XII Tonale Winter School in Cosmology

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December 2018

1 M33 X-7: natal or accretion produced spin?

The dimensionless energy and angular momentum of a particle with mass m at the ISCO of a BH with spin a and mass M are

$$\tilde{E}_{\rm ISCO}(a) = \sqrt{1 - \frac{2}{3 r_{\rm ISCO}(a)}}\,, \tag{1}$$

$$\tilde{L}_{\rm ISCO}(a) = \frac{2}{3\sqrt{3}} \left[1 + 2\sqrt{3 r_{\rm ISCO}(a) - 2}\right]\,, \tag{2}$$

$$r_{\rm ISCO}(a) = 3 + Z_2 - \frac{a}{|a|}\sqrt{(3 - Z_1)(3 + Z_1 + 2Z_2)}\,, \tag{3}$$

$$Z_1 = 1 + (1 - a^2)^{1/3}\left[(1 + a)^{1/3} + (1 - a)^{1/3}\right]\,, \tag{4}$$

$$Z_2 = \sqrt{3a^2 + Z_1^2}\,. \tag{5}$$

where $\tilde{E}_{\rm ISCO} \equiv E_{\rm ISCO}/m$ and $\tilde{L}_{\rm ISCO} \equiv L_{\rm ISCO}/(mM)$.

Using these formulae, compute the accretion rate corresponding to the Eddington luminosity $L_{\rm edd} = 1.26 \times 10^{38}$ erg/s. Then compute how long it would take to spin up (via coherent accretion) the BH candidate M33 X-7 to its current spins ($a = 0.84$) from $a = 0$. Also compute what needs to be the initial mass of the BH considering that its present day mass is $15.65 M_\odot$.

Knowing that the stellar companion (with mass $\approx 70 M_\odot$) is not older than 2-3 million years, is the spin of M33 X-7 natal or accretion produced?

2 Chaotic vs Coherent accretion

Calculate the time evolution of mass, spin and radiative efficiency for a BH with initial mass $M = 10^4 M_\odot$ and initial spin $a = 0$. Consider both the case in which accretion takes place coherently (on a thin prograde accretion disk) and chaotically (with half of the mass accreting on prograde orbits and half on retrograde orbit). Compare this to the compilation of spin measurements for massive BHs shown in the lectures and draw your conclusions on which accretion scenario is most likely.

3 The final spin of the BH remnant of GW150914

Consider a "particle" falling into a much larger BH from a quasicircular equatorial inspiral. Show that if one neglects the energy loss via GWs the final spin of the BH is

$$a_{\rm fin} = \frac{1}{(1+q)^2}\left(a_1 + a_2 q^2 + \tilde{L}_{\rm ISCO}(a_1) q\right) \tag{6}$$

where $q = m_2/m_1 \leq 1$. Compute the final spin for $|a_1| = |a_2| = 0$ extrapolating to the case $q = 1$. Iterate the procedure by using the final spin so obtained in the ISCO angular momentum in place of a_1, and proceed until the final spin has converged. Compared to the final spin of 0.68 predicted by numerical relativity simulations of the merger of two equal mass non-spinning BHs. Interpret the result in terms of the "effective one-body" model discussed in the lectures.

Generalize the calculation to isotropically distributed spins. Use that fact that for a particle falling into the BH from an orbit with angle $\hat{\mathbf{L}} \cdot \hat{\mathbf{a}}_1 = \cos\iota$

$$\tilde{L}_{\rm ISCO} = |\tilde{L}_{\rm ISCO}(\iota = 0)| \frac{1+\cos\iota}{2} + |\tilde{L}_{\rm ISCO}(\iota = \pi)| \frac{1-\cos\iota}{2} \tag{7}$$

and the fact that the resulting formula must be symmetric under exchanges of the two BHs.

Consider now a "merger tree" of massive BHs of equal mass $10^5 M_\odot$ merging every $\Delta z = 1$ from $z = 15$ to $z = 0$ into a BH of final mass $\sim 3.3 \times 10^9 M_\odot$ (we are neglecting accretion). Assign randomly oriented initial spins (with uniform distribution between 0 and 0.9) to the BH population and compute the final BH spin at $z = 0$. Perform the same calculation for different realizations/distributions of the initial spins and determine how the results change as function of redshift. [Hint: our simple formula captures the main physics but fails at large spins, so never allow spins to exceed ≈ 0.9]