Exercise 1: From the action of a point particle $S = -m\int d\tau$, derive the geodesics equations by varying with respect to the trajectory, and the stress energy tensor by varying with respect to the metric. Show that the geodesics equations are equivalent to the covariant conservation of the stress energy tensor.

Exercise 2: Consider a Keplerian binary (i.e. a binary with large separation, for which the laws of Newtonian mechanics are applicable) on a circular orbit on the (x, y) plane, and an observer (far from the source) along the z axis. Compute the gravitational-wave signal according to the "Green" formula of the lectures, and according to the quadrupole formula. Show that the amplitudes of the two predictions differ by a factor 2.

Exercise 3: By using the ansatz

$$\begin{aligned}
g_{00} &= -1 - 2\frac{\phi}{c^2} + \mathcal{O}\Big(\frac{1}{c^4}\Big), \\
g_{0i} &= \mathcal{O}\Big(\frac{1}{c^3}\Big), \\
g_{ij} &= \Big(1 - 2\frac{\phi}{c^2}\Big)\delta_{ij} + \mathcal{O}\Big(\frac{1}{c^4}\Big), \qquad (1)
\end{aligned}$$

(with $x^0 = ct$ and x^i Cartesian spatial coordinates) in the relaxed form of the Einstein equations (eqs 2.3 through 2.7 of gr-qc/0007087), show that

$$\begin{aligned}
\tau^{00}(t, \boldsymbol{x}) &= \sum_A m_A c^2 \delta^3\left(\boldsymbol{x} - \boldsymbol{x}_A(t)\right) + \mathcal{O}\Big(\frac{1}{c^0}\Big), \qquad (2) \\
\tau^{0i}(t, \boldsymbol{x}) &= \sum_A m_A c\, \dot{x}^i_A \delta^3\left(\boldsymbol{x} - \boldsymbol{x}_A(t)\right) + \mathcal{O}\Big(\frac{1}{c}\Big), \qquad (3) \\
\tau^{ij}(t, \boldsymbol{x}) &= \sum_A m_A \dot{x}^i_A \dot{x}^j_A \delta^3\left(\boldsymbol{x} - \boldsymbol{x}_A(t)\right) \\
&+ \frac{1}{4\pi G}\Big(\partial^i\phi\,\partial^j\phi - \frac{1}{2}\delta^{ij}\partial_k\phi\,\partial^k\phi\Big) + \mathcal{O}\Big(\frac{1}{c^2}\Big), \qquad (4)
\end{aligned}$$

Exercise 4: Using Maple, Mathematica or similar, consider an interferometer on the (x, y) plane and a gravitational wave coming from the sky position (ϕ, θ) (spherical coordinates). Define the wave plus and cross polarizations with respect to two unit vectors $\boldsymbol{e}_x$ and $\boldsymbol{e}_y$ orthogonal to the propagation direction $\boldsymbol{n}$, and such that the triad $(\boldsymbol{e}_x, \boldsymbol{e}_y, \boldsymbol{n})$ is right-handed. Show that the detector's pattern functions are

$$F_+ = \frac{1}{2}\cos 2\zeta\left(\cos^2\theta + 1\right)\cos 2\phi - \sin 2\zeta\cos\theta\sin 2\phi$$

and

$$F_\times = \frac{1}{2}\sin 2\zeta\left(\cos^2\theta + 1\right)\cos 2\phi + \cos 2\zeta\cos\theta\sin 2\phi$$

where $\zeta < \pi/2$ is the angle between $\boldsymbol{e}_y$ and the intersection between the plane of the detector and the plane orthogonal to the wave propagation direction.