

and therefore

$$\langle \tilde{T}(\mathbf{l}) \tilde{T}(\mathbf{l}') \rangle \sim \langle \tilde{T}(\mathbf{l}) \delta\tilde{T}(\mathbf{l}') \rangle \propto \psi(\mathbf{l} - \mathbf{l}')$$

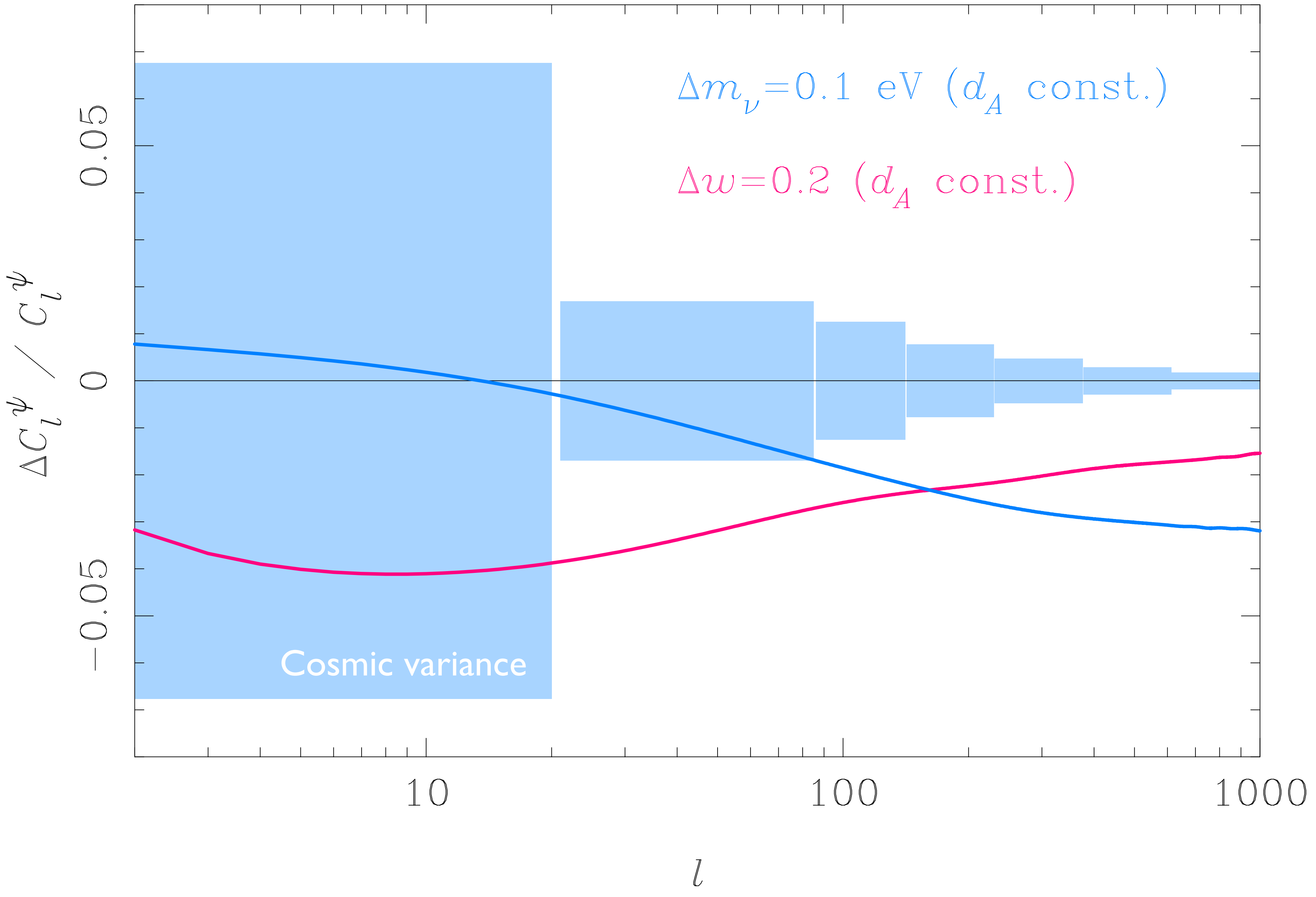
➡ The lensing potential can be estimated using off-diagonal correlations

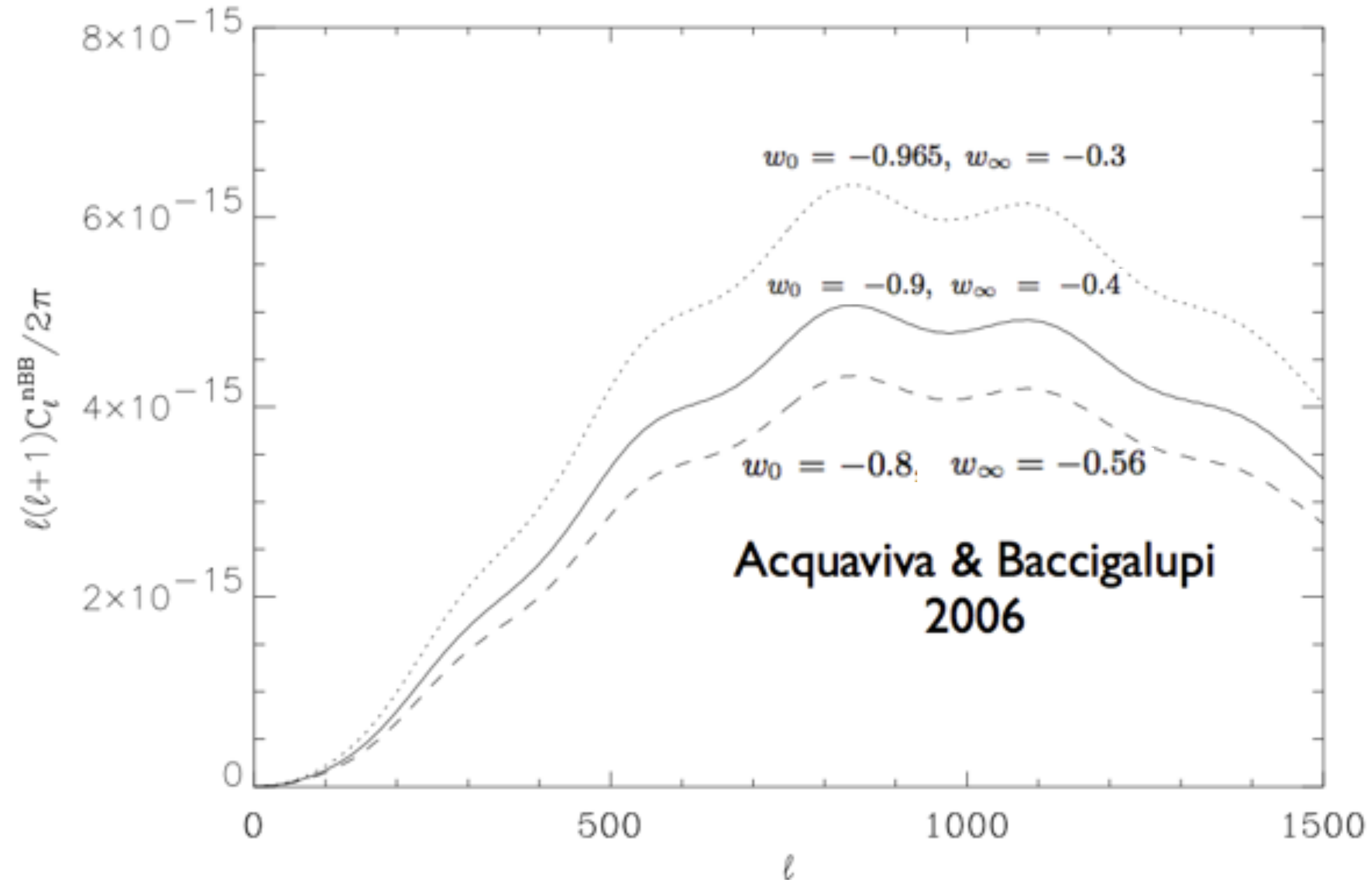
➡ This holds for polarization as well, EB eventually will give the best constraints

- Hu and Okamoto (2002), Okamoto and Hu (2003), Hirata and Seljak (2003)

Dark energy and Neutrinos

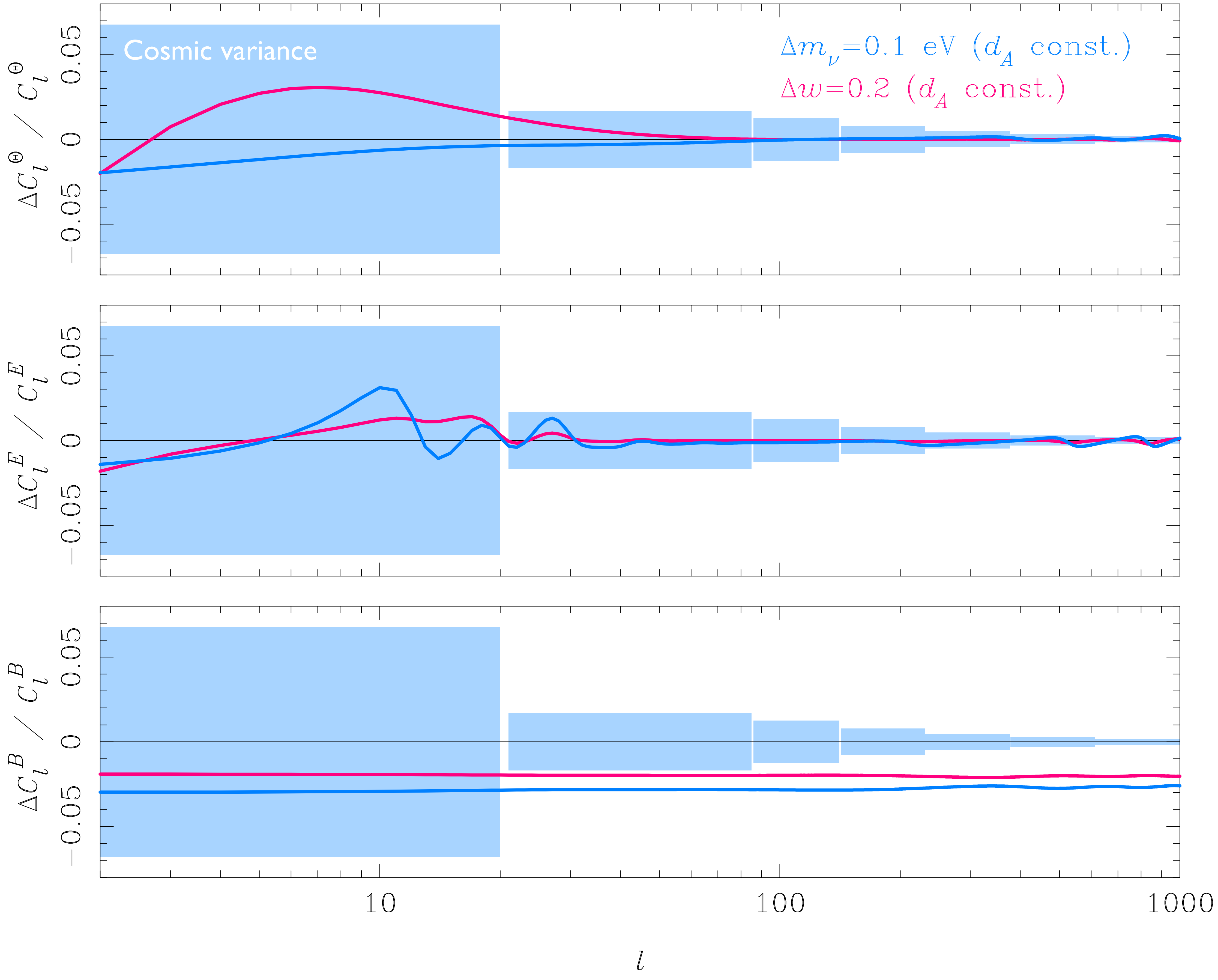
Lewis & Challinor (2006)





Acquaviva & Baccigalupi 2006

Lewis & Challinor (2006)



- EE lensing effects: just a fraction of the total signal, which determines the cosmic variance
- BB: lensing is the whole signal (so far), thus very sensitive to structure formation

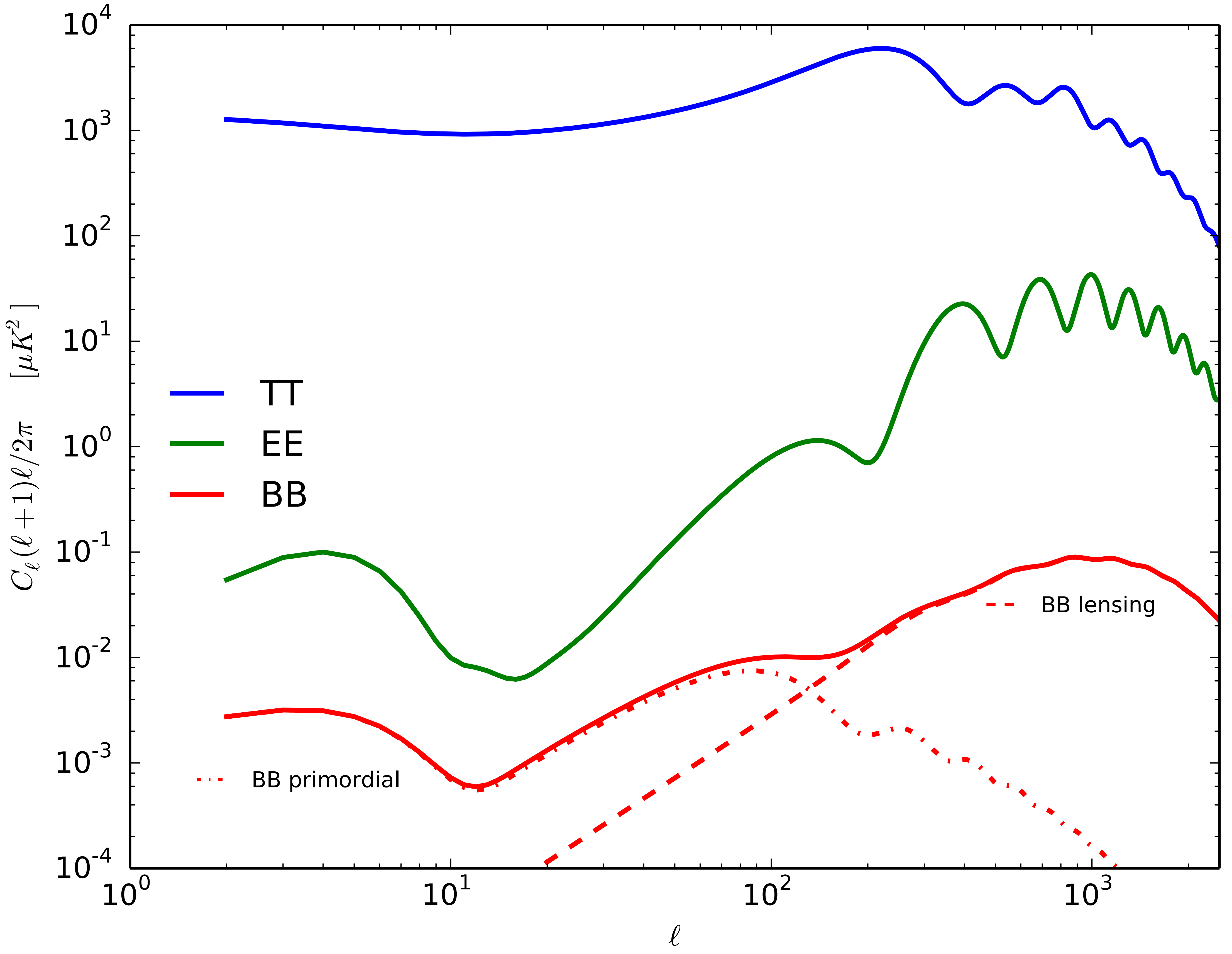
Conclusions

Intervening structures bend the path of CMB photons

- ➡ observed CMB is a “distorted picture” of the last scattering surface

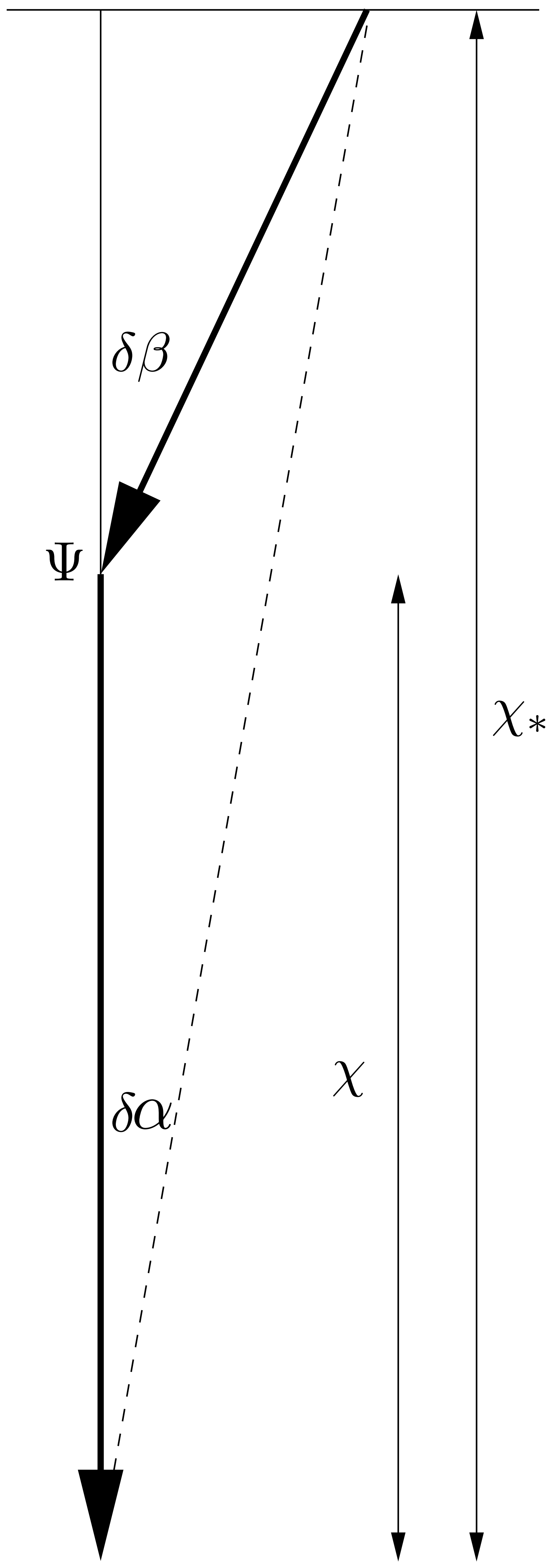
Main effects

- ➡ statistics is modified
  - Power spectrum, smoothed and small-scales power
  - Non gaussianities
- ➡ Conversion of primordial E to observed B
  - ➡ Obstacle for primordial GW detection
- ➡ Information about structure formation
  - Dark energy
  - Neutrino mass
  - …



Backup

…more precisely



Displacement for any direction on the sky $\mathbf{x}$

$$\alpha(\mathbf{x}) = -2 \int_0^{\chi_*} d\chi \, \frac{\chi_* - \chi}{\chi_*} \nabla_\perp \Psi(\chi \mathbf{x}; \eta_0 - \chi)$$

- photon at $(\chi \mathbf{x}; \eta_0 - \chi)$ gets deflected by $\delta\beta = -2\, d\chi\, \nabla_\perp \Psi$
- its contribution to α is given by $\delta\beta\, (\chi_* - \chi)/\chi_*$
- Integration over the path of the photon

Born approximation: integration over the undeflected path

➡ $\nabla_{\mathbf{x}} = \chi^{-1} \nabla_\perp$

➡ $\alpha(\mathbf{x}) = \nabla \psi(\mathbf{x})$ $\quad\quad$ $\psi(\mathbf{x}) = -2 \int_0^{\chi_*} d\chi \left(\frac{\chi_* - \chi}{\chi_* \chi} \right) \Psi(\chi \mathbf{x}; \eta_0 - \chi)$

Lensing potential