ICA based component separation and CMB polarization

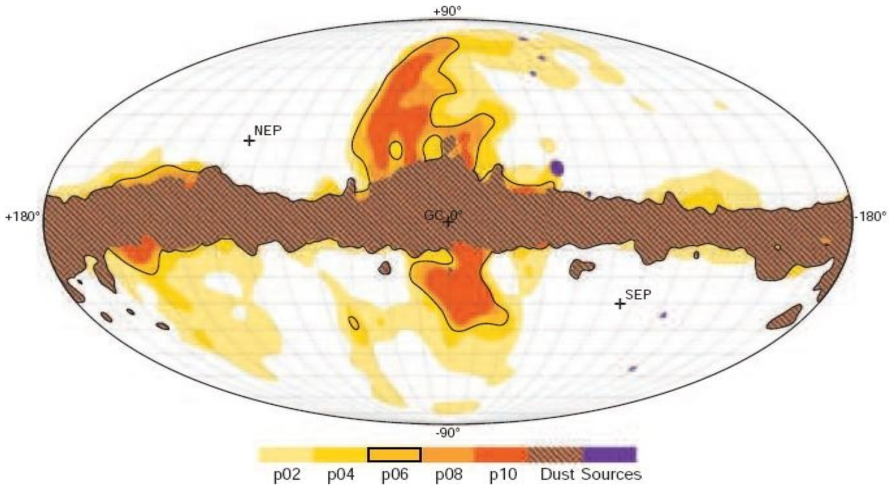
Carlo Baccigalupi & Federico Stivoli

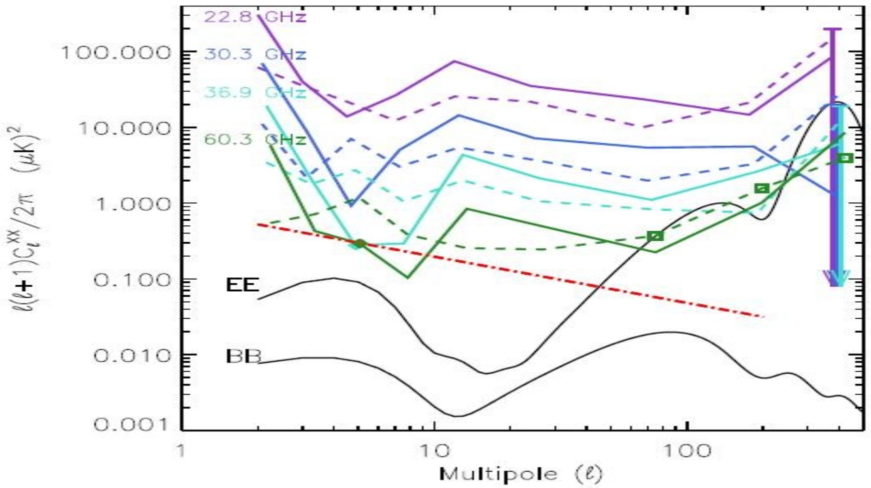
outline

- aim of this talk: transmit the experience we gained by trying ICA component separation in polarization (mnras 2004, 2006, astro-ph/0505381,0209591)
- foreground expectations in polarization
- an example of the ideal ICA performance, why we believe it's worth studying this
- quantitative tests

WMAP polarized foregrounds

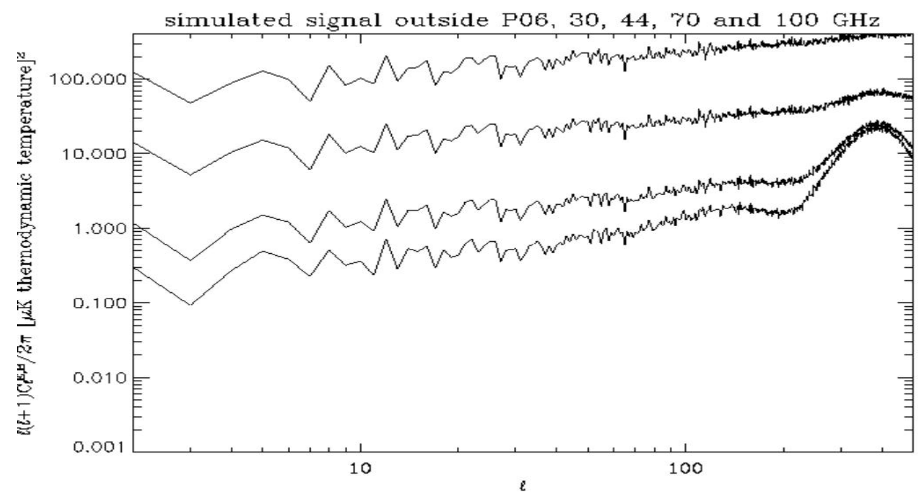
- up to the degree scale, the sky is foreground dominated at all WMAP frequencies after the Galaxy has been cut out
- the E and B foreground signals are comparable as expected (Zaldarriaga 2001)

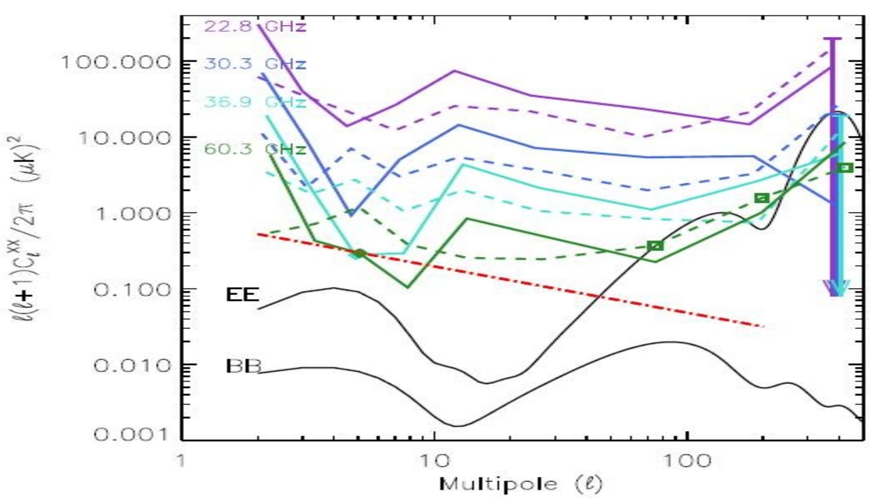




WMAP polarized foregrounds

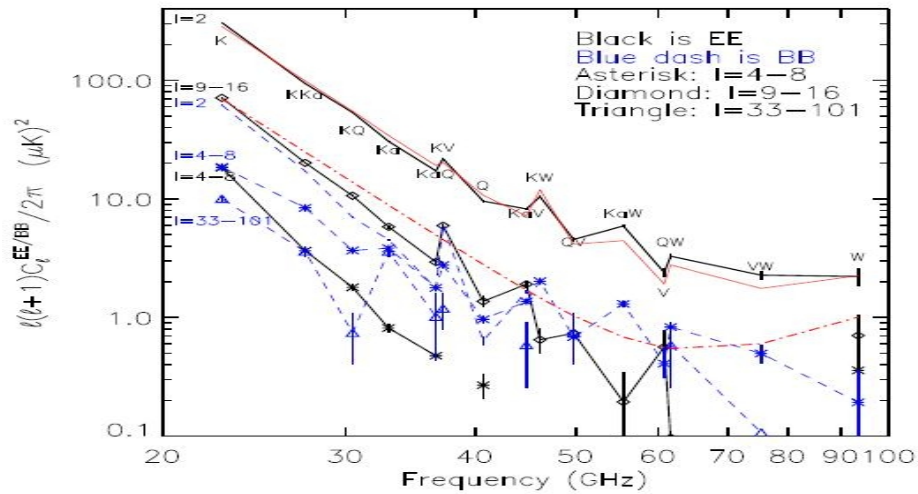
- up to the degree scale, the sky is foreground dominated at all WMAP frequencies after the Galaxy has been cut out
- the E and B foreground signals are comparable as expected (Zaldarriaga 2001)
- confirmation of a nasty expectation

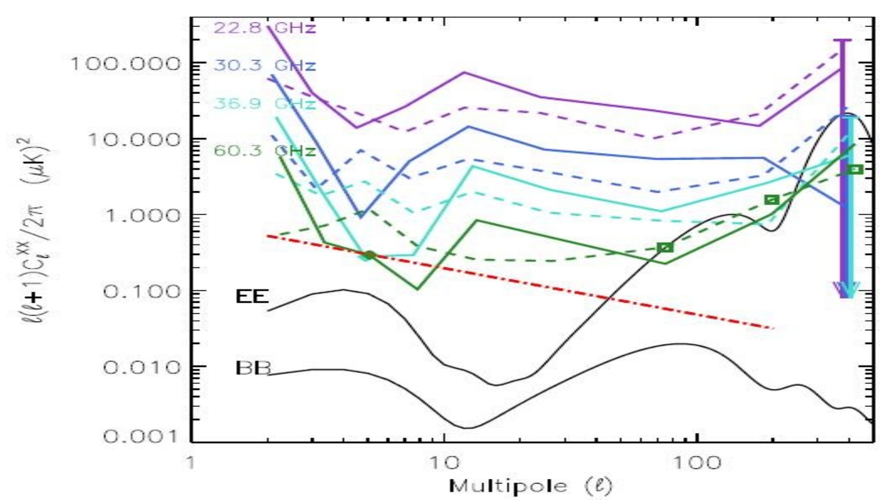




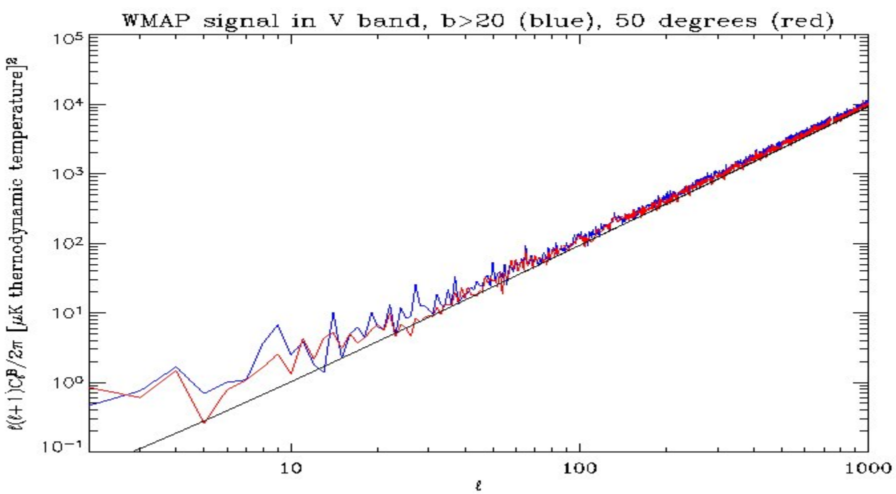
WMAP polarized foregrounds

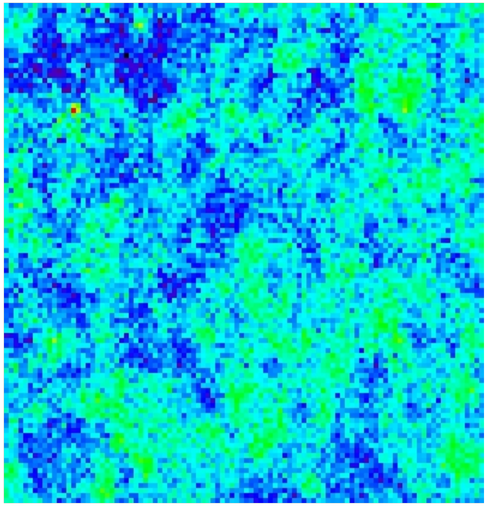
- up to the degree scale, the sky is foreground dominated at all WMAP frequencies after the Galaxy has been cut out
- the E and B foreground signals are comparable as expected (Zaldarriaga 2001)
- confirmation of a nasty expectation
- indications that the minimum is found



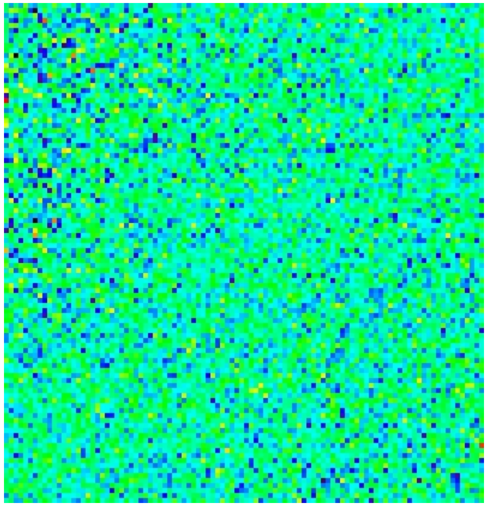


BOOMERANG, EBEx patch, RA=62°, dec=-50°, V band

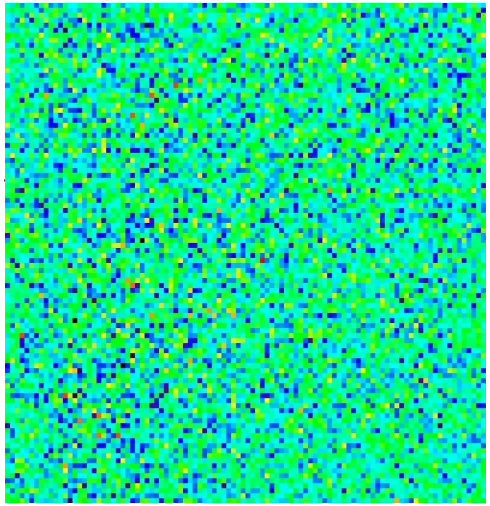




T



Q



U

the ICA algorithm

Component Separation

$$\mathbf{x}(n_f, n_p) = \mathbf{B} \cdot \mathbf{A}(n_f, n_c) \cdot \mathbf{s}(n_c, n_p) + \mathbf{n}(n_f, n_p)$$

$\mathbf{x}$: data at the n_f frequencies
$\mathbf{s}$: the n_c emission processes to recover
$\mathbf{n}$: the instrumental noise
$\mathbf{A}$: mixing matrix
$\mathbf{B}$: beam convolution
n_p: the number of observed pixels or harmonic modes

Goal: find **s**

the ICA algorithm

The ICA Concept: the Central Limit Upside-down

Assume Statistical Independence
The statistics of a mix of statistical independent components tends to be Gaussian

Reverse Path
Combine linearly the data to maximize a suitable measure of non-Gaussianity

Extract and Iterate
If the hypotheses are verified, the maxima correspond to the independent components present into the data

Relevant Works:

Hyvärinen & Oja 2000, Neural Networks 13, 411
Amari & Chichocki 1998, Proc. IEEE 86, 2026

the ICA algorithm

- in polarization, the separation is performed separately on Q and U, which are then compared as usual for getting the E and B modes

FastICA Main Loop

out of $\mathbf{x} = \mathbf{B} \cdot \mathbf{A} \cdot \mathbf{s} + \mathbf{n}$ find $\mathbf{W}$ such that $\mathbf{W} \cdot \mathbf{x} = \mathbf{B} \cdot \mathbf{s} + \mathbf{W} \cdot \mathbf{n}$

Whitening: $\mathbf{C} = E\{\mathbf{x}\mathbf{x}^T\}$, $\Sigma = E\{\mathbf{n}\mathbf{n}^T\}$,
$\hat{\Sigma} = (\mathbf{C} - \Sigma)^{-1/2} \Sigma (\mathbf{C} - \Sigma)^{-1/2}$, $\hat{\mathbf{x}} = (\mathbf{C} - \Sigma)^{-1/2} \mathbf{x}$

Row by Row:

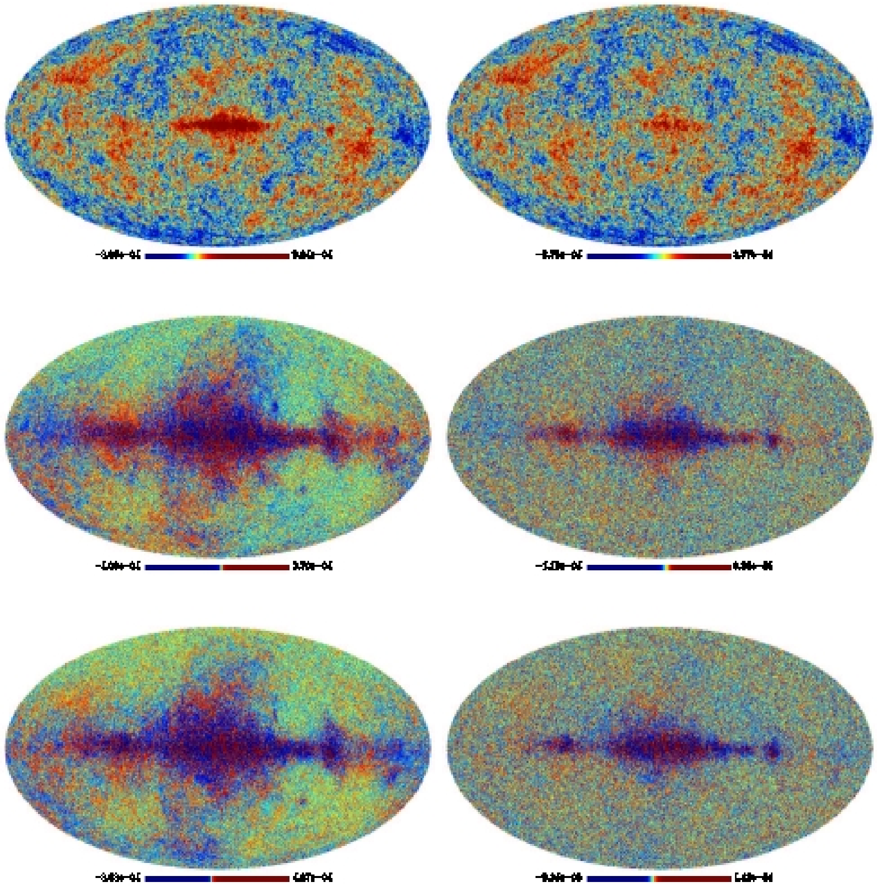
1. Choose an initial vector $\mathbf{w}$;
2. update it through
$$\mathbf{w}_{new} = E\{\hat{\mathbf{x}} g(\mathbf{w}^T \hat{\mathbf{x}})\} - (I + \hat{\Sigma}) \mathbf{w} E\{ g'(\mathbf{w}^T \hat{\mathbf{x}})\}$$
where $g(u) = u^3$, $g(u) = \tanh u$, $g(u) = \exp(-u^2)$...
3. let
$$\mathbf{w}_{new} = \frac{\mathbf{w}_{new}}{||\mathbf{w}_{new}||}$$
4. Compare $\mathbf{w}_{new}$ with the old one; if not converged, go back to 2, if converged, begin another process keeping orthogonality between rows

Computationally Costless:
A few full sky channels at 3.5' resolution run in a few minutes on a normal PC

an example of the ICA nominal performance

- mix all sky simulated synchrotron and CMB at two frequencies
- perform separation on T, Q and U
- check the foreground cleaning on the power spectra

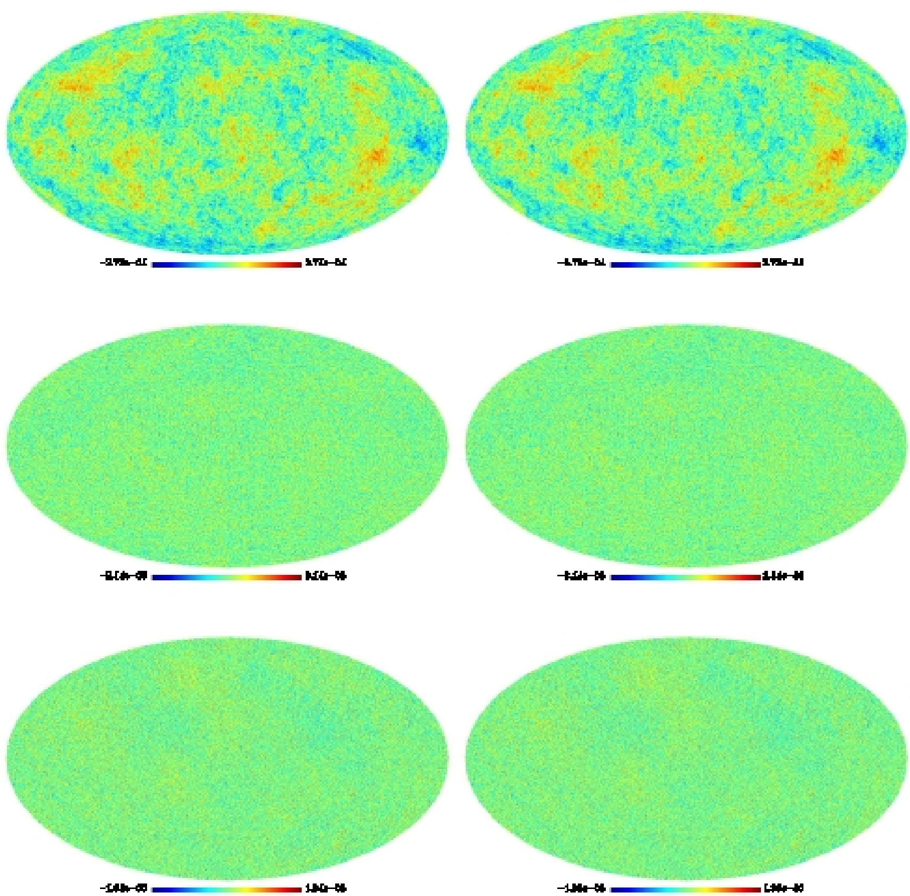
Mix CMB and Synchrotron at 50 and 80 GHz:



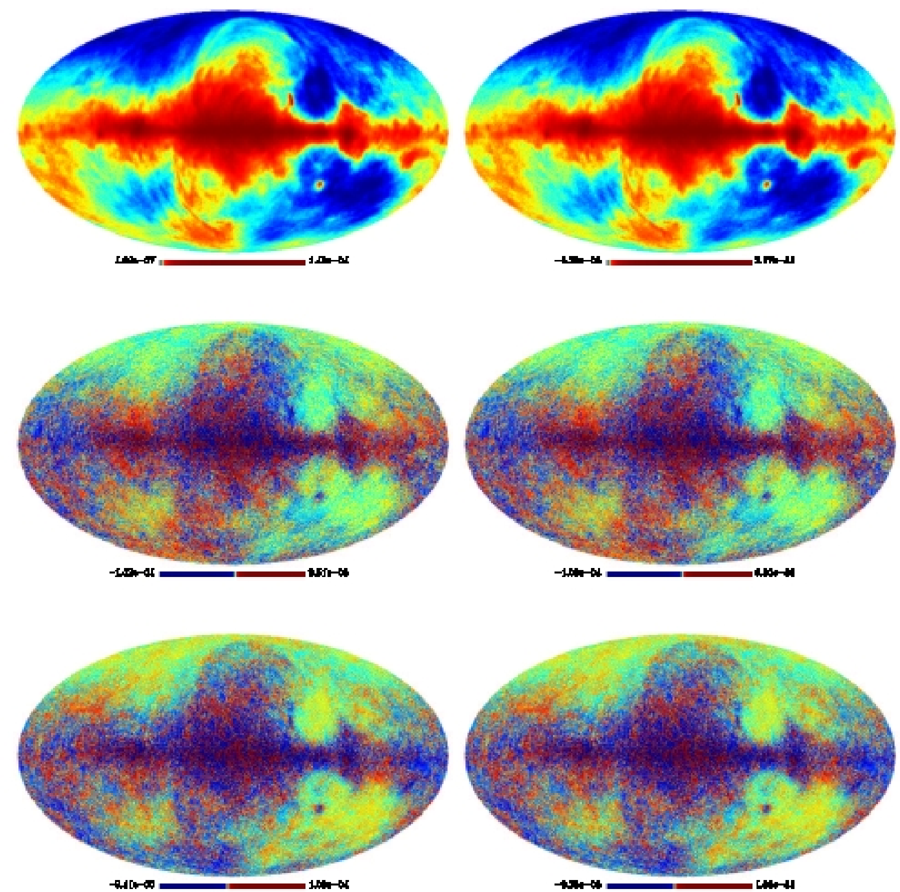
Ask the code to recover the independent components

and example of the ICA nominal performance

FastICA Input/Output: CMB

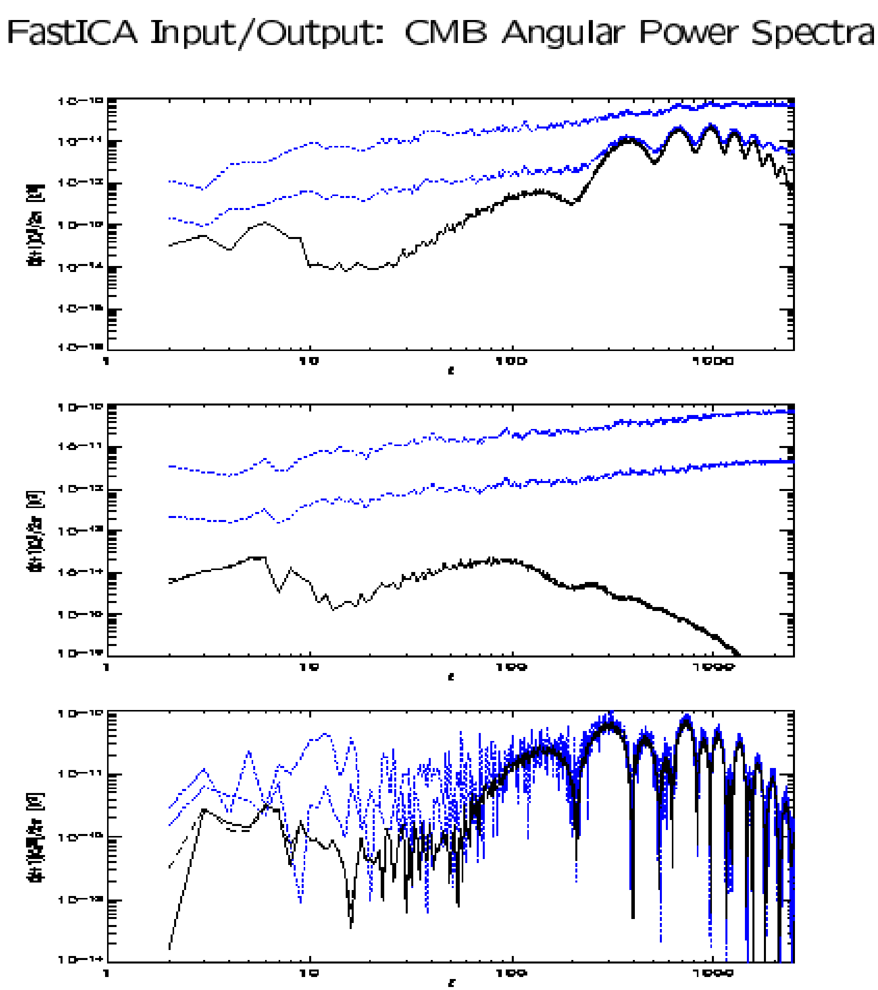


FastICA Input/Output: Synchrotron



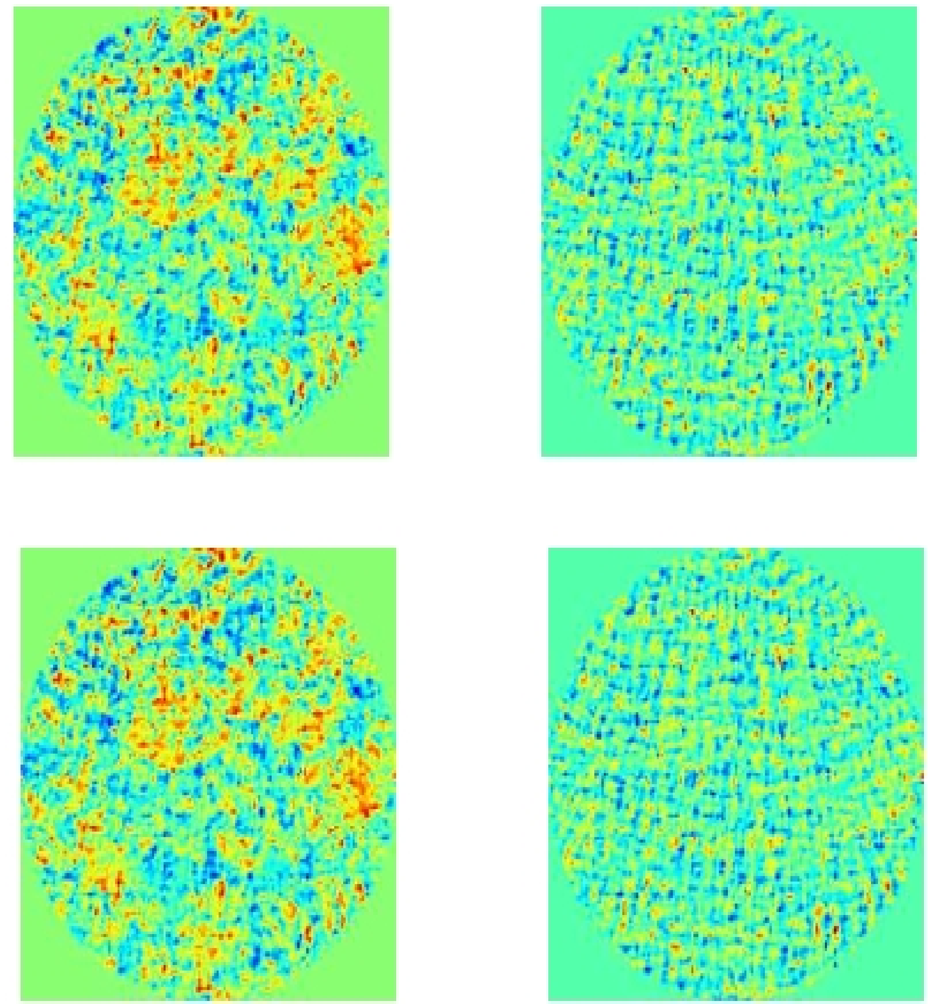
an example of the ICA nominal performance

- warnings:
- no noise
- no systematics
- one foreground only
- uniform spectral index
- ...
- but it's worth studying this (NASA LTSA 2004-2009)



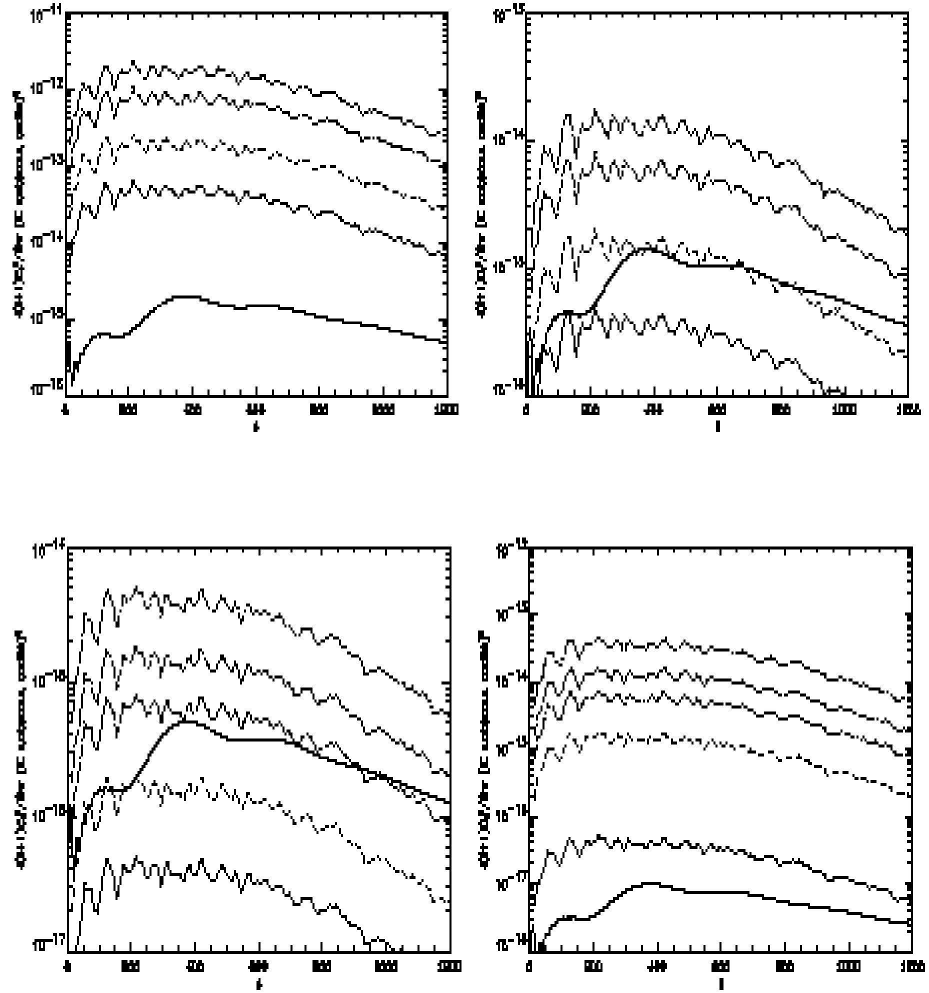
quantitative analysis

- Stivoli et al. (2006) considered a case which might be of interest for forthcoming probes
- two frequency combinations, 40, 90 GHz and 150, 350 GHz
- BOOMERANG, EBEx patch centered at RA=62^{o}, dec=-50^{o}
- 1% of the sky
- Gaussian, uniform noise comparable to the CMB signal

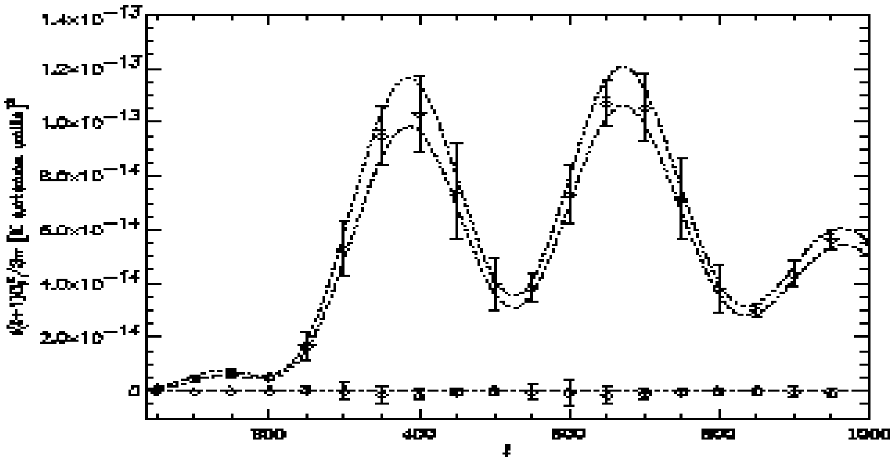


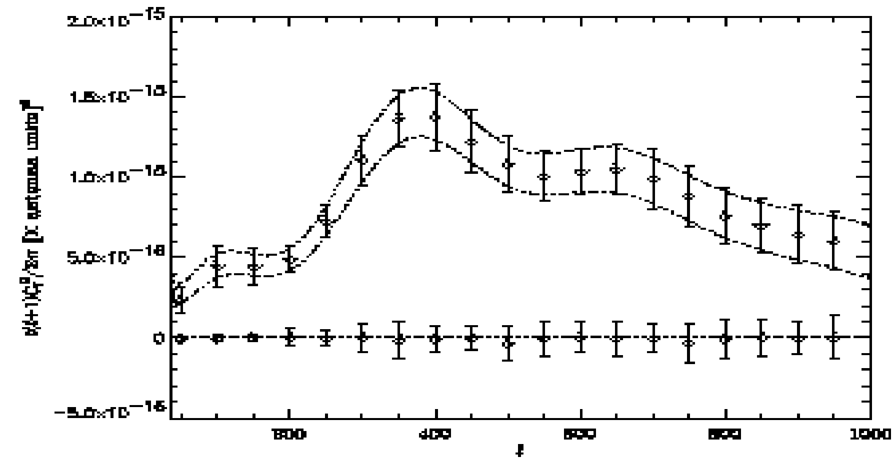
quantitative analysis

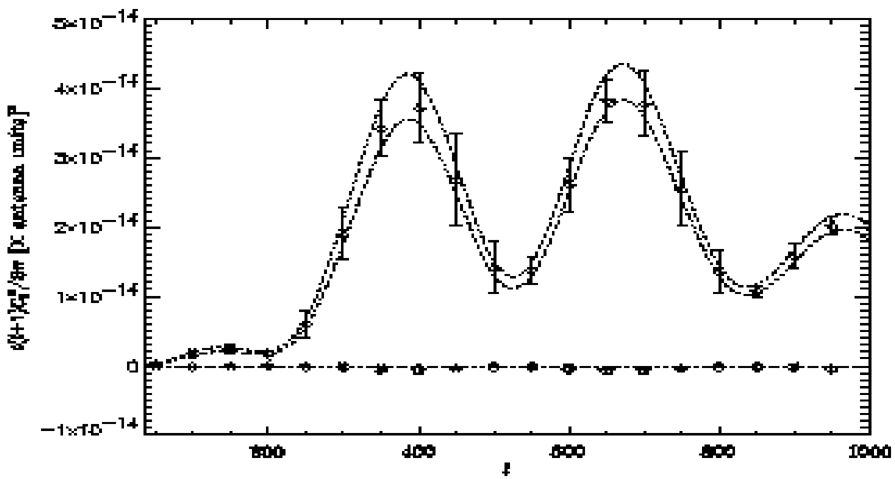
- marked foreground contamination for B modes
- foreground fluctuation amplitude artificially increased to study the stability of the results
- variation of other parameters, sky area, noise amplitude, ...

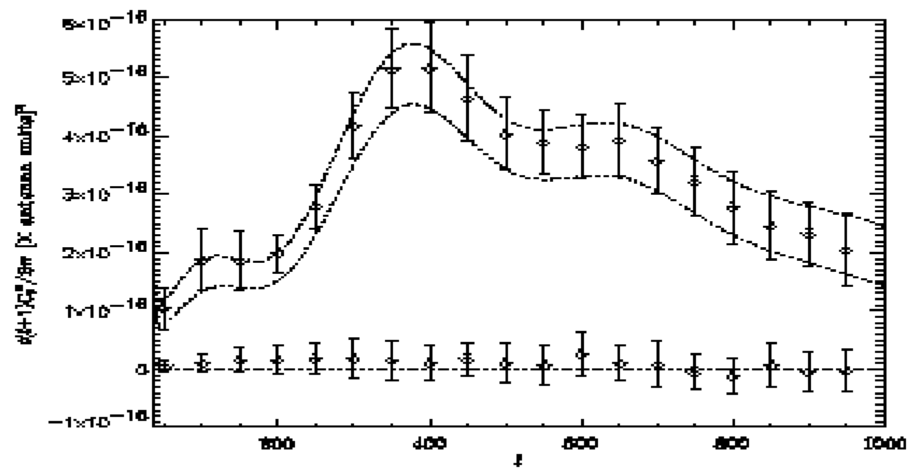


quantitative analysis









quantitative analysis

- work in progress:
- is ICA able to recover the non-Gaussianity from lensing or primordial distortion (ricciardi et al.)
- relevant systematics from specific instruments, (EBEx, stivoli et al.)
- minimum detectable r in increasingly realistic conditions
- ...don't know if this will be useful or not, but worth studying!