Cosmic Microwave Background Polarisation:

Foreground Constrast & Component Saparation

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Total Intensity Diffuse Galactic Foregrounds

Synchrotron

Haslam et al. (1982): all sky, 408 MHz, 1° resolution
Reich & Reich (1986): northern sky, 1.4 GHz, $36'$
Jonas et al. (1998): southern sky, 2.3 GHz, $20'$

Free-Free

Finkbeiner (2003): all sky $\mathrm{H}\alpha$ emission, 0.65 μm, $6'$

Thermal Dust

Schlegel, Finkbeiner, Davis (1998): all sky, 100 μm, $6'$
Finkbeiner et al. (1999): updated microwave frequency scaling

WMAP

Maximum Entropy Based Component Separation using the above data as priors, Bennett et al. 2003

Polarised Diffuse Galactic Foregrounds: Synchrotron

Radio Band Polarisation Database:

Brouw & Spoelstra 1976: 1° resolution, half sky

Duncan et al., Uyaniker et al. 1999: 10 arcmin. resolution, $|b| \leq 15°$

Works:

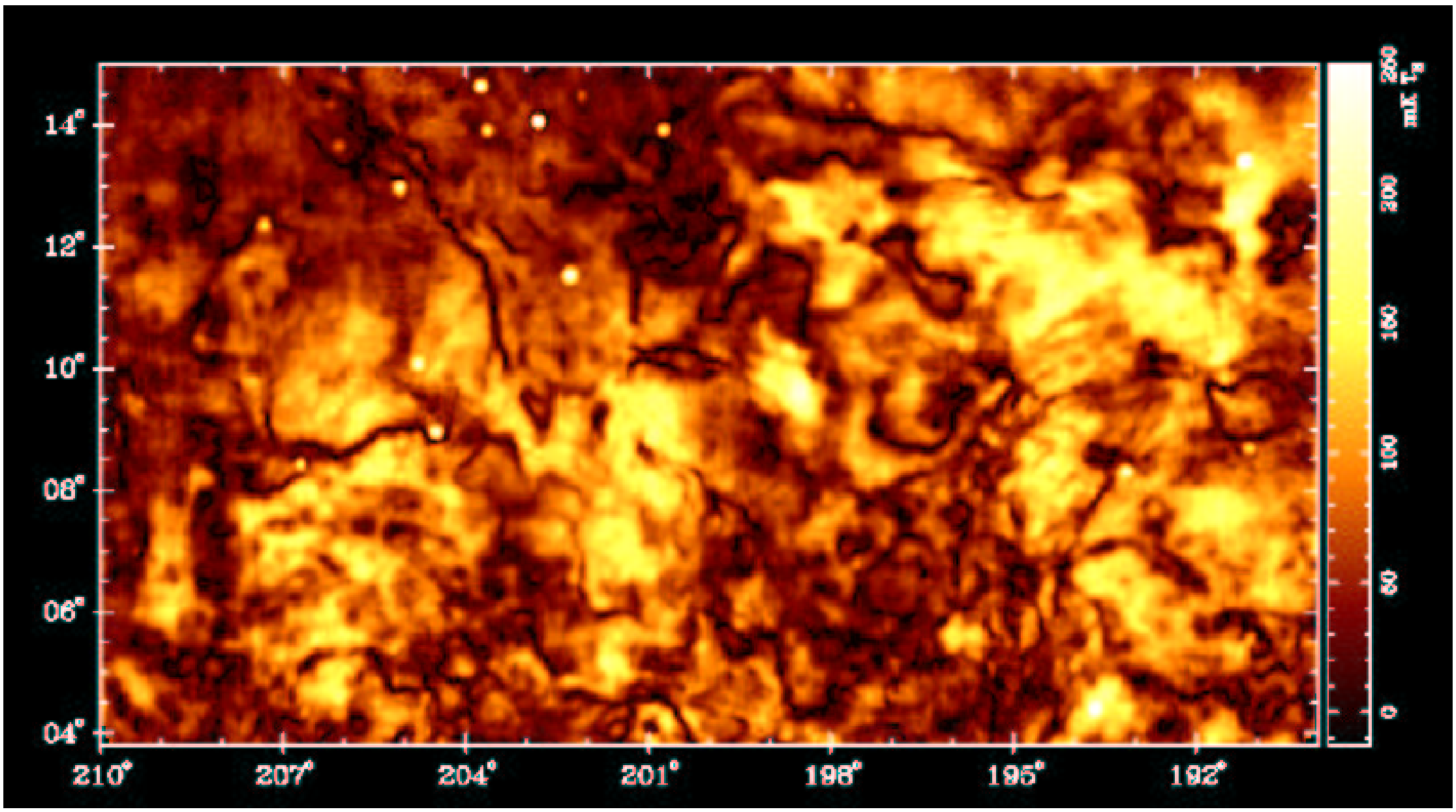
Tegmark et al., Tucci et al. 2000: first evidence for $C_\ell^{E,B} \propto \ell^{-1.5 \div -2}$

Baccigalupi et al. 2001: slope and amplitude relatively independent on the sky region: in particular, no amplitude decrease up to the highest $|b|$; Faraday depolarisation relevant, but not substantial

Giardino et al. 2002: polarisation angle analysis, sky synchrotron template assuming theoretical polarisation level in Haslam et al. synchrotron template

Tucci et al. 2002 find that $C_\ell^{E,B} \propto \ell^{-1.5 \div -2}$ hold up to arcmin. scales.

A snapshot of the diffuse Galactic synchrotron emission

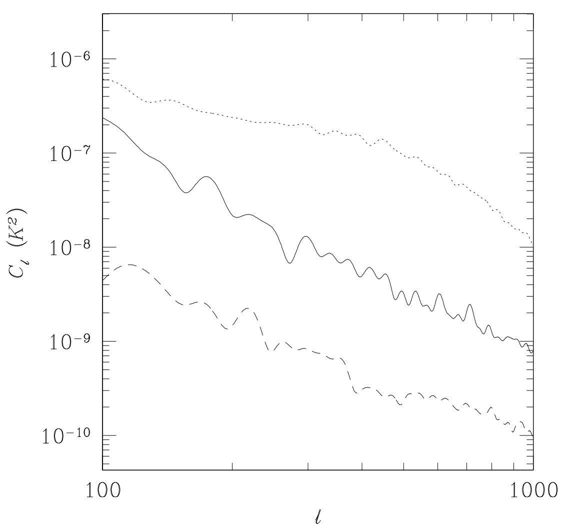


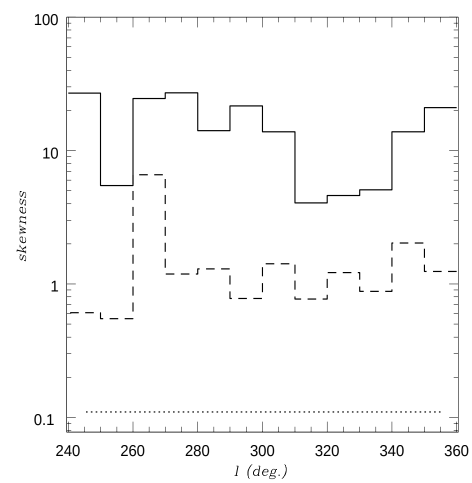
Baccigalupi et al. (2001): synchrotron is synchrotron

Claiming Moderate Faraday Depolarisation
The Fan Region signal is the same as in other directions
The high total vs. polarised intensity on the plane is due to unpolarised HII regions (Paladini et al. 2003)

Marked non-Gaussianity

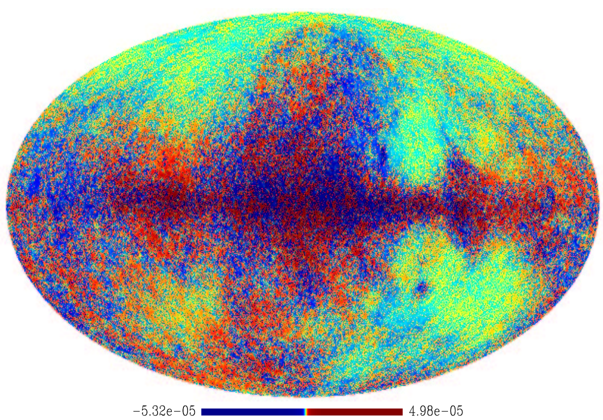
Two Angular Regimes
The super-degree power is steeper ($\sim \ell^{-3}$) than the sub-degree one, ($\sim \ell^{-1.5 \div -2}$) but they match at the crossover scale

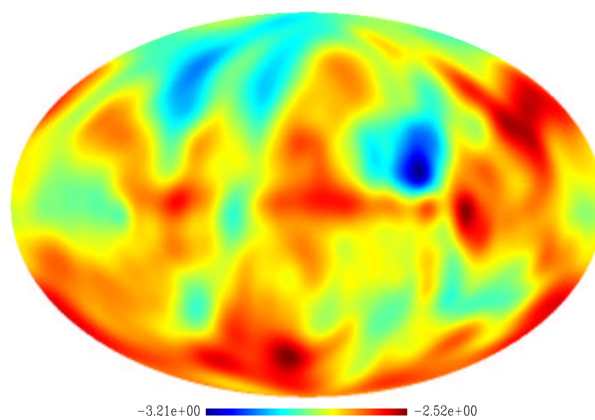


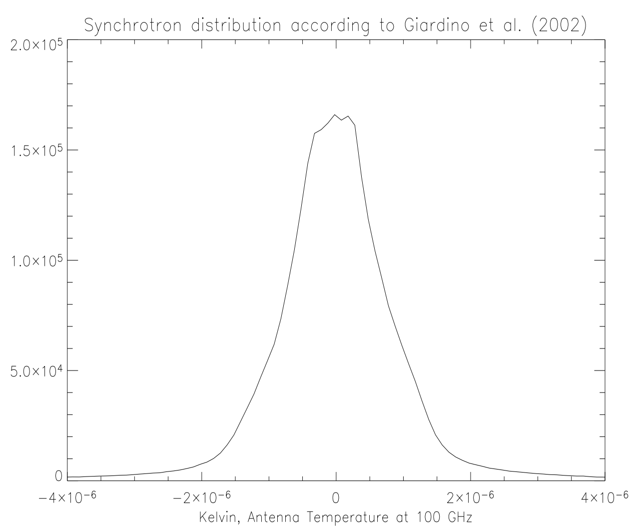


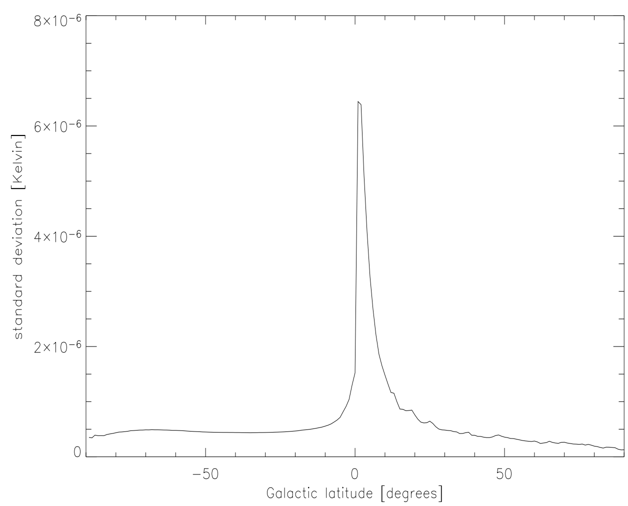
Giardino et al. (2002): it's all synchrotron

Compute $C_\ell^{E,B}$ of cos and sin of 2×polarisation angle
Build a Gaussian template out of that average power spectrum and convolve with the Haslam et al. (1982) intensity theoretically polarised at 75%
Scale in frequency matching Haslam et al. (1982) and Reich et al. (1986)



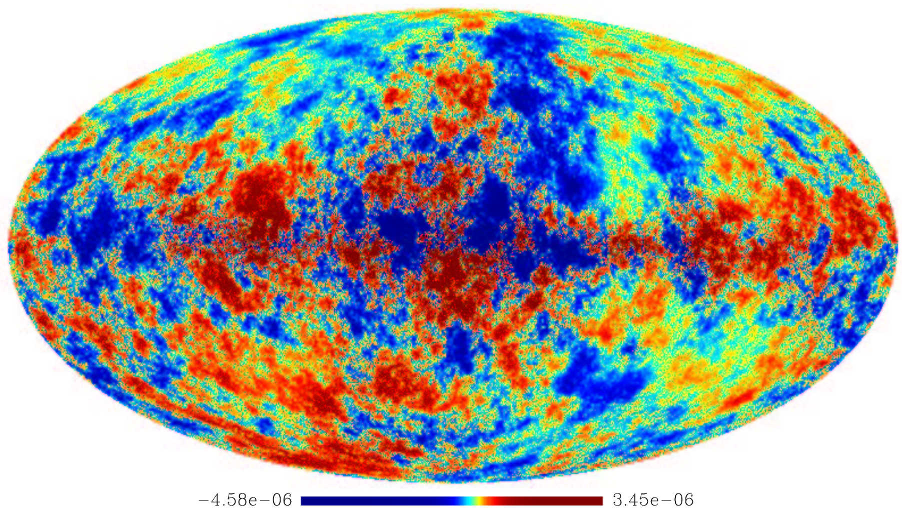


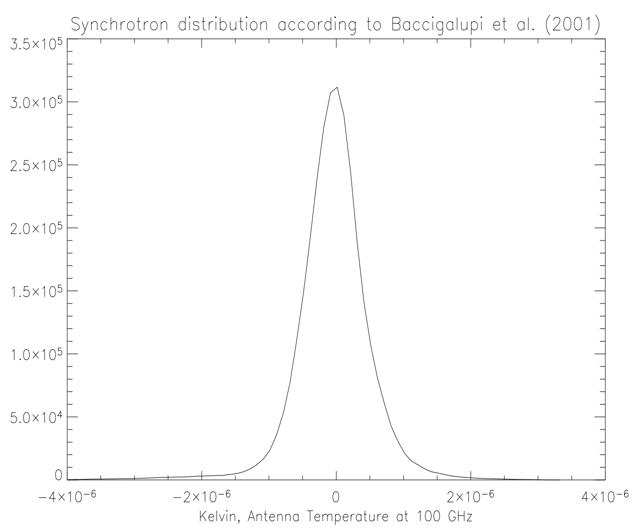


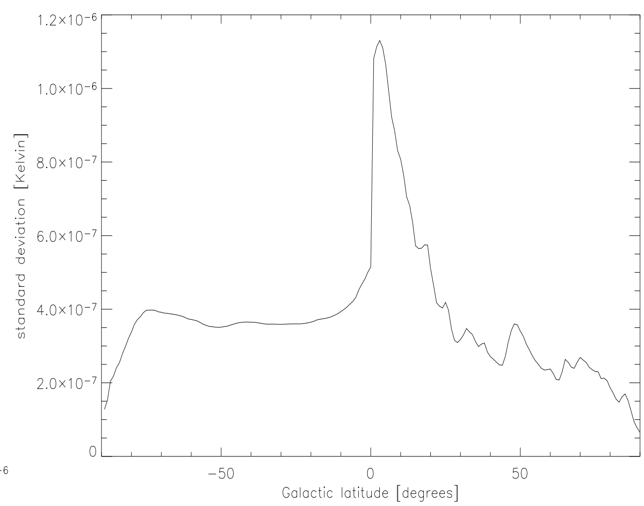


Baccigalupi et al. (2001): yes it is

Scale the angular power in the Giardino et al. (2002) template to match the desired one







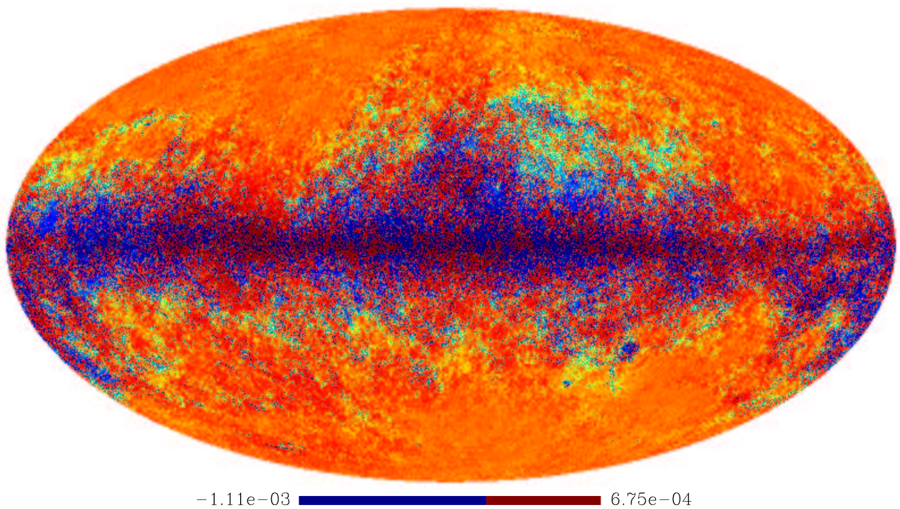
Polarised Diffuse Galactic Foregrounds: Dust

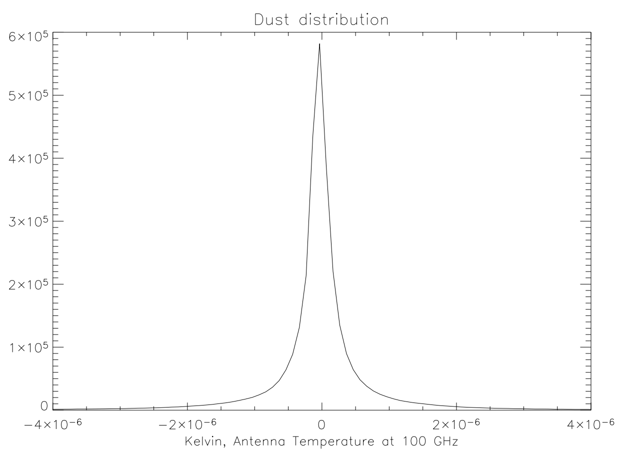
$3 \div 5\%$ polarisation degree detected by Archeops on the Galactic plane

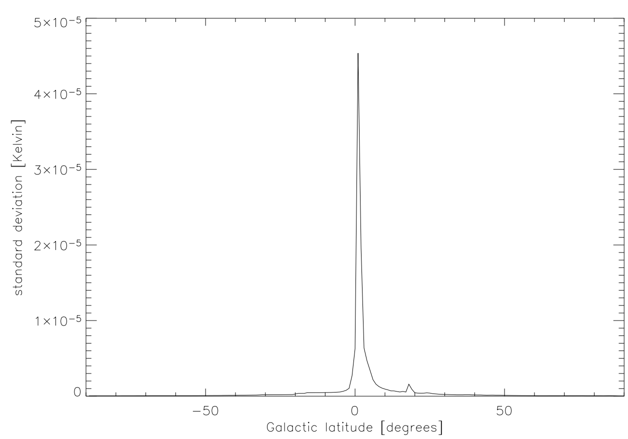
Assume it constant over the template by Finkbeiner et al. (1999)

Assume the same polarisation angle pattern as synchrotron

Resulting angular power specrum in agreement with earlier estimates (Prunet et al. 2000)







CMB Contamination

Frequency Scalings

CMB fluctuations $\sim x^2 \exp(x)/[\exp(x){-}1]^2$, $x = h\nu/kT_{CMB}$

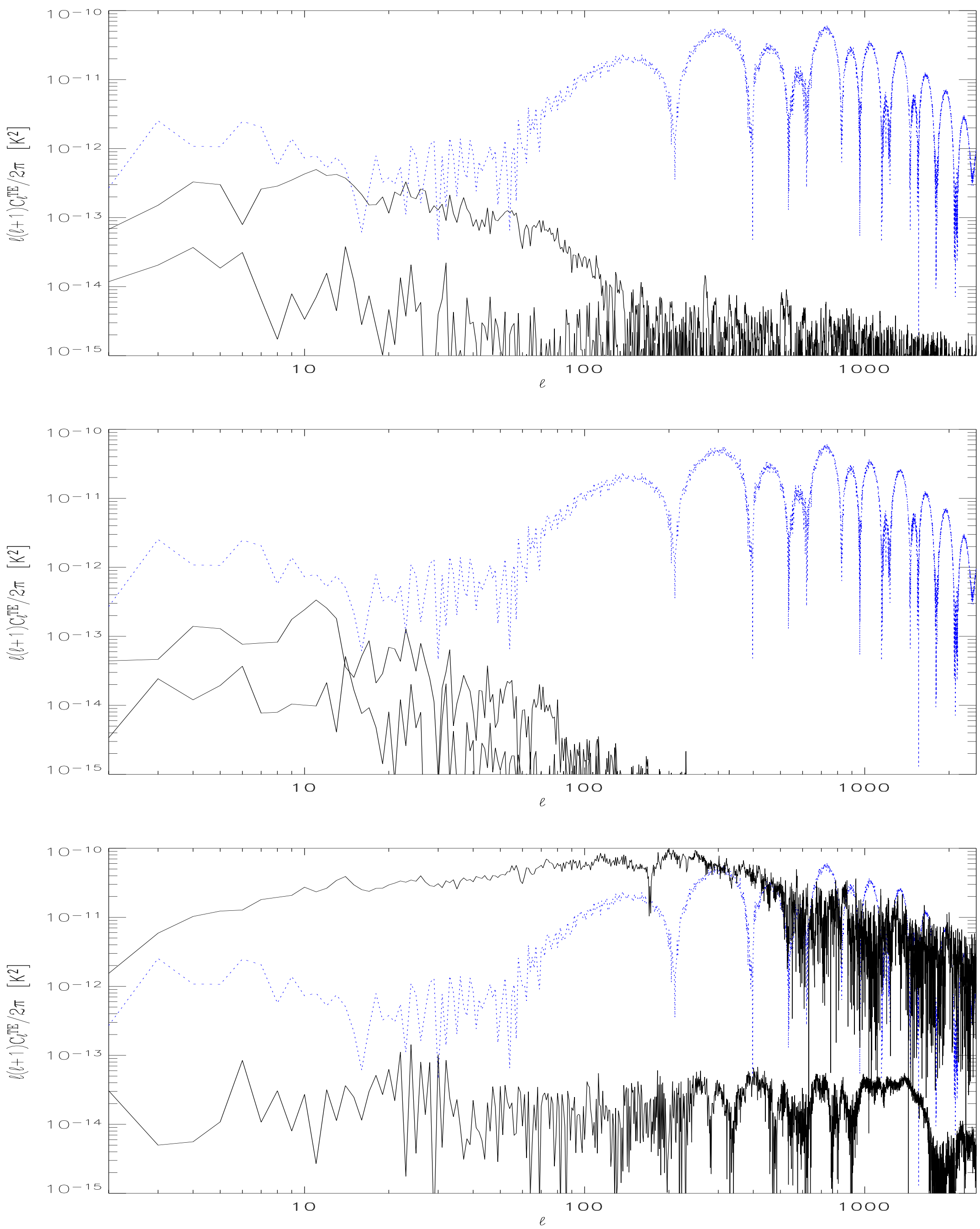
Synchrotron $\sim \nu^{-2.75}$

Dust, two component model (Finkbeiner et al. 2003), well fitted by $y^{4.7}/[\exp(y) - 1]$, $y = h\nu/kT_d$, $T_d = 18$ K

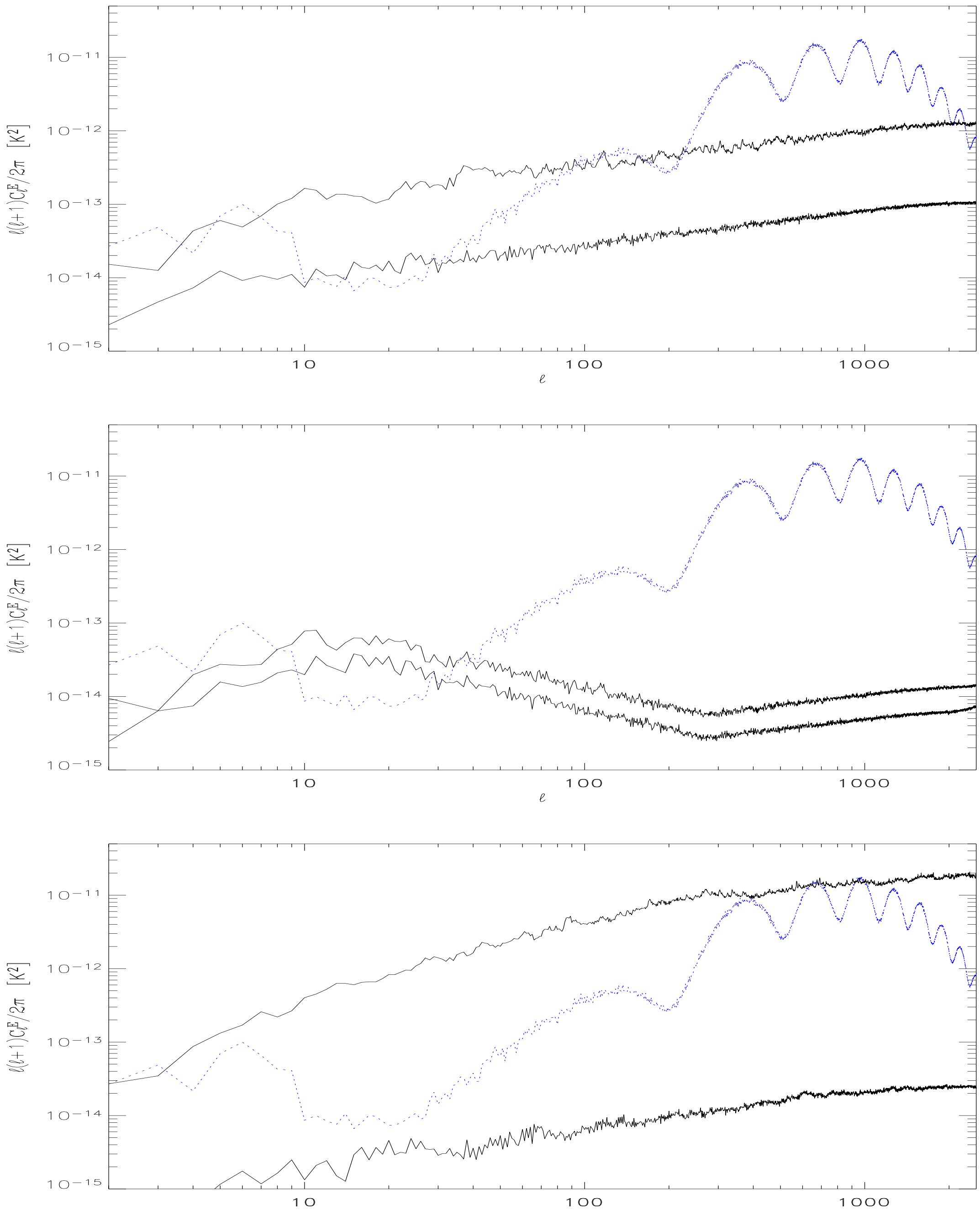
Minimum contamination in the range 70-150 GHz

WMAP first year: evidence for steeper synchrotron and dust, minimum contamination at the lower bound

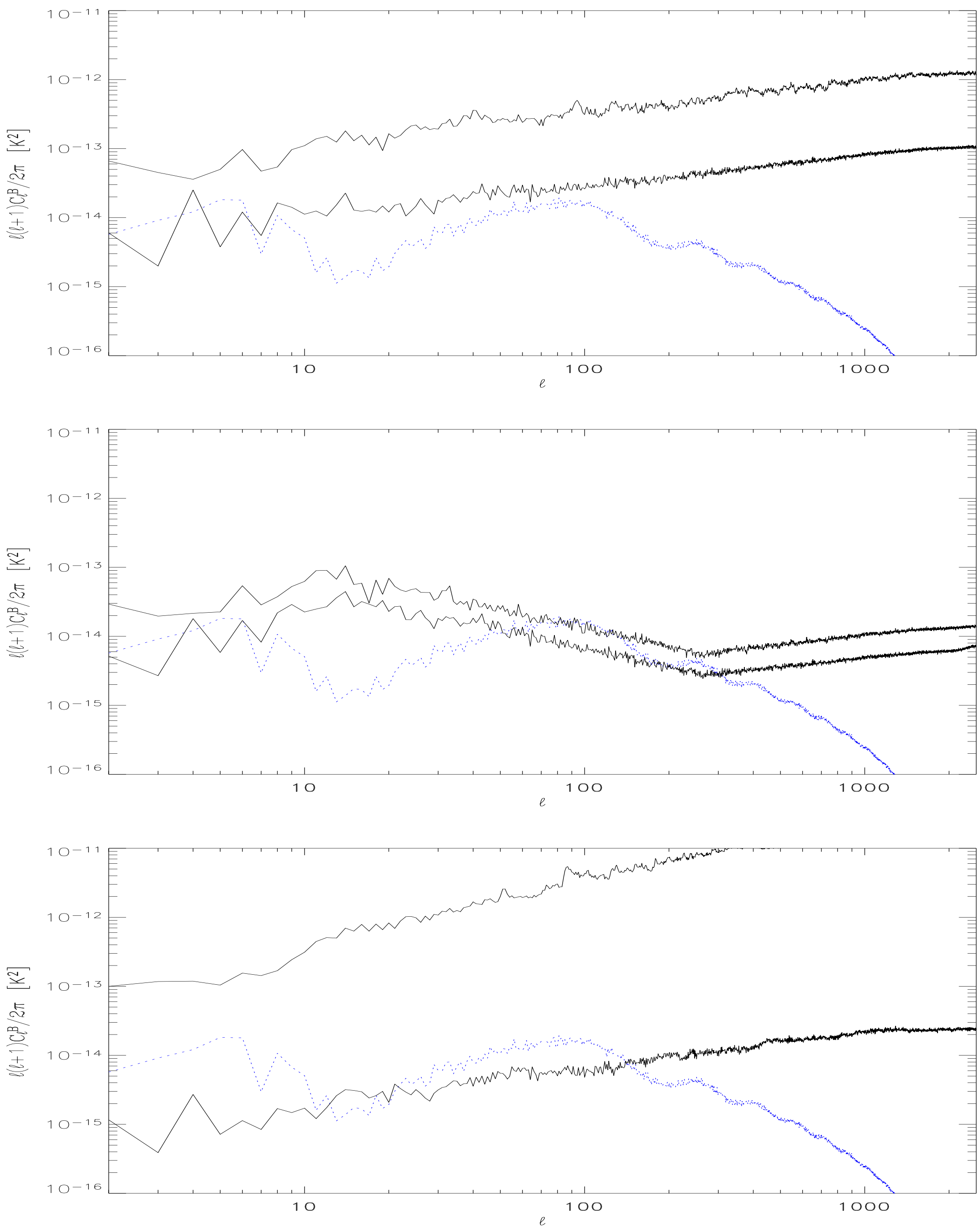
TE Contamination to the CMB



E Contamination to the CMB



B Contamination to the CMB



Component Separation

$$\mathbf{x}(n_f, n_p) = \mathbf{B} \cdot \mathbf{A}(n_f, n_c) \cdot \mathbf{s}(n_c, n_p) + \mathbf{n}(n_f, n_p)$$

$\mathbf{x}$: data at the n_f frequencies
$\mathbf{s}$: the n_c emission processes to recover
$\mathbf{n}$: the instrumental noise
$\mathbf{A}$: mixing matrix
$\mathbf{B}$: beam convolution
n_p: the number of observed pixels or harmonic modes

Goal: find $\mathbf{s}$

Work Plan:
Find some algorithm which works
Implement it in your data analysis

Blind & non-Blind Approaches

$$\mathbf{x}(n_f, n_p) = \mathbf{B} \cdot \mathbf{A}(n_f, n_c) \cdot \mathbf{s}(n_c, n_p) + \mathbf{n}(n_f, n_p)$$

Non-Blind: all the instrumental features are known and under control, $\mathbf{A}$ is known, and also the power spectra of all but one component in $\mathbf{s}$

Blind: recover $\mathbf{s}$ assuming their statistical independence

Blind & non-Blind: all the instrumental features are known and under control

Total & Polarised Intensity A full non-Blind approach for polarisation is unlikely, due to the lack of reliable priors

Non-Blind Works, Total Intensity:
Tegmark & Efstathiou 1996, Bouchet & Gispert 1998, Hobson et al. 1998, Stolyarov et al. 2001

Blind Works, Total Intensity, Polarisation:
Baccigalupi et al. 2000, Delabrouille et al. 2002, Maino et al. 2002, Delabrouille et al. 2003, Baccigalupi et al. 2003

On Data (COBE), Non-Blind, Blind:
Barreiro et al. 2003, Maino et al. 2003

The ICA Concept: the Central Limit Upside-down

Assume Statistical Independence
The statistics of a mix of statistical independent components tends to be Gaussian

Reverse Path
Combine linearly the data to maximize a suitable measure of non-Gaussianity

Extract and Iterate
If the hypotheses are verified, the maxima correspond to the independent components present into the data

Relevant Works:
Hyvärinen & Oja 2000, Neural Networks 13, 411
Amari & Chichocki 1998, Proc. IEEE 86, 2026

FastICA Main Loop

out of $\mathbf{x} = \mathbf{B}\cdot\mathbf{A}\cdot\mathbf{s} + \mathbf{n}$ find $\mathbf{W}$ such that $\mathbf{W}\cdot\mathbf{x} = \mathbf{B}\cdot\mathbf{s} + \mathbf{W}\cdot\mathbf{n}$

Whitening: $\mathbf{C} = E\{\mathbf{x}\mathbf{x}^T\}$, $\mathbf{\Sigma} = E\{\mathbf{n}\mathbf{n}^T\}$,
$\hat{\mathbf{\Sigma}} = (\mathbf{C} - \mathbf{\Sigma})^{-1/2}\mathbf{\Sigma}(\mathbf{C} - \mathbf{\Sigma})^{-1/2}$, $\hat{\mathbf{x}} = (\mathbf{C} - \mathbf{\Sigma})^{-1/2}\mathbf{x}$

Row by Row:

1. Choose an initial vector $\mathbf{w}$;

2. update it through

$$\mathbf{w}_{new} = E\{\hat{\mathbf{x}}g(\mathbf{w}^T\hat{\mathbf{x}})\} - (I + \hat{\mathbf{\Sigma}})\mathbf{w}E\{\ g'(\mathbf{w}^T\hat{\mathbf{x}})\}$$

   where $g(u) = u^3$, $g(u) = \tanh u$, $g(u) = \exp(-u^2)$...

3. let

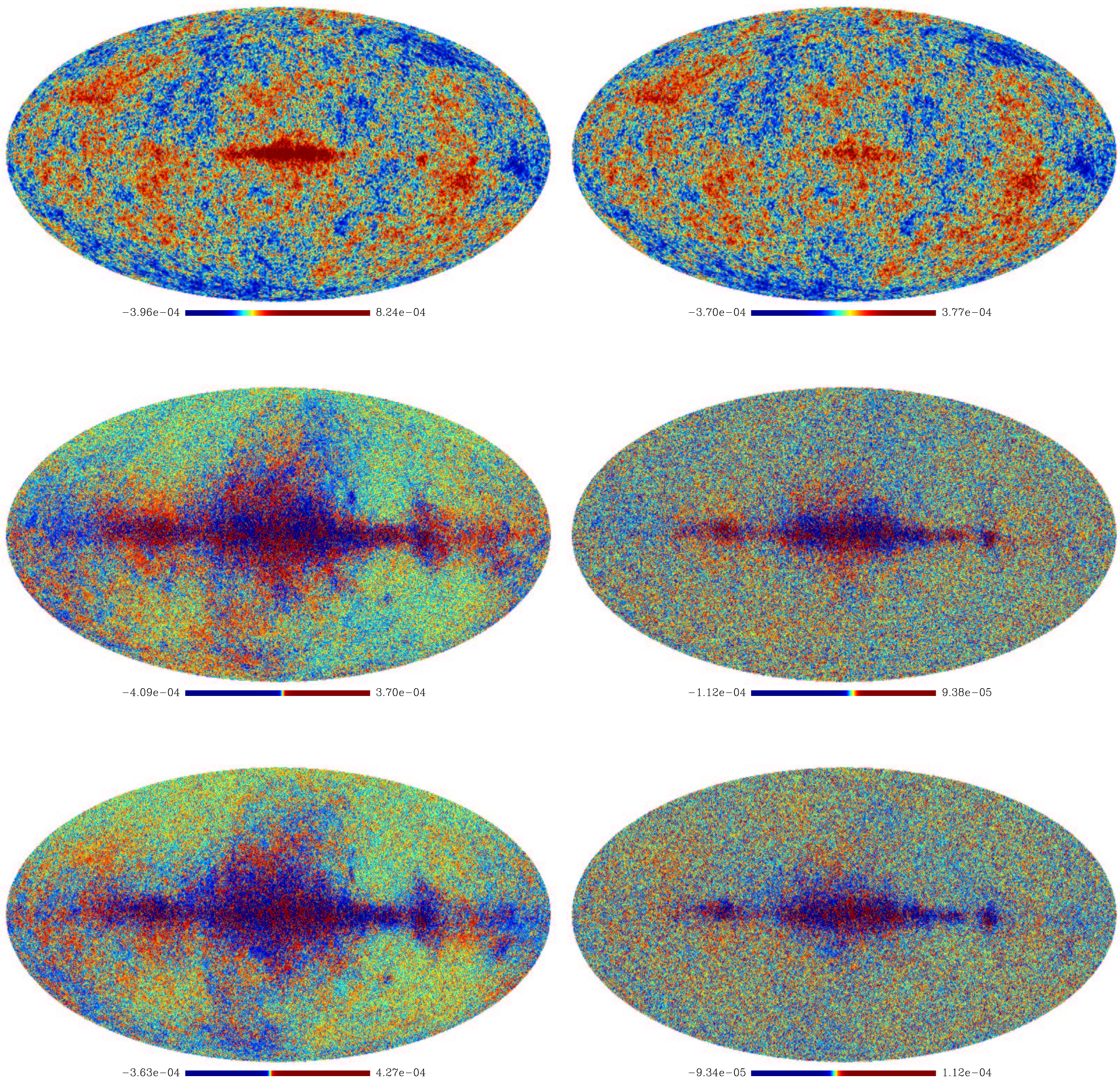
$$\mathbf{w}_{new} = \frac{\mathbf{w}_{new}}{||\mathbf{w}_{new}||}$$

4. Compare $\mathbf{w}_{new}$ with the old one; if not converged, go back to 2, if converged, begin another process keeping orthogonality between rows

Computationally Costless:
A few full sky channels at 3.5' resolution run in a few minutes on a normal PC

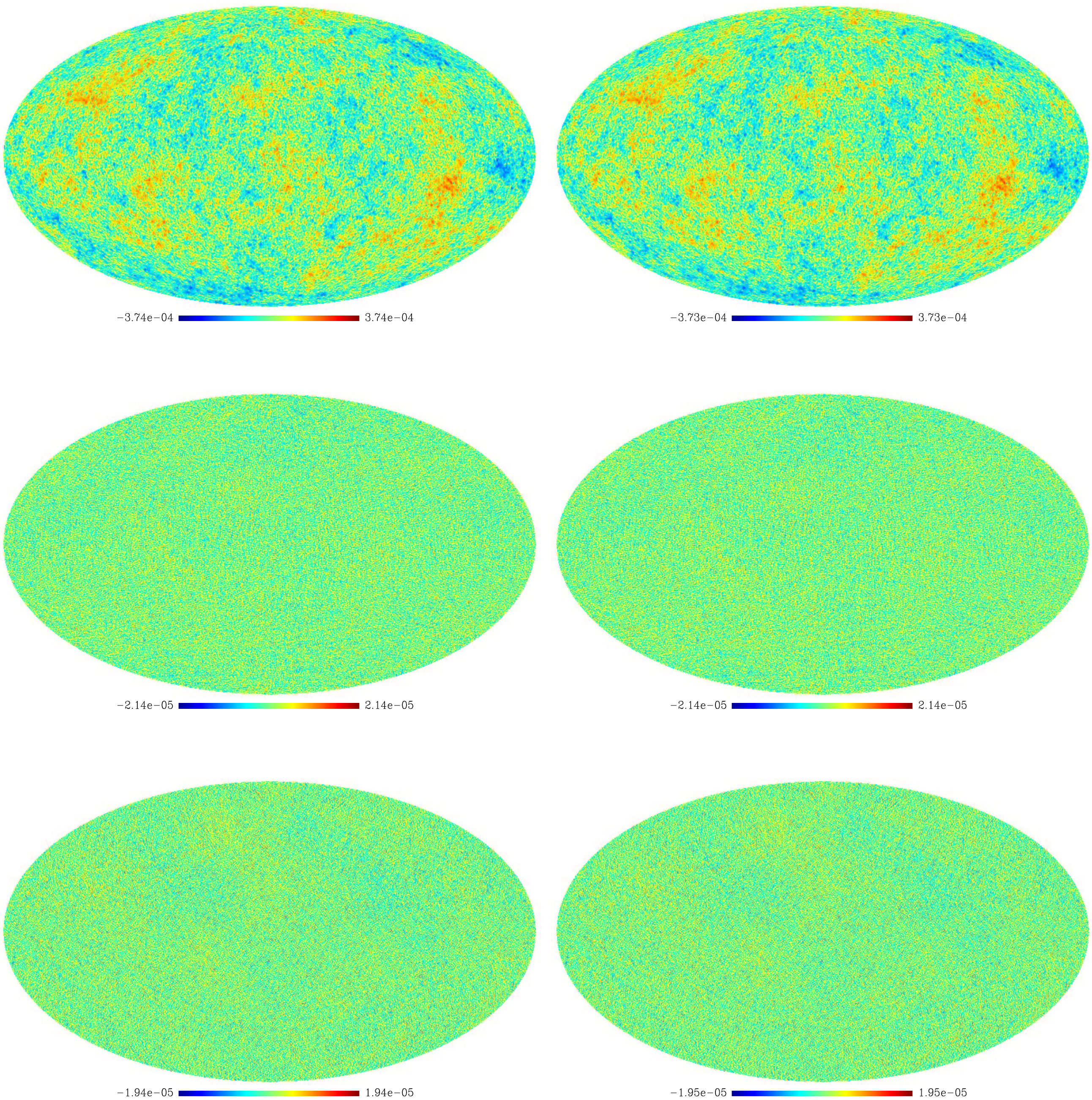
The impressive FastICA nominal performance

Mix CMB and Synchrotron at 50 and 80 GHz:

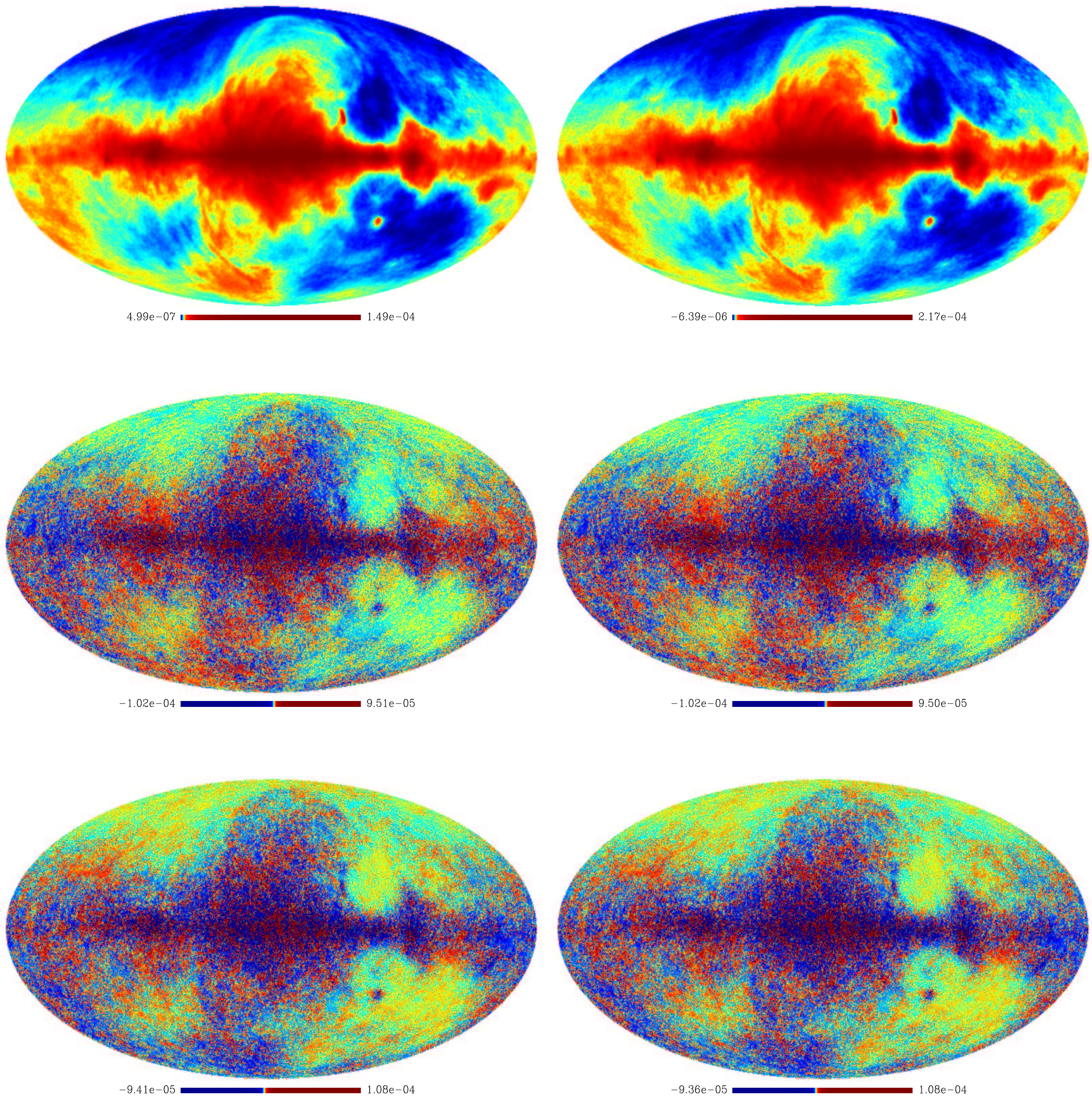


Ask the code to recover the independent components

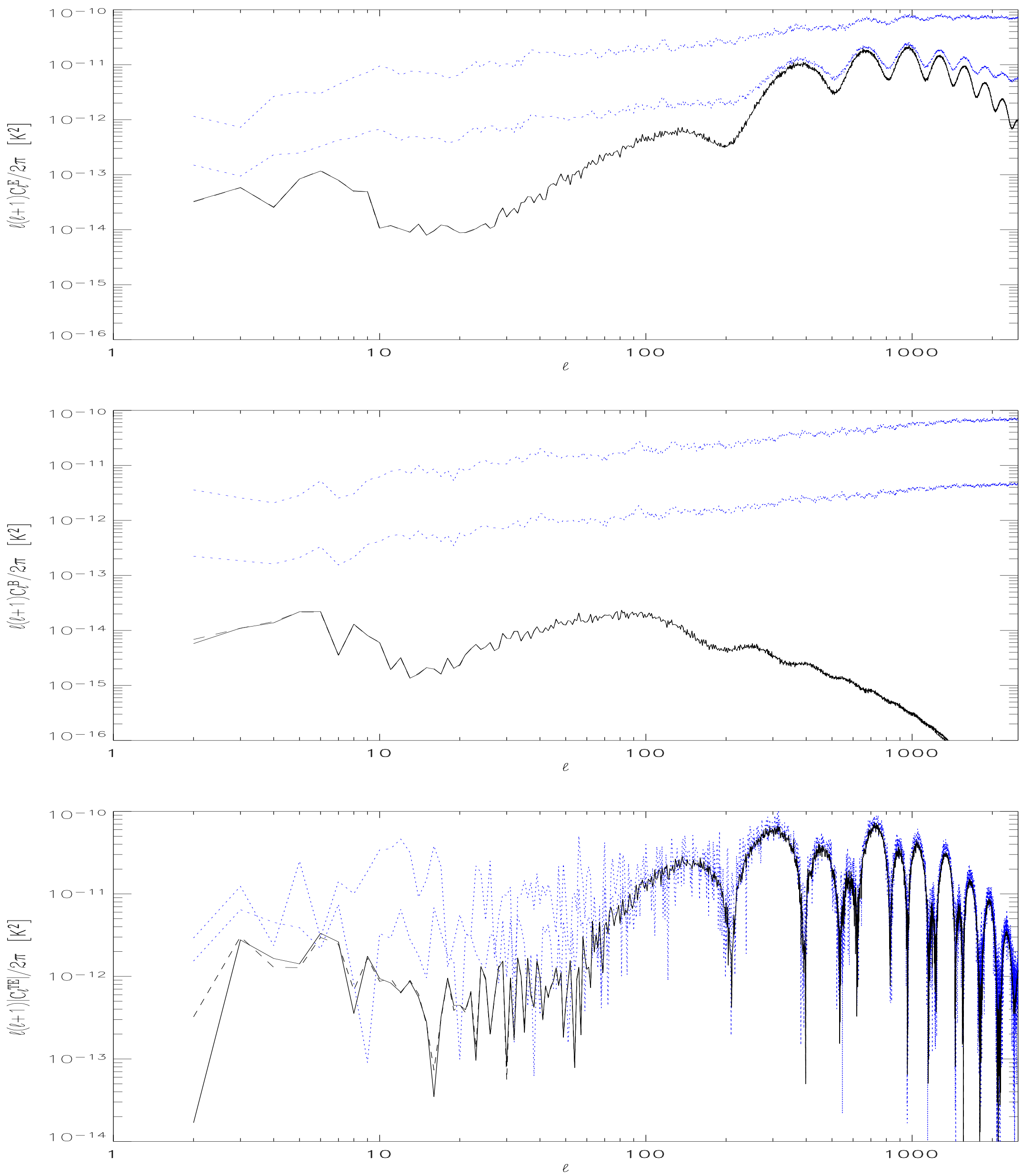
FastICA Input/Output: CMB



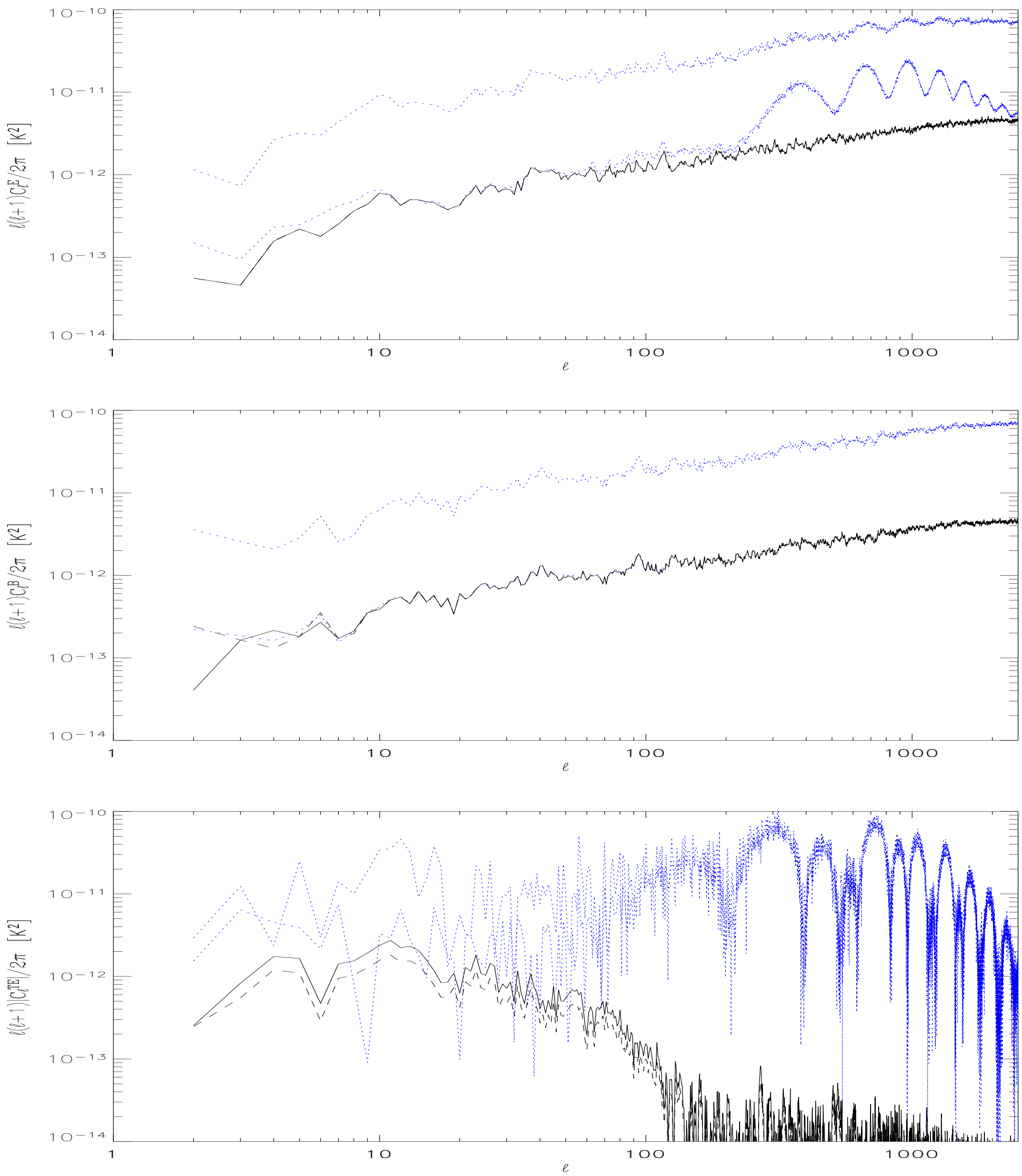
FastICA Input/Output: Synchrotron



FastICA Input/Output: CMB Angular Power Spectra



FastICA Input/Output: Synchrotron Angular Spectra



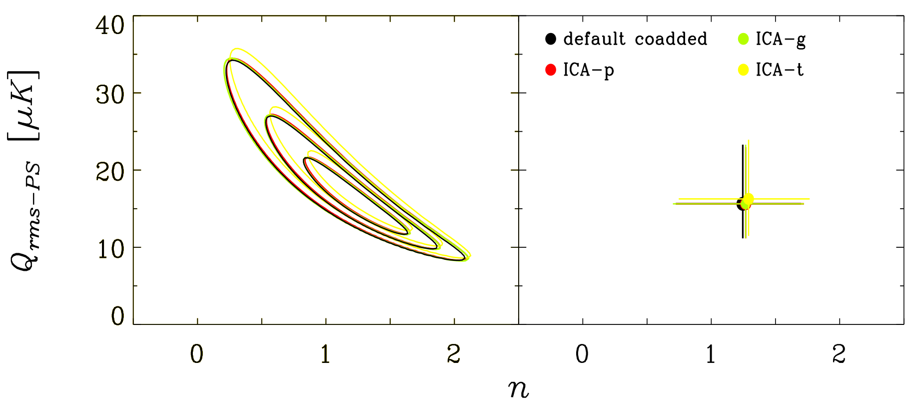
Accomplishments

Baccigalupi et al. 2000
Adaptive ICA on sky patches

Maino et al. 2002
FastICA performance with the Planck nominal features: recovering all sky CMB up to $\ell \simeq 2000$

Baccigalupi et al. 2003
Component Separation in polarisation, FastICA with the Planck nominal features: contamination removal on super-degree angular scales, B modes detectable if they are at least 30% of the density perturbations

Maino et al. 2003
FastICA does not crash on COBE! Instead, it reproduces correct normalization and spectral index of primordial perturbations, with no more CMB map though



Frontiers

No Point in Not Having These Tools in Your Toolkit

Effective, Fast & Computationally Costless

Object-Oriented, User-Friendly Public Version Desirable...

WMAP

Ideal Systematics Gym: check performance in presence of non-stationary noise, realistic beams, space varying spectral indexes, ...

Unveil New CMB: avoid large sky cut when looking for CMB science, check WMAP results independently, gain insight into the CMB signal statistics, ...