density-functional perturbation theory

response functions, phonons, and all that

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response functions

$$\text{property} = \frac{\partial(\text{variable})}{\partial(\text{strength})}$$

response functions

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- ☛ polarizability, dielectric constant $\quad \dfrac{\partial \mathsf{P}_i}{\partial \mathsf{E}_j}$
- ☛ elastic constants $\quad \dfrac{\partial \sigma_{ij}}{\partial \epsilon_{kl}}$
- ☛ piezoelectric constants $\quad \dfrac{\partial \mathsf{P}_i}{\partial \epsilon_{kl}}$
- ☛ interatomic force constants $\quad \dfrac{\partial f_i^s}{\partial u_j^t}$
- ☛ Born effective charges $\quad \dfrac{\partial d_i^s}{\partial u_j^s}$
- ☛ $\cdots$ $\quad \cdots$

susceptibilities as energy derivatives

$$\hat{H}_\alpha = \hat{H}^\circ + \alpha \hat{A}$$

$$\chi_{BA} = \frac{\partial \langle \hat{B} \rangle_\alpha}{\partial \alpha}$$

susceptibilities as energy derivatives

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$$\chi_{BA} = \frac{\partial \langle \hat{B} \rangle_\alpha}{\partial \alpha}$$

$$\langle \hat{B} \rangle = \frac{\partial E_\beta}{\partial \beta}$$

(Hellmann & Feynman)

$$\hat{H}_\beta = \hat{H}^\circ + \beta \hat{B}$$

susceptibilities as energy derivatives

$$\hat{H}_\alpha = \hat{H}^\circ + \alpha\hat{A}$$

$$\chi_{BA} = \frac{\partial\langle\hat{B}\rangle_\alpha}{\partial\alpha}$$

$$\langle\hat{B}\rangle = \frac{\partial E_\beta}{\partial\beta}$$

(Hellmann & Feynman)

$$\hat{H}_\beta = \hat{H}^\circ + \beta\hat{B}$$

$$\chi_{BA} = \frac{\partial^2 E_{\alpha\beta}}{\partial\alpha\partial\beta}$$

$$\hat{H}_{\alpha\beta} = \hat{H}^\circ + \alpha\hat{A} + \beta\hat{B}$$

energy derivatives

$$H = H_0 + \sum_i \lambda_i v_i$$

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$$E[\lambda] = E_0 - \sum_i f_i \lambda_i + \frac{1}{2} \sum_{ij} h_{ij} \lambda_i \lambda_j + \cdots$$

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$$H = H_0 + \sum_i \lambda_i v_i$$

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☛ structural optimization & molecular dynamics

energy derivatives

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$$E[\lambda] = E_0 - \sum_i f_i \lambda_i + \frac{1}{2} \sum_{ij} h_{ij} \lambda_i \lambda_j + \cdots$$

- ☛ structural optimization & molecular dynamics
- ☛ (static) response functions
  - elastic constants
  - dielectric tensor
  - piezoelectric tensor
  - ...
- ☛ vibrational modes in the adiabatic approximation
  - interatomic force constants
  - Born effective charges
  - ...

energy derivatives from perturbation theory

$$H = H_0 + \sum_i \lambda_i v_i$$

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energy derivatives from perturbation theory

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$$f_i = - \left. \frac{\partial E}{\partial \lambda_i} \right|_{\lambda=0} = -\langle \Psi_0 | v_i | \Psi_0 \rangle$$

energy derivatives from perturbation theory

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$$f_i = - \left. \frac{\partial E}{\partial \lambda_i} \right|_{\lambda=0} = -\langle \Psi_0 | v_i | \Psi_0 \rangle = \int v_i(\mathbf{r}) \rho_0(\mathbf{r}) d\mathbf{r}$$

energy derivatives from perturbation theory

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$$h_{ij} = \left. \frac{\partial^2 E}{\partial \lambda_i \partial \lambda_j} \right|_{\lambda=0} = 2 \sum_n \frac{\langle \Psi_0 | v_i | \Psi_n \rangle \langle \Psi_n | v_j | \Psi_0 \rangle}{\epsilon_0 - \epsilon_n}$$

energy derivatives from perturbation theory

$$H = H_0 + \sum_i \lambda_i v_i \qquad E[\lambda] = E_0 - \sum_i f_i \lambda_i + \frac{1}{2} \sum_{ij} h_{ij} \lambda_i \lambda_j + \cdots$$

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$$= 2 \langle \Psi_0 | v_i | \Psi'_j \rangle$$

energy derivatives from perturbation theory

$$H = H_0 + \sum_i \lambda_i v_i \qquad E[\lambda] = E_0 - \sum_i f_i \lambda_i + \frac{1}{2}\sum_{ij} h_{ij}\lambda_i\lambda_j + \cdots$$

$$f_i = -\left.\frac{\partial E}{\partial \lambda_i}\right|_{\lambda=0} = -\langle \Psi_0 | v_i | \Psi_0 \rangle = \int v_i(\mathbf{r})\rho_0(\mathbf{r})d\mathbf{r}$$

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$$= 2\langle \Psi_0 | v_i | \Psi_j' \rangle = \int v_i(\mathbf{r})\rho_j'(\mathbf{r})d\mathbf{r}$$

energy derivatives from perturbation theory

$$H = H_0 + \sum_i \lambda_i v_i \qquad E[\lambda] = E_0 - \sum_i f_i \lambda_i + \frac{1}{2} \sum_{ij} h_{ij} \lambda_i \lambda_j + \cdots$$

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$$\begin{aligned} h_{ij} = \left. \frac{\partial^2 E}{\partial \lambda_i \partial \lambda_j} \right|_{\lambda=0} &= 2 \sum_n \frac{\langle \Psi_0 | v_i | \Psi_n \rangle \langle \Psi_n | v_j | \Psi_0 \rangle}{\epsilon_0 - \epsilon_n} \\ &= 2 \langle \Psi_0 | v_i | \Psi'_j \rangle = \int v_i(\mathbf{r}) \rho'_j(\mathbf{r}) d\mathbf{r} \\ &= 2 \langle \Psi'_i | v_j | \Psi_0 \rangle = \int v_j(\mathbf{r}) \rho'_i(\mathbf{r}) d\mathbf{r} \end{aligned}$$

the "2n+1" theorem

$$\Phi = \Phi_0 + \mathcal{O}(\epsilon) \Rightarrow E = E_0 + \mathcal{O}(\epsilon^2)$$

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$$E = \frac{\langle \Phi_0 + \Phi' | (H_0 + V') | \Phi_0 + \Phi' \rangle}{\langle \Phi_0 + \Phi' | \Phi_0 + \Phi' \rangle} + \mathcal{O}(V'^4)$$

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$$E^{(3)} = \langle \Phi' | V' | \Phi' \rangle - \langle \Phi' | \Phi' \rangle \langle \Phi_0 | V' | \Phi_0 \rangle$$

density-functional perturbation theory

$$V_\lambda(\mathbf{r}) = V_0(\mathbf{r}) + \sum_i \lambda_i v_i(\mathbf{r})$$

density-functional perturbation theory

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$$E(\lambda) = \min_n \left(F[n] + \int V_\lambda(\mathbf{r}) n(\mathbf{r}) \right) \quad \int n(\mathbf{r}) d\mathbf{r} = N$$

DFT

density-functional perturbation theory

$$V_\lambda(\mathbf{r}) = V_0(\mathbf{r}) + \sum_i \lambda_i v_i(\mathbf{r})$$

$$E(\lambda) = \min_n \left(F[n] + \int V_\lambda(\mathbf{r}) n(\mathbf{r}) \right) \quad \int n(\mathbf{r}) d\mathbf{r} = N$$ DFT

$$\frac{\partial E(\lambda)}{\partial \lambda_i} = \int n_\lambda(\mathbf{r}) v_i(\mathbf{r}) d\mathbf{r}$$ HF

density-functional perturbation theory

$$V_\lambda(\mathbf{r}) = V_0(\mathbf{r}) + \sum_i \lambda_i v_i(\mathbf{r})$$

$$E(\lambda) = \min_n \left(F[n] + \int V_\lambda(\mathbf{r}) n(\mathbf{r}) \right) \quad \int n(\mathbf{r}) d\mathbf{r} = N$$ DFT

$$\frac{\partial E(\lambda)}{\partial \lambda_i} = \int n_\lambda(\mathbf{r}) v_i(\mathbf{r}) d\mathbf{r}$$ HF

$$\frac{\partial^2 E(\lambda)}{\partial \lambda_i \partial \lambda_j} = \int \frac{\partial n_\lambda(\mathbf{r})}{\partial \lambda_j} v_i(\mathbf{r}) d\mathbf{r}$$ DFPT

calculating the response

$$n(\mathbf{r}) = \sum_v |\phi_v(\mathbf{r})|^2$$

$$n'(\mathbf{r}) = 2\mathrm{Re}\sum_v \phi_v^{\circ *}(\mathbf{r})\phi_v'(\mathbf{r})$$

calculating the response

$$n(\mathbf{r}) = \sum_v |\phi_v(\mathbf{r})|^2$$

$$n'(\mathbf{r}) = 2\mathrm{Re}\sum_v \phi_v^{\circ *}(\mathbf{r})\phi_v'(\mathbf{r})$$

$$= 2\mathrm{Re}\sum_{cv} \rho'_{vc}\phi_v^{\circ *}(\mathbf{r})\phi_c^{\circ}(\mathbf{r})$$

$$\phi_v' = \sum_c \phi_c^\circ \frac{\langle \phi_c^\circ | V' | \phi_v^\circ \rangle}{\epsilon_v^\circ - \epsilon_c^\circ}$$

calculating the response

$$n(\mathbf{r}) = \sum_v |\phi_v(\mathbf{r})|^2$$

$$n'(\mathbf{r}) = 2\mathrm{Re} \sum_v \phi_v^{\circ *}(\mathbf{r}) \phi_v'(\mathbf{r})$$

$$= 2\mathrm{Re} \sum_{cv} \rho_{vc}' \phi_v^{\circ *}(\mathbf{r}) \phi_c^{\circ}(\mathbf{r})$$

$$\phi_v' = \sum_c \phi_c^{\circ} \frac{\langle \phi_c^{\circ} | V' | \phi_v^{\circ} \rangle}{\epsilon_v^{\circ} - \epsilon_c^{\circ}}$$

$$(H^{\circ} - \epsilon_v^{\circ}) \phi_v' = -P_c V' \phi_v^{\circ}$$

calculating the response

$$n'(\mathbf{r}) = 2\mathrm{Re}\sum_v \phi_v^{\circ *}(\mathbf{r})\phi_v'(\mathbf{r})$$

$$(H^\circ - \epsilon_v^\circ)\phi_v' = -P_c V' \phi_v^\circ$$

DFPT: the equations

DFT

$$V_0(\mathbf{r}) \leftrightarrows n(\mathbf{r})$$

$$V_{SCF}(\mathbf{r}) = V_0(\mathbf{r}) + \int \frac{n(\mathbf{r}')}{|\mathbf{r} - \mathbf{r}'|} d\mathbf{r}' + \mu_{xc}(\mathbf{r})$$

$$n(\mathbf{r}) = \sum_{\epsilon_v < E_F} |\phi_v(\mathbf{r})|^2$$

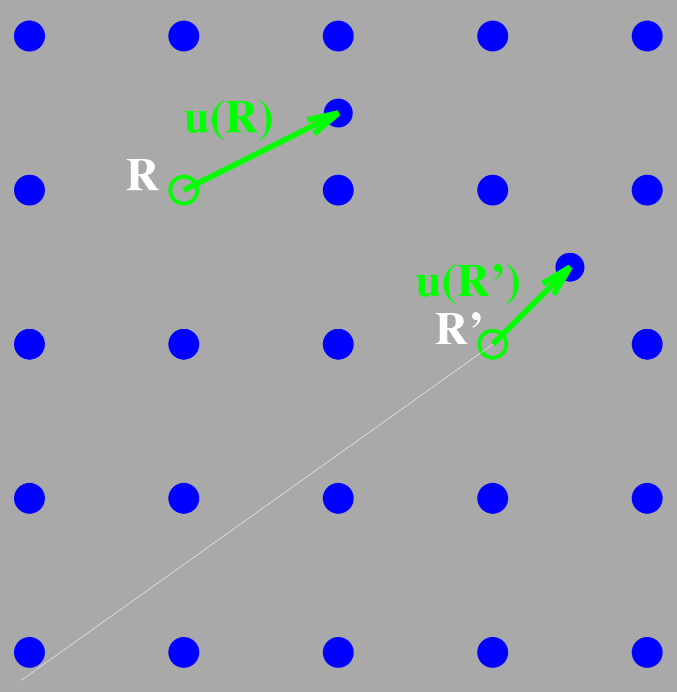
$$\big(-\Delta + V_{SCF}(\mathbf{r})\big)\phi_v(\mathbf{r}) = \epsilon_v \phi_v(\mathbf{r})$$

DFPT: the equations

DFT	DFPT
$V_0(\mathbf{r}) \leftrightarrows n(\mathbf{r})$	$V'(\mathbf{r}) \leftrightarrows n'(\mathbf{r})$
$V_{SCF}(\mathbf{r}) = V_0(\mathbf{r}) + \int \frac{n(\mathbf{r}')}{\lvert\mathbf{r}-\mathbf{r}'\rvert} d\mathbf{r}' + \mu_{xc}(\mathbf{r})$	$V'_{SCF}(\mathbf{r}) = V'(\mathbf{r}) + \int \frac{n'(\mathbf{r}')}{\lvert\mathbf{r}-\mathbf{r}'\rvert} d\mathbf{r}' + \mu'_{xc}(\mathbf{r})$
$n(\mathbf{r}) = \sum_{\epsilon_v < E_F} \lvert\phi_v(\mathbf{r})\rvert^2$	$n'(\mathbf{r}) = 2 \text{ Re} \sum_{\epsilon_v < E_F} \phi_v^*(\mathbf{r})\phi'_v(\mathbf{r})$
$(-\Delta + V_{SCF}(\mathbf{r}))\phi_v(\mathbf{r}) = \epsilon_v \phi_v(\mathbf{r})$	$(-\Delta + V_{SCF}(\mathbf{r}) - \epsilon_v)\phi'_v(\mathbf{r}) = P_c V'_{SCF}(\mathbf{r})\phi_v(\mathbf{r})$

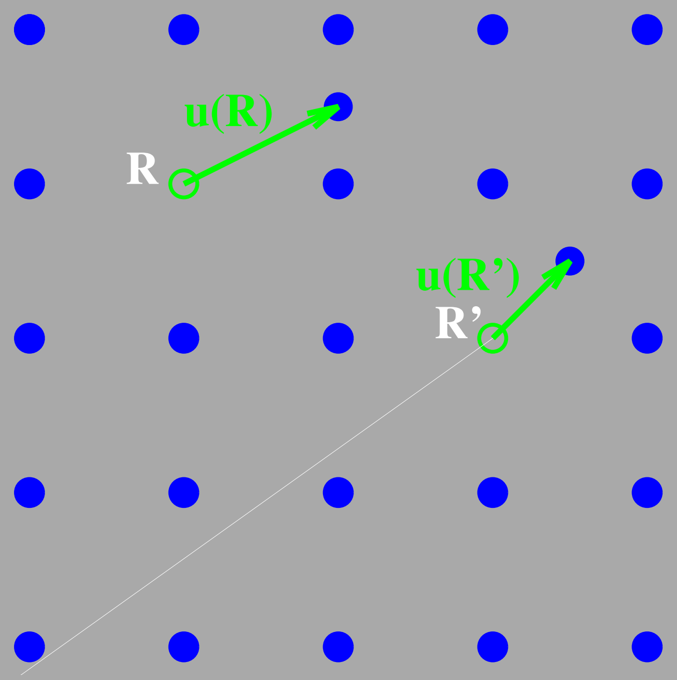
SB, P. Giannozzi, and A. Testa, Phys. Rev. Lett. **58**, 1861 (1987) simulating atomic vibrations ...

lattice dynamics



$$V(\mathbf{r}) = V_0(\mathbf{r}) + \sum_{\mathbf{R}} \mathbf{u}(\mathbf{R}) \cdot \frac{\partial v(\mathbf{r}-\mathbf{R})}{\partial \mathbf{R}} + \cdots$$

lattice dynamics

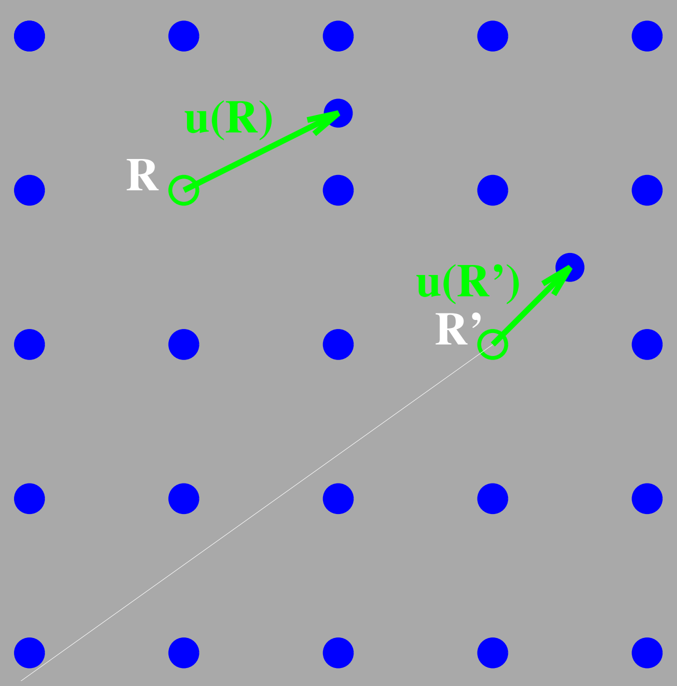


$$V(\mathbf{r}) = V_0(\mathbf{r}) + \sum_{\mathbf{R}} \mathbf{u}(\mathbf{R}) \cdot \frac{\partial v(\mathbf{r}-\mathbf{R})}{\partial \mathbf{R}} + \cdots$$

$$E = E_0 + \frac{1}{2} \sum_{\mathbf{R},\mathbf{R}'} \mathbf{u}(\mathbf{R}) \cdot \frac{\partial^2 E}{\partial \mathbf{u}(\mathbf{R}) \partial \mathbf{u}(\mathbf{R}')} \cdot \mathbf{u}(\mathbf{R}')$$

$$+ \cdots$$

lattice dynamics



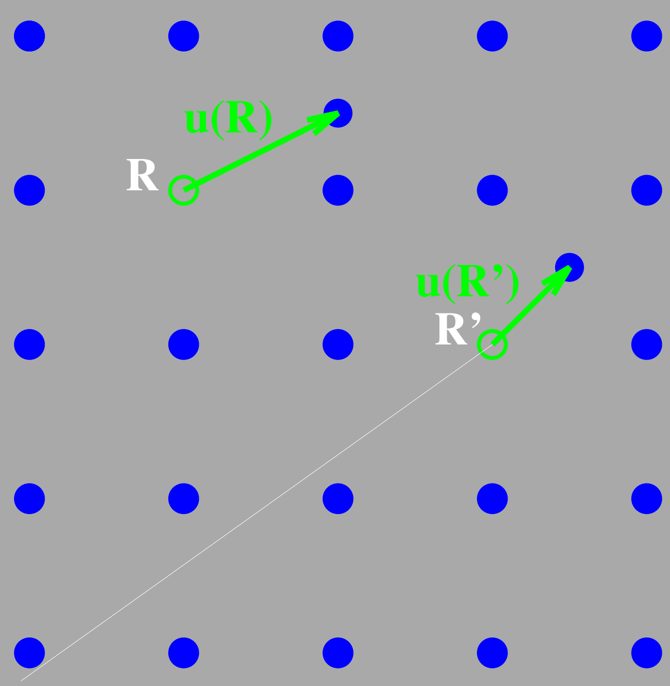
$$V(\mathbf{r}) = V_0(\mathbf{r}) + \sum_{\mathbf{R}} \mathbf{u}(\mathbf{R}) \cdot \frac{\partial v(\mathbf{r}-\mathbf{R})}{\partial \mathbf{R}} + \cdots$$

$$-\frac{\partial \mathbf{F}(\mathbf{R})}{\partial \mathbf{u}(\mathbf{R}')}$$

$$E = E_0 + \frac{1}{2} \sum_{\mathbf{R},\mathbf{R}'} \mathbf{u}(\mathbf{R}) \cdot \frac{\partial^2 E}{\partial \mathbf{u}(\mathbf{R}) \partial \mathbf{u}(\mathbf{R}')} \cdot \mathbf{u}(\mathbf{R}')$$

$$+ \cdots$$

lattice dynamics



$$V(\mathbf{r}) = V_0(\mathbf{r}) + \sum_{\mathbf{R}} \mathbf{u}(\mathbf{R}) \cdot \frac{\partial v(\mathbf{r}-\mathbf{R})}{\partial \mathbf{R}} + \cdots$$

DFT

DPFT

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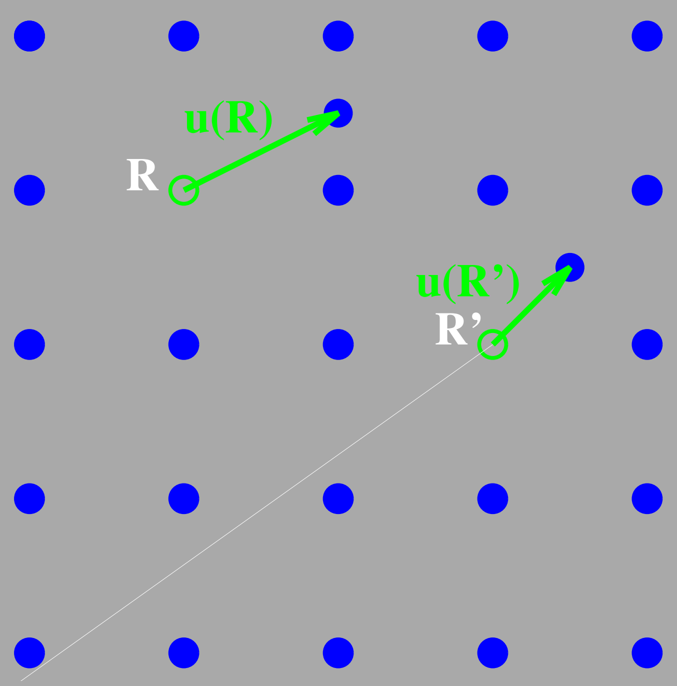
$$+ \cdots$$

lattice dynamics



$$V(\mathbf{r}) = V_0(\mathbf{r}) + \sum_{\mathbf{R}} \mathbf{u}(\mathbf{R}) \cdot \frac{\partial v(\mathbf{r}-\mathbf{R})}{\partial \mathbf{R}} + \cdots$$

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$$-\frac{\partial \mathbf{F}(\mathbf{R})}{\partial \mathbf{u}(\mathbf{R}')}$$

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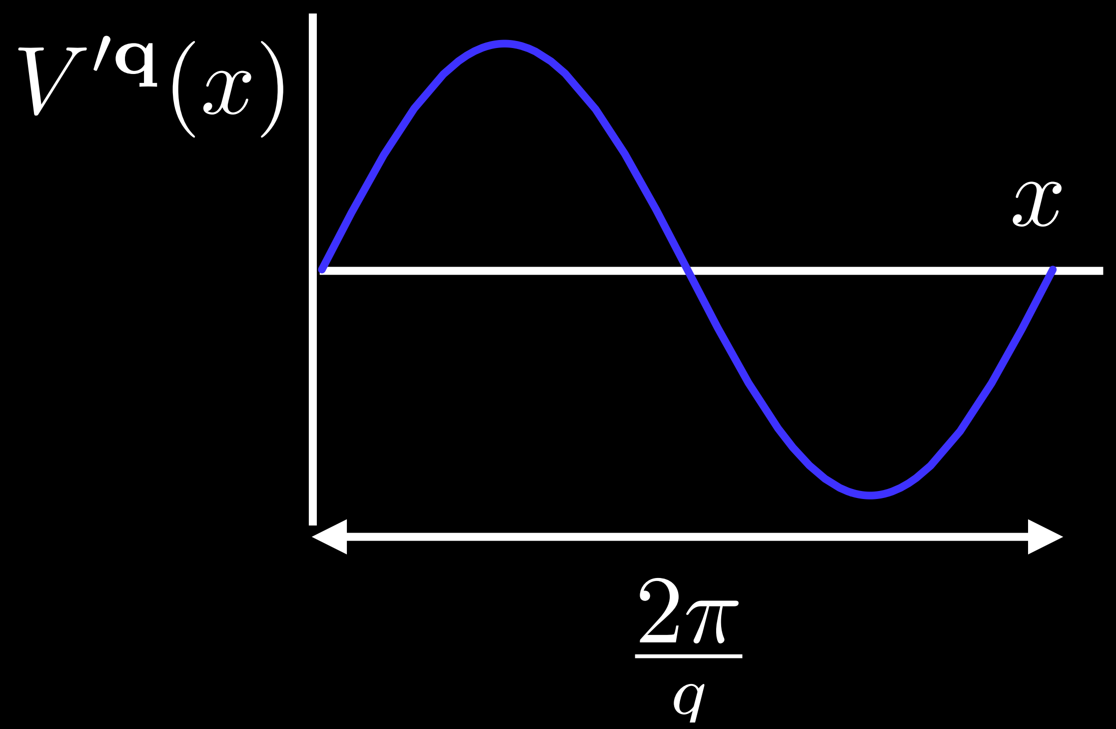
$$+ \cdots$$

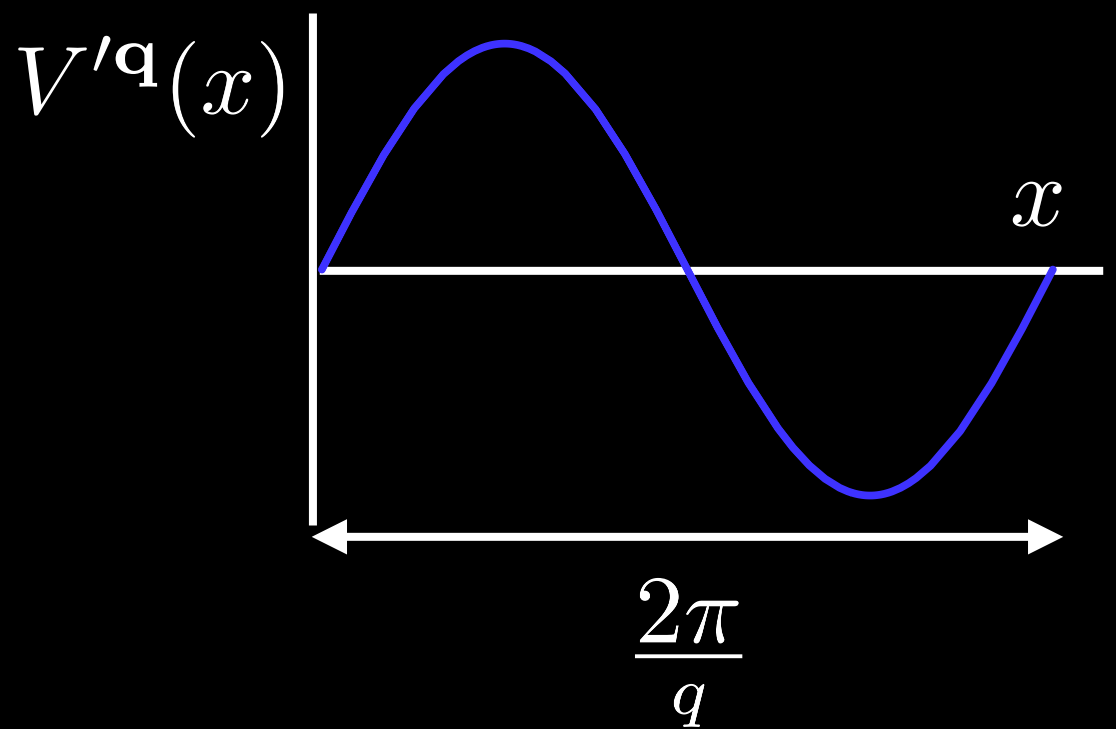
$$\det \left[\frac{\partial^2 E}{\partial \mathbf{u}(\mathbf{R}) \partial \mathbf{u}(\mathbf{R}')} - \omega^2 M(\mathbf{R}) \delta_{\mathbf{R},\mathbf{R}'} \right] = 0$$

monochromatic perturbations



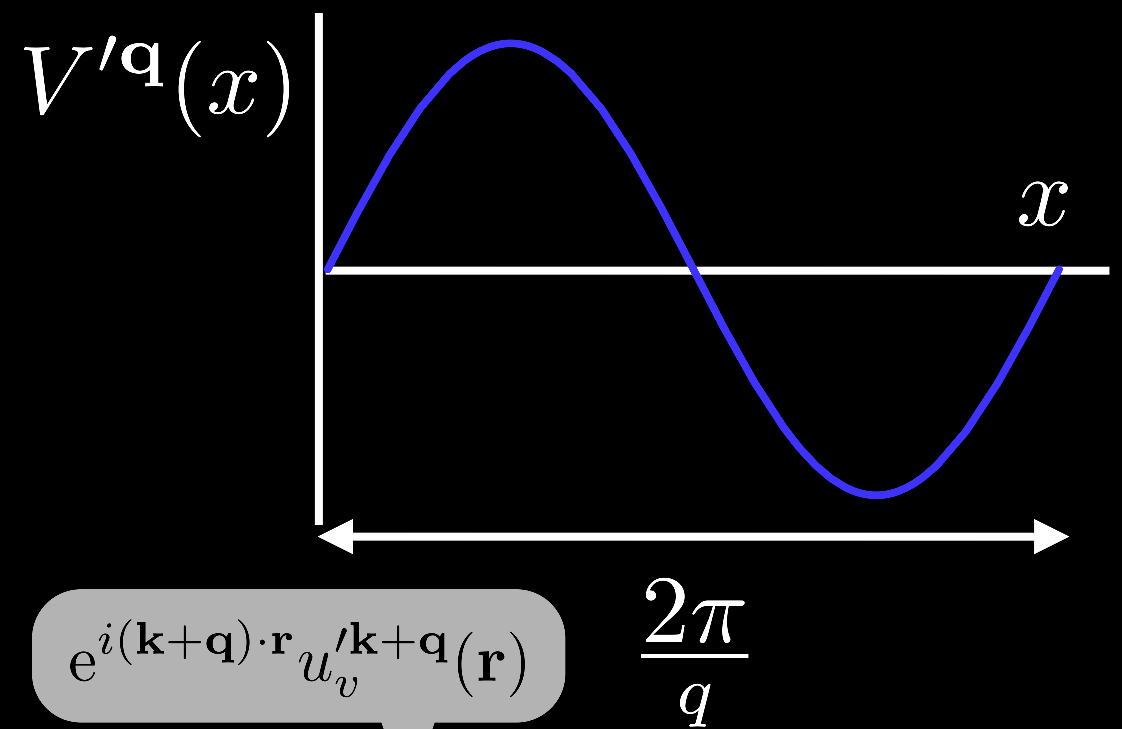
DFPT rhs: $-P_c V'^{\mathbf{q}}_{SCF} \phi^{\mathbf{k}}_v(\mathbf{r})$

monochromatic perturbations



$$\left(H_0 - \epsilon_v^{\mathbf{k}}\right)\phi_v'^{\mathbf{k}+\mathbf{q}}(\mathbf{r}) = -P_c V_{SCF}'^{\mathbf{q}} \phi_v^{\mathbf{k}}(\mathbf{r})$$

monochromatic perturbations

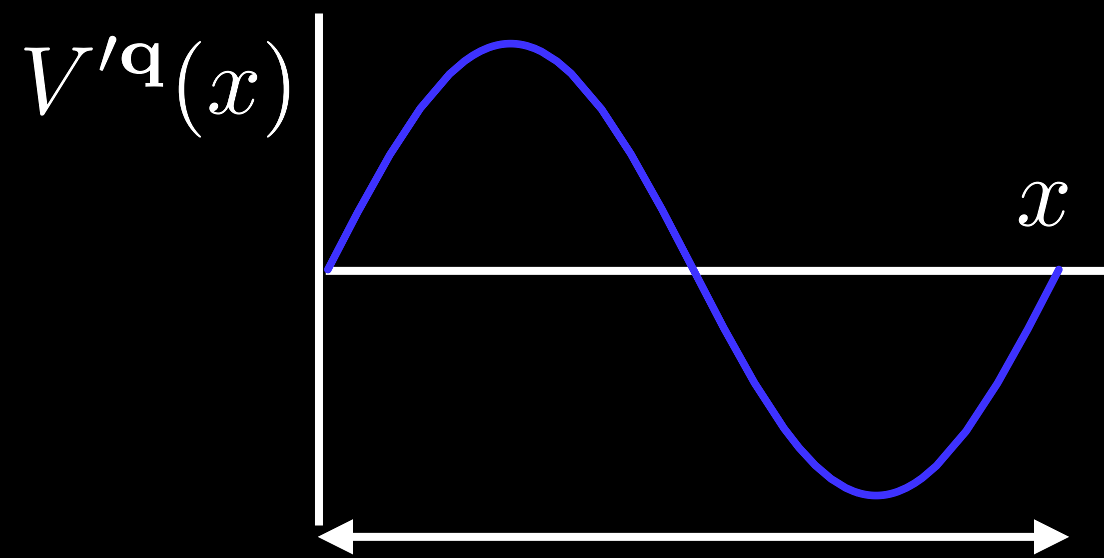


$\mathrm{e}^{i(\mathbf{k}+\mathbf{q})\cdot\mathbf{r}} u_v'^{\mathbf{k}+\mathbf{q}}(\mathbf{r})$

$\mathrm{e}^{i\mathbf{k}\cdot\mathbf{r}} u_v^{\circ\mathbf{k}}(\mathbf{r})$

$$\left(H_0 - \epsilon_v^{\mathbf{k}}\right)\phi_v'^{\mathbf{k}+\mathbf{q}}(\mathbf{r}) = -P_c V_{SCF}'^{\mathbf{q}} \phi_v^{\mathbf{k}}(\mathbf{r})$$

monochromatic perturbations



$$\frac{2\pi}{q}$$

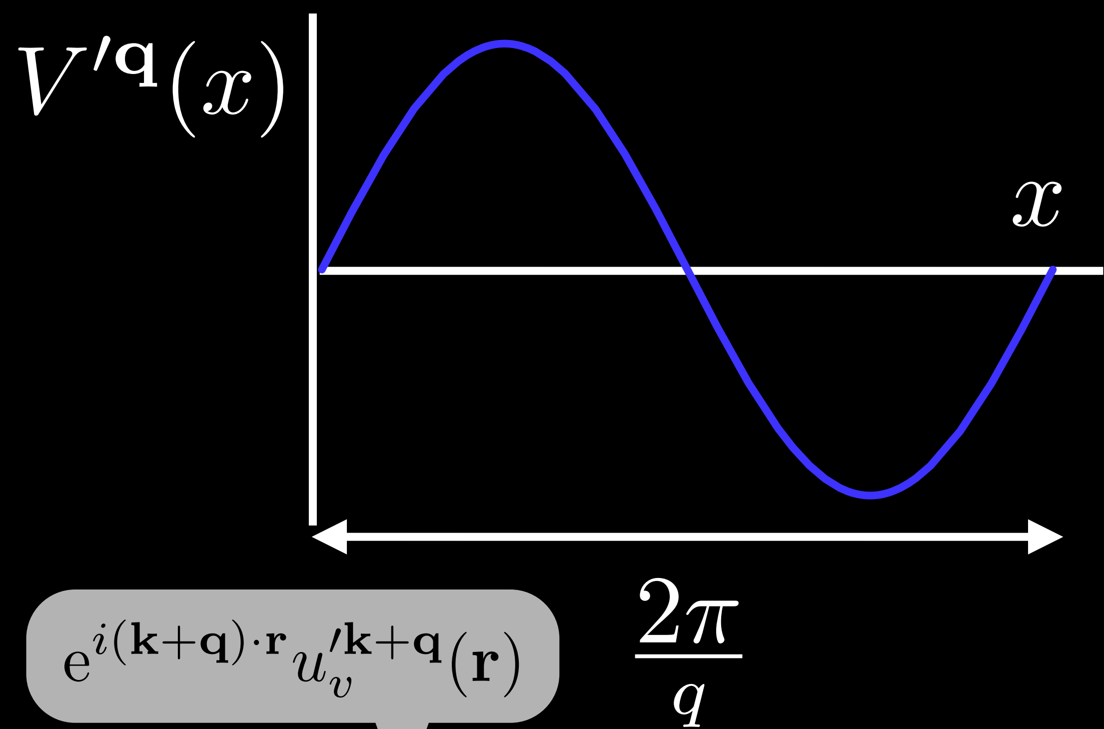
$\mathrm{e}^{i(\mathbf{k}+\mathbf{q})\cdot\mathbf{r}}u'^{\mathbf{k}+\mathbf{q}}_v(\mathbf{r})$

$\mathrm{e}^{i\mathbf{k}\cdot\mathbf{r}}u^{\circ\mathbf{k}}_v(\mathbf{r})$

$$\left(H_0 - \epsilon^{\mathbf{k}}_v\right)\phi'^{\mathbf{k}+\mathbf{q}}_v(\mathbf{r}) = -P_c V'^{\mathbf{q}}_{SCF}\phi^{\mathbf{k}}_v(\mathbf{r})$$

$$n'^{\mathbf{q}}(\mathbf{r}) = \mathrm{e}^{i\mathbf{q}\cdot\mathbf{r}}\sum_{v,\mathbf{k}} u^{\circ\mathbf{k}*}_v(\mathbf{r})u'^{\mathbf{k}+\mathbf{q}}_v(\mathbf{r})$$

monochromatic perturbations



$e^{i(\mathbf{k}+\mathbf{q})\cdot\mathbf{r}} u'^{\mathbf{k}+\mathbf{q}}_v(\mathbf{r})$

$e^{i\mathbf{k}\cdot\mathbf{r}} u^{\circ\mathbf{k}}_v(\mathbf{r})$

$$\left(H_0 - \epsilon^{\mathbf{k}}_v\right)\phi'^{\mathbf{k}+\mathbf{q}}_v(\mathbf{r}) = -P_c V'^{\mathbf{q}}_{SCF}\phi^{\mathbf{k}}_v(\mathbf{r})$$

$$n'^{\mathbf{q}}(\mathbf{r}) = e^{i\mathbf{q}\cdot\mathbf{r}} \sum_{v,\mathbf{k}} u^{\circ\mathbf{k}*}_v(\mathbf{r}) u'^{\mathbf{k}+\mathbf{q}}_v(\mathbf{r})$$

$$V'^{\mathbf{q}}(\mathbf{r}) = V'^{\mathbf{q}}_{ext}(\mathbf{r}) + \int \left(\frac{e^2}{|\mathbf{r}-\mathbf{r}'|} + \kappa_{xc}(\mathbf{r},\mathbf{r}')\right) n'^{\mathbf{q}}(\mathbf{r}')d\mathbf{r}'$$

phonons in polar materials

$$E(\mathbf{u} \quad) = \frac{1}{2} M \omega_0^2 u^2$$

phonons in polar materials

$$E(\mathbf{u}, \mathsf{E}) = \frac{1}{2} M \omega_0^2 u^2 - \frac{\Omega}{8\pi} \epsilon_\infty \mathsf{E}^2 - eZ^* \mathbf{u} \cdot \mathsf{E}$$

phonons in polar materials

$$E(\mathbf{u}, \mathsf{E}) = \frac{1}{2} M \omega_0^2 u^2 - \frac{\Omega}{8\pi} \epsilon_\infty \mathsf{E}^2 - eZ^* \mathbf{u} \cdot \mathsf{E}$$

$$\mathbf{F} \equiv -\frac{\partial E}{\partial \mathbf{u}} = -M\omega_0^2 \mathbf{u} + Z^* \mathsf{E}$$

$$\mathsf{D} \equiv -\frac{4\pi}{\Omega} \frac{\partial E}{\partial \mathsf{E}} = \frac{4\pi}{\Omega} Z^* \mathbf{u} + \epsilon_\infty \mathsf{E}$$

phonons in polar materials

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$$\mathrm{rot}\ \mathsf{E} \sim i\mathbf{q} \times \mathsf{E} = 0$$

phonons in polar materials

$$E(\mathbf{u},\mathsf{E}) = \frac{1}{2}M\omega_0^2 u^2 - \frac{\Omega}{8\pi}\epsilon_\infty \mathsf{E}^2 - eZ^*\mathbf{u}\cdot\mathsf{E}$$

$$\mathbf{F} \equiv -\frac{\partial E}{\partial \mathbf{u}} = -M\omega_0^2\mathbf{u} + Z^*\mathsf{E}$$

$$\mathsf{D} \equiv -\frac{4\pi}{\Omega}\frac{\partial E}{\partial \mathsf{E}} = \frac{4\pi}{\Omega}Z^*\mathbf{u} + \epsilon_\infty\mathsf{E}$$

$\mathrm{rot}\ \mathsf{E} \sim i\mathbf{q}\times\mathsf{E} = 0$ $\quad\quad$ $\mathbf{u}\perp\mathbf{q} \Rightarrow \mathsf{E} = 0$ $\quad\quad$ (T)

phonons in polar materials

$$E(\mathbf{u}, \mathsf{E}) = \frac{1}{2} M \omega_0^2 u^2 - \frac{\Omega}{8\pi} \epsilon_\infty \mathsf{E}^2 - e Z^* \mathbf{u} \cdot \mathsf{E}$$

$$\mathbf{F} \quad \equiv \quad -\frac{\partial E}{\partial \mathbf{u}} = -M\omega_0^2 \mathbf{u} + Z^* \mathsf{E}$$

$$\mathsf{D} \quad \equiv \quad -\frac{4\pi}{\Omega} \frac{\partial E}{\partial \mathsf{E}} = \frac{4\pi}{\Omega} Z^* \mathbf{u} + \epsilon_\infty \mathsf{E}$$

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phonons in polar materials

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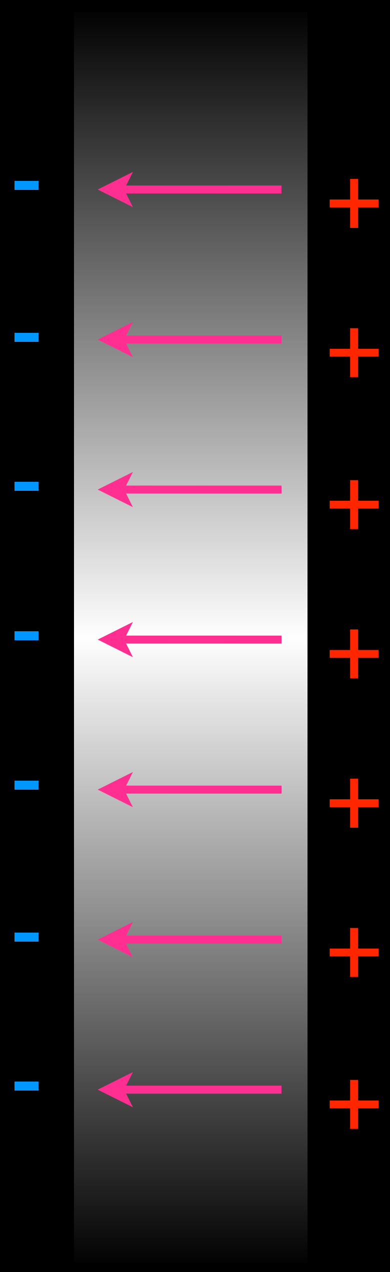
$$\mathbf{F}_T = -M\omega_0^2 \mathbf{u} \qquad \mathbf{F}_L = -M\left(\omega_0^2 + \frac{4\pi Z^*}{M\Omega\epsilon_\infty}\right)\mathbf{u}$$

macroscopic electric fields

$\mathbf{E} = \text{const}$



$V'(\mathbf{r}) = \mathbf{E} \cdot \mathbf{r}$

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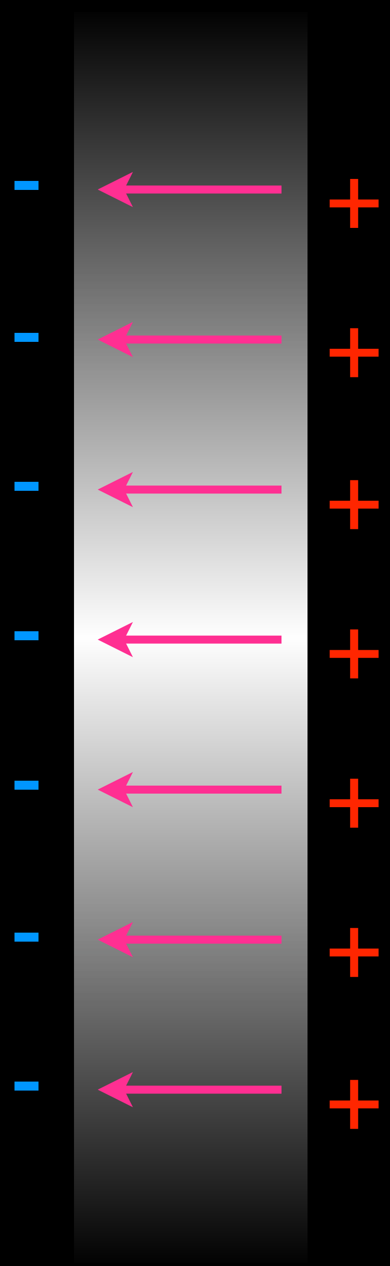

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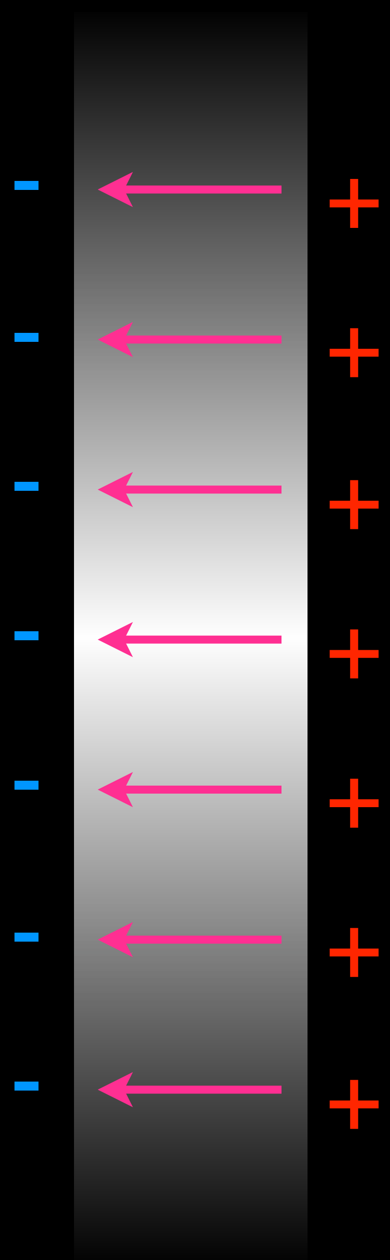

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$$
\begin{aligned}
\phi_v^0(\mathbf{r}) &= \mathrm{e}^{i\mathbf{k}\cdot\mathbf{r}} u_{v,\mathbf{k}}(\mathbf{r}) \\
V'(\mathbf{r})\phi_v^0(\mathbf{r}) &= ??
\end{aligned}
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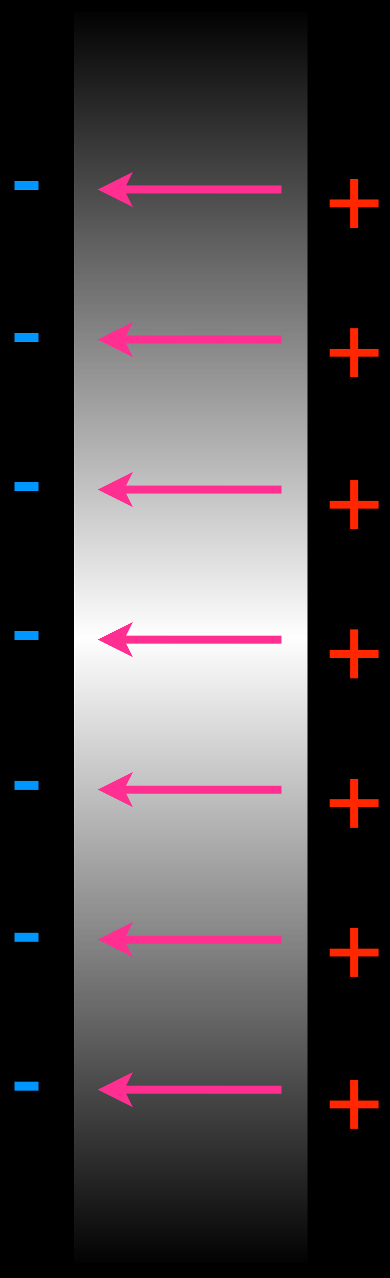

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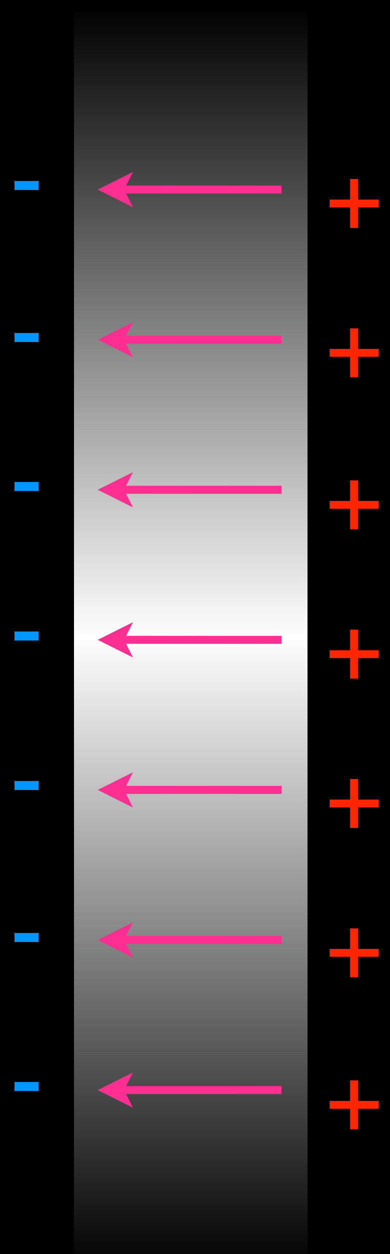

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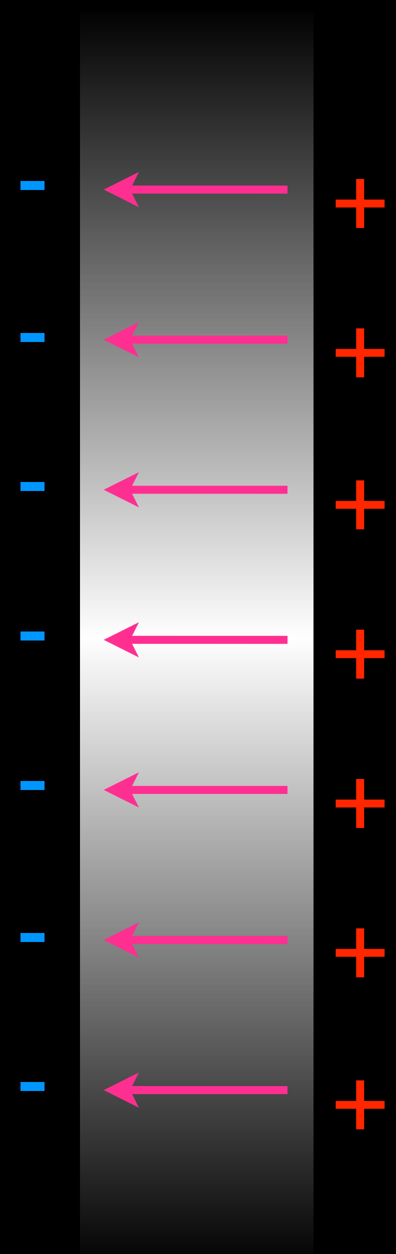

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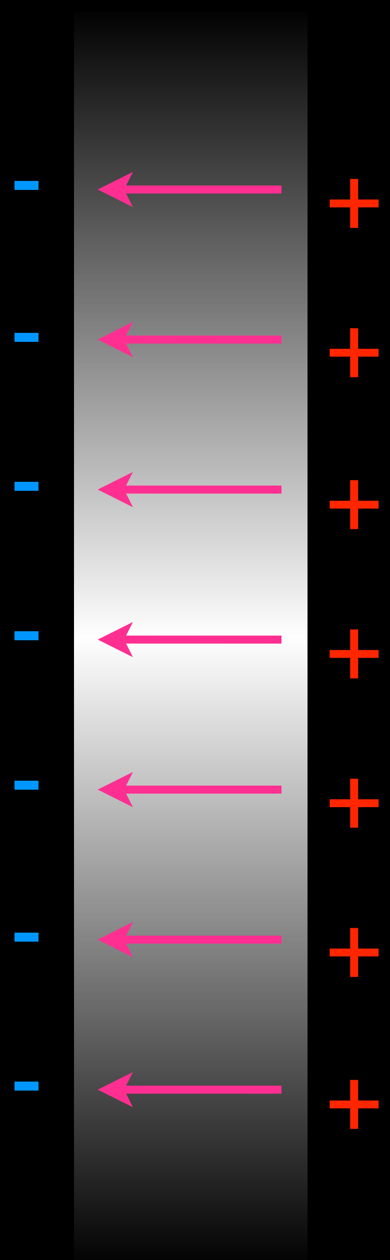

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$$(H_0 - \epsilon_v^0)\psi_v' = -\mathsf{E} P_c [H_0, x] \phi_v^0$$

DFPT rhs

interatomic force constants

$$\Phi_{st}^{\alpha\beta}(\mathbf{R}-\mathbf{R}') \quad = \quad -\frac{\partial^2 E}{\partial u_s^{\alpha}(\mathbf{R})\partial u_t^{\beta}(\mathbf{R}')}$$

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short ranged +
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short ranged +
dipole-dipole

- **remove singularities** in D(q)
- **do FFT's** (# R's = # q's - the shorter the range, the coarser the grid)
- **store information**
- **interpolate** D(q) on any finer mesh (pad Φ with 0's and do FFT^{-1}: # q's = # R's)
- **calculate phonon bands**

DFPT: the main features

☛ response functions calculated in terms of response orbitals, $\{\phi'_v\}$

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- ☛ non-local perturbations: OK
- ☛ non-periodic perturbations: OK
- ☛ macroscopic electric fields: OK

VOLUME 62, NUMBER 24 PHYSICAL REVIEW LETTERS 12 JUNE 1989

Piezoelectric Properties of III-V Semiconductors from First-Principles Linear-Response Theory

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Stefano Baroni
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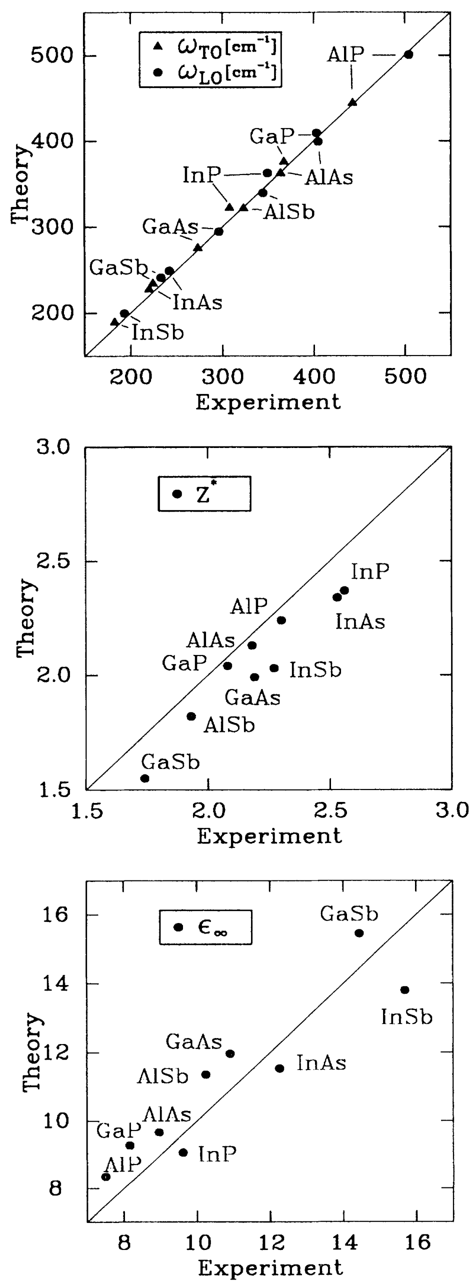
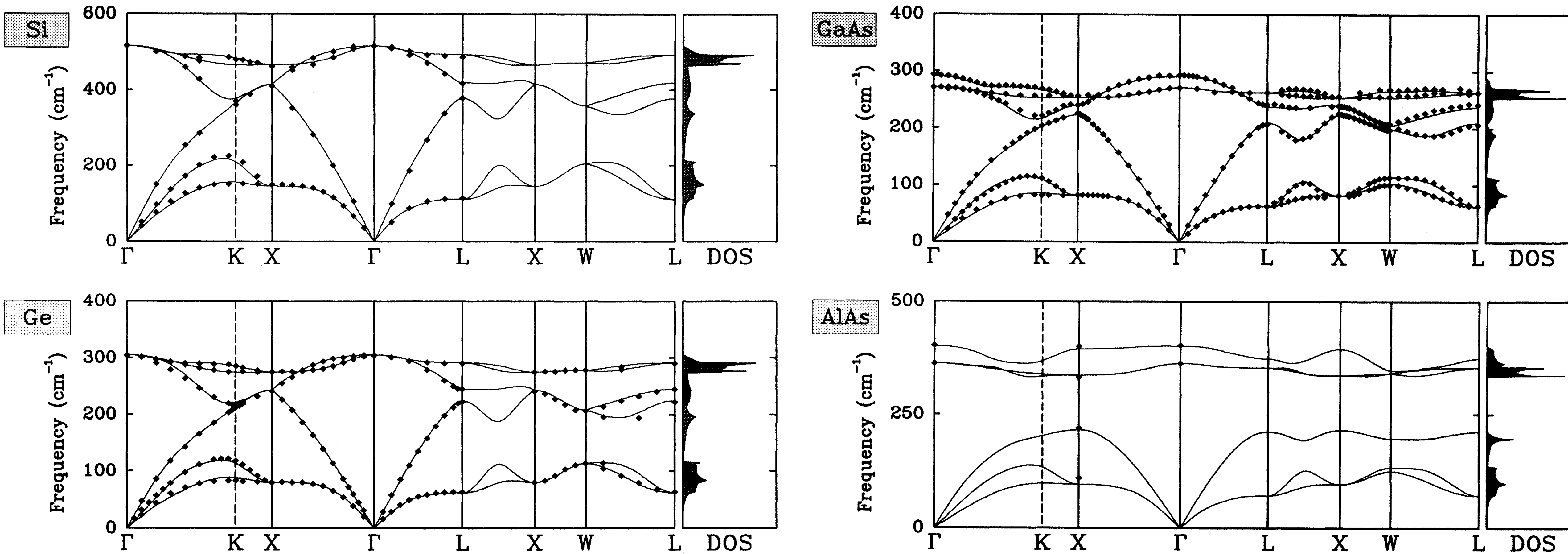
Raffaele Resta [(b)]
Institut Romand de Recherche Numérique en Physique des Matériaux (IRRMA), Ecole Polytechnique Fédérale de Lausanne, CH-1015, Lausanne, Switzerland

(Received 7 November 1988)



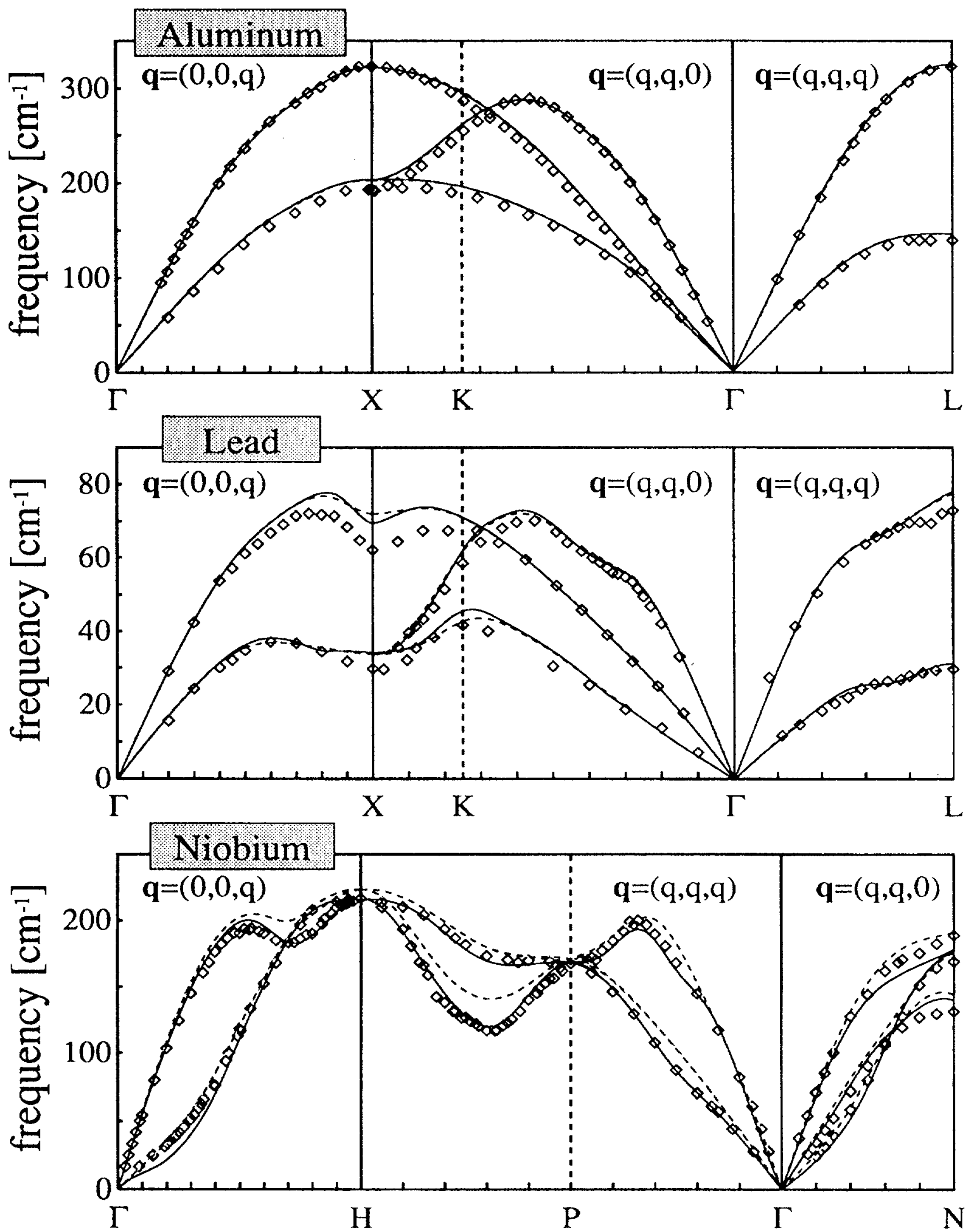
$\bar{\gamma}_{14}$	P	As	Sb
Al	0.11 (···)	−0.03 (···)	−0.13 (−0.16)
Ga	−0.18 (−0.18)	−0.35 (−0.32)	−0.40 (−0.39)
In	0.12 (|0.09|)	−0.08 (−0.10)	−0.20 (−0.18)

phonons from DFPT



P. Giannozzi, S. de Gironcoli, P. Pavone, and SB, Phys. Rev. B **43**, 7231 (1991)

DFPT phonons in metals



Stefano de Gironcoli,
Phys. Rev. B **51**, 6773 (1995)

applications done so far

- Dielectric properties
- Piezoelectric properties
- Elastic properties
- Phonon in crystals and alloys
- Phonon at surfaces, interfaces, super-lattices, and nano-structures
- Raman and infrared activities
- Anharmonic couplings and vibrational line widths
- Mode softening and structural transitions
- Electron-phonon interaction and superconductivity
- Thermal expansion
- Isotopic effects on structural and dynamical properties
- Thermo-elasticity and other thermal properties of minerals
- …

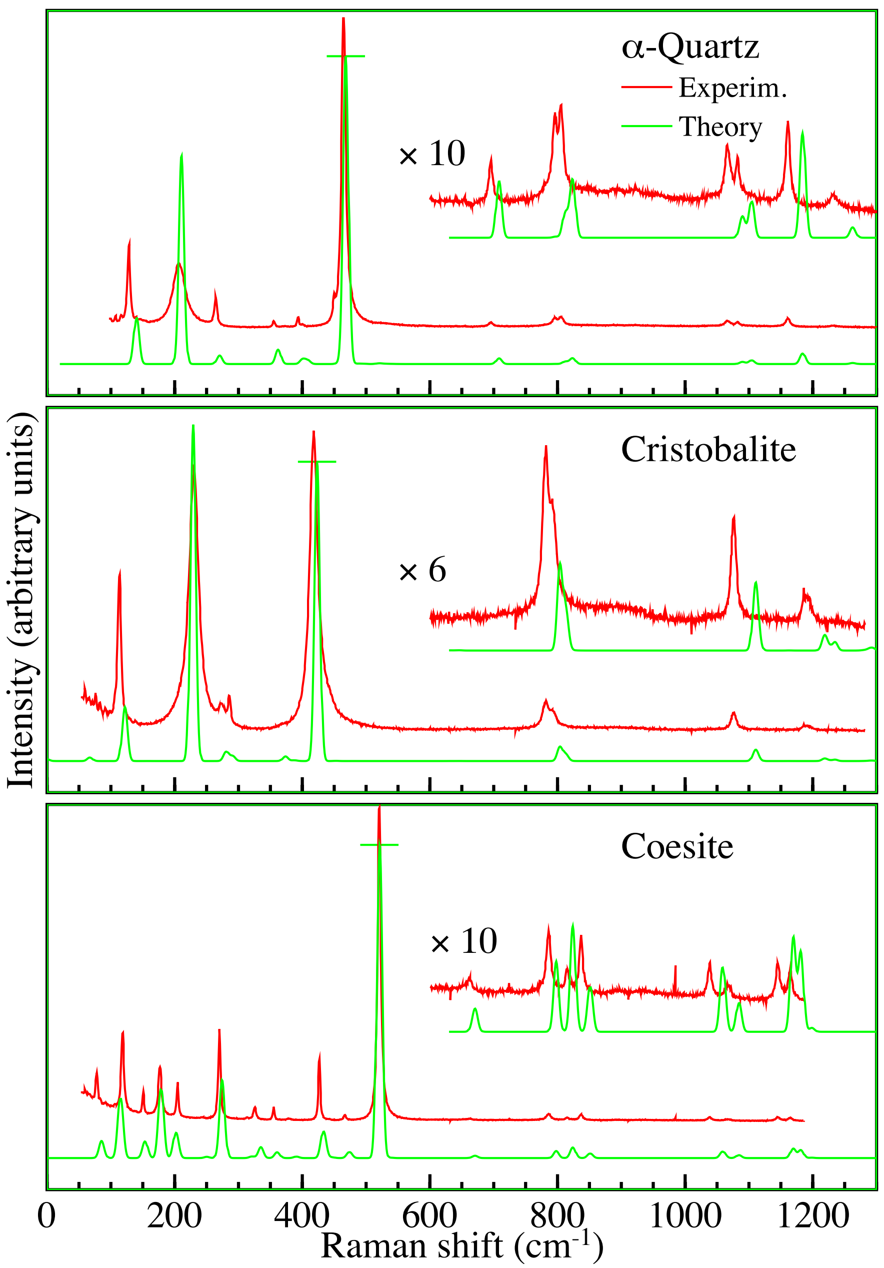
SB, A. Dal Corso, S. de Gironcoli, and P. Giannozzi, *Phonons and related crystal properties from density-functional perturbation theory*, Rev. Mod. Phys. **73**, 515 (2001)

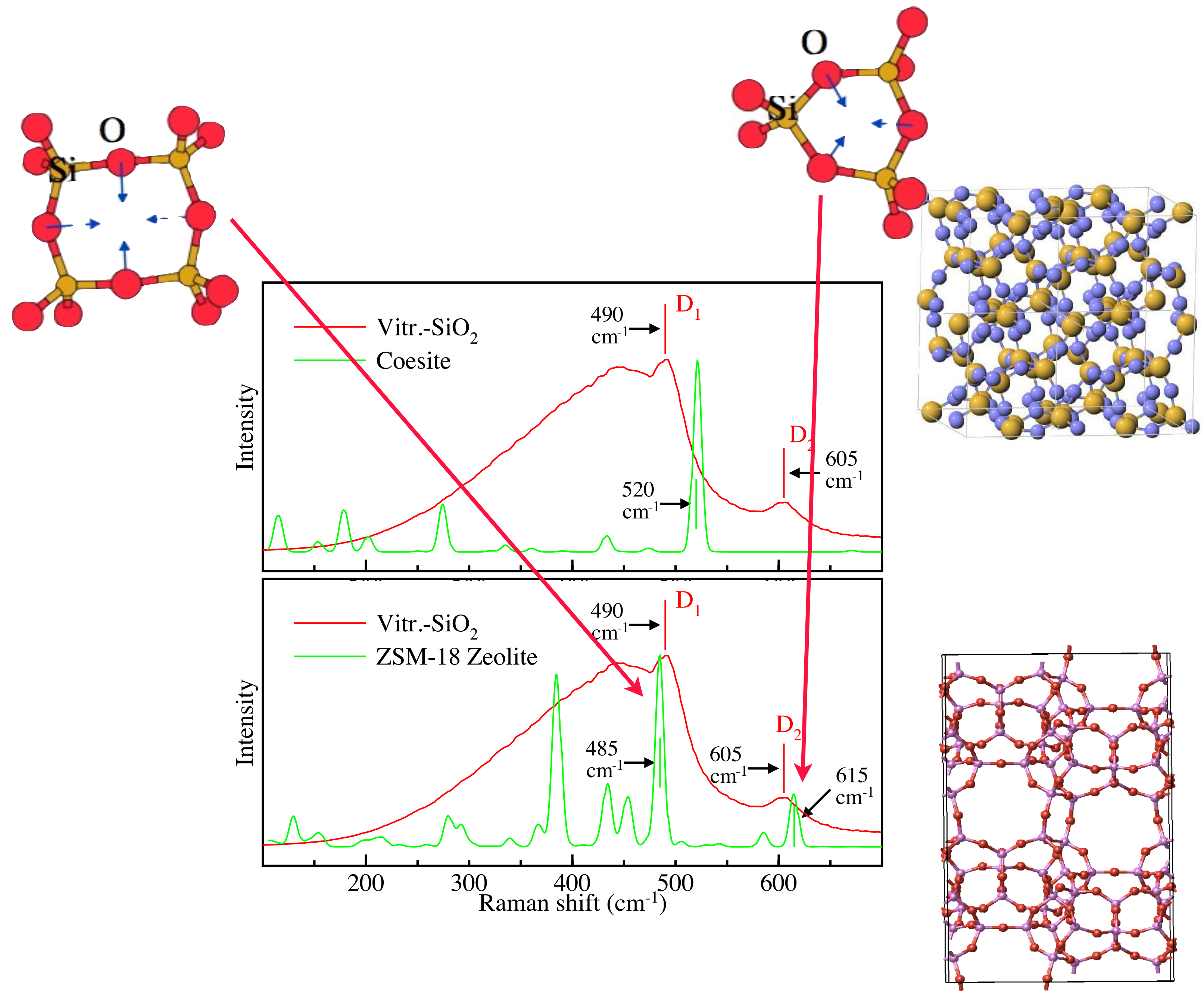
a sampler of recent applications

VOLUME 90, NUMBER 3 PHYSICAL REVIEW LETTERS week ending 24 JANUARY 2003

First-Principles Calculation of Vibrational Raman Spectra in Large Systems: Signature of Small Rings in Crystalline SiO_2

Michele Lazzeri and Francesco Mauri





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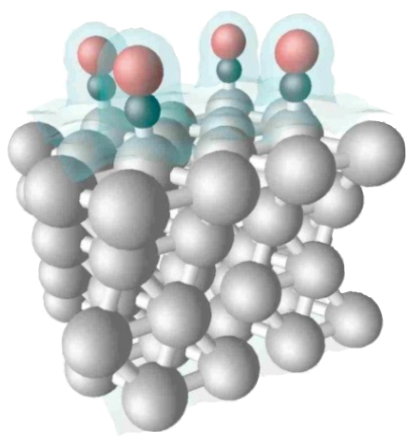
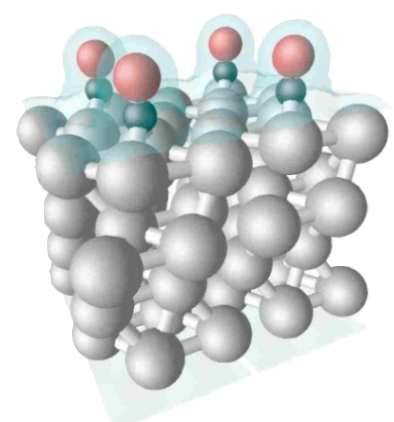
Vibrational Recognition of Adsorption Sites for CO on Platinum and Platinum−Ruthenium Surfaces

Ismaila Dabo,*,† Andrzej Wieckowski,‡ and Nicola Marzari†

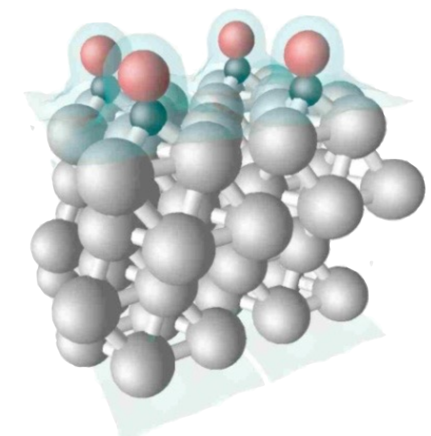
11046 J. AM. CHEM. SOC. ■ VOL. 129, NO. 36, 2007



atop ($CO@Pt_1$)
E_{DFT} = +0.10 eV
ν_{DFT} = 2050 cm^{-1}
ν_{exp} = 2070 cm^{-1}



bridge ($CO@Pt_2$)
E_{DFT} = +0.03 eV
ν_{DFT} = 1845 cm^{-1}
ν_{exp} = 1830 cm^{-1}



fcc ($CO@Pt_3$)
E_{DFT} = 0 eV
ν_{DFT} = 1743 cm^{-1}
ν_{exp} = 1780 cm^{-1}

expt

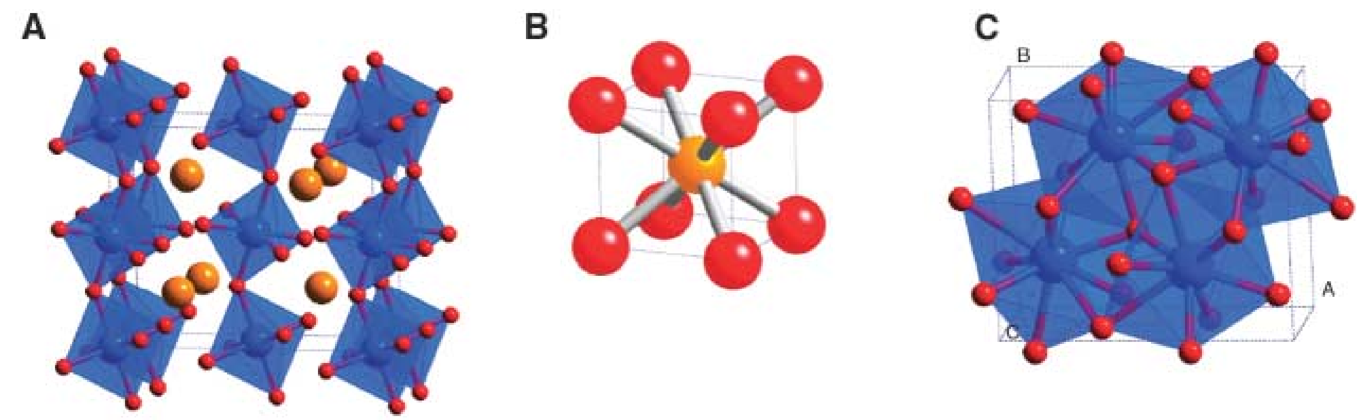
DFT

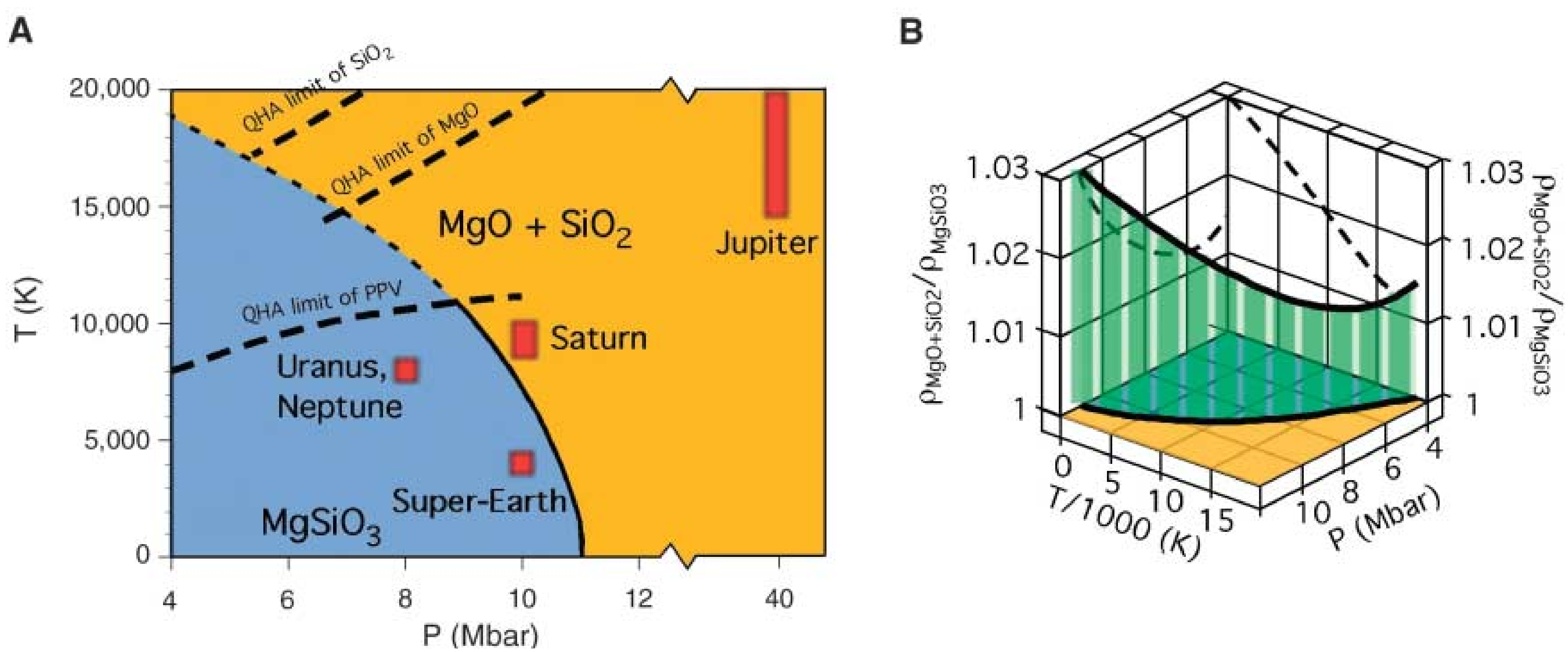
a sampler of recent applications

Dissociation of $MgSiO_3$ in the Cores of Gas Giants and Terrestrial Exoplanets

Koichiro Umemoto,[1] Renata M. Wentzcovitch,[1*] Philip B. Allen[2]

www.sciencemag.org SCIENCE VOL 311 17 FEBRUARY 2006 983





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PRL **100,** 257001 (2008) PHYSICAL REVIEW LETTERS week ending 27 JUNE 2008

Ab Initio Description of High-Temperature Superconductivity in Dense Molecular Hydrogen

P. Cudazzo,[1] G. Profeta,[1] A. Sanna,[2,3] A. Floris,[3] A. Continenza,[1] S. Massidda,[2] and E. K. U. Gross[3]

[1]*CNISM - Dipartimento di Fisica, Università degli Studi dell'Aquila, Via Vetoio 10, I-67010 Coppito (L'Aquila) Italy*
[2]*SLACS-INFM/CNR—Dipartimento di Fisica, Università degli Studi di Cagliari, I-09124 Monserrato (CA), Italy*
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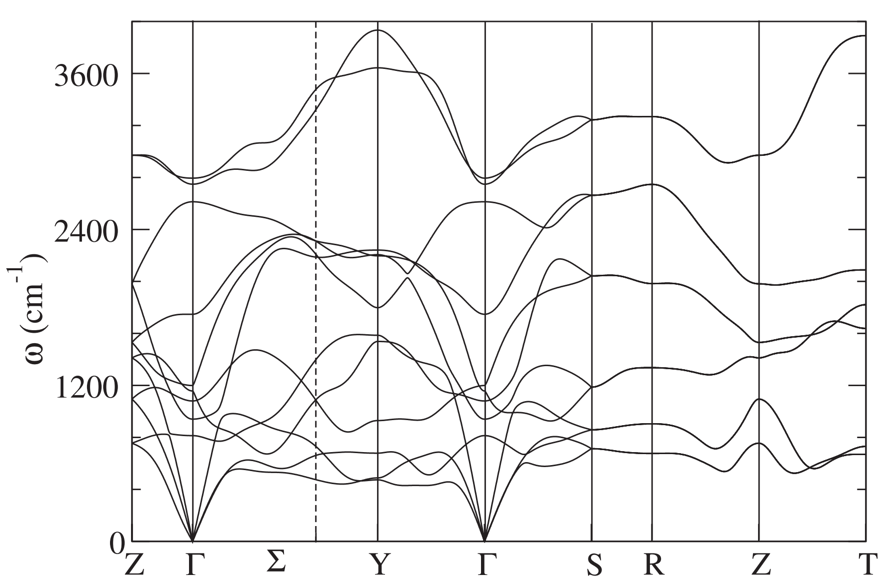
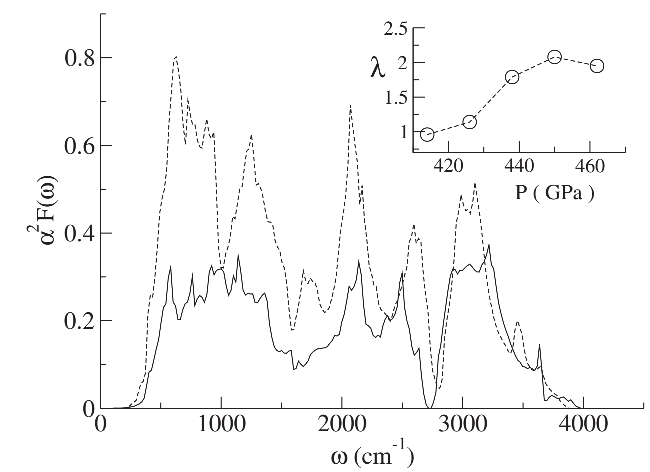
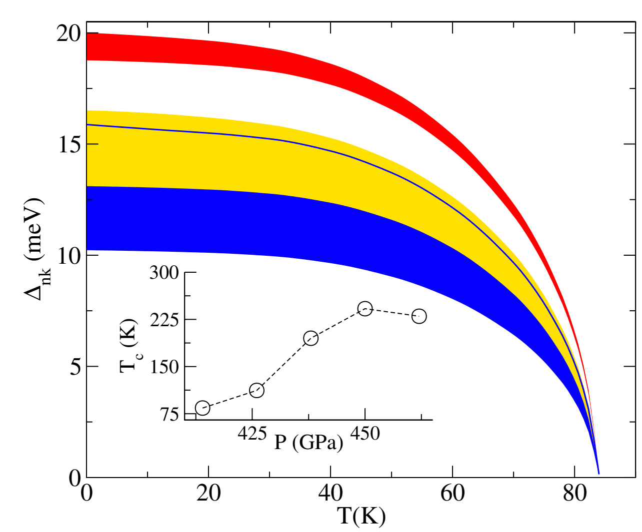
(Received 7 December 2007; published 23 June 2008; corrected 27 June 2008)

414 GPa







a sampler of recent applications

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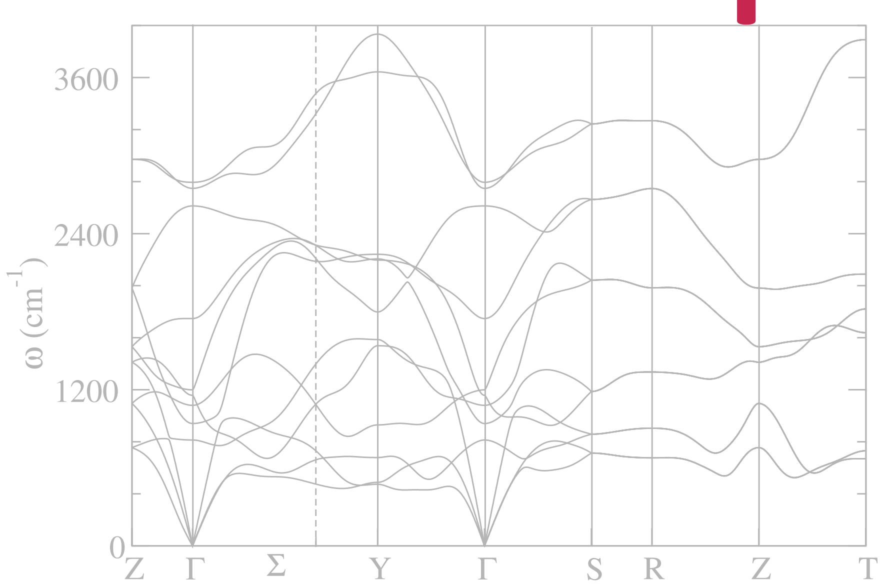

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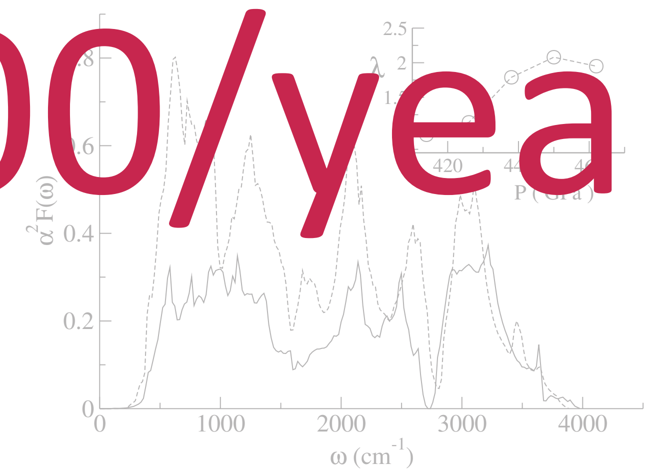

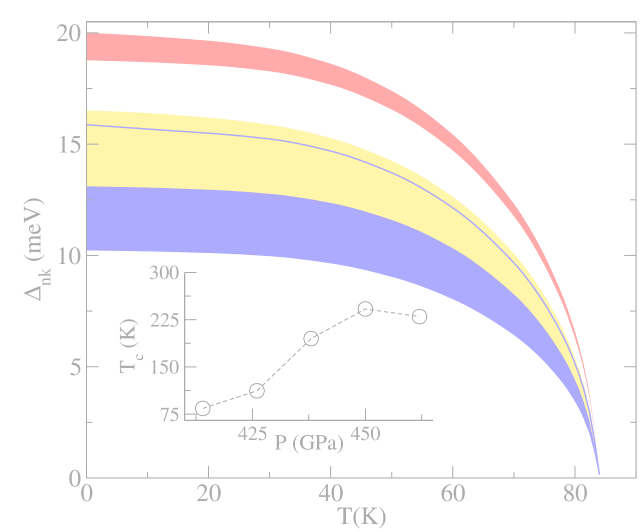

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