1 Supersymmetry: a bird eyes view

Why should a theoretical particle physicist ever know about supersymmetry? Why embark on reading a book as long as this? There are several reasons why it's worth it. Let me mention the ones I think are the most relevant.

The first such reasons, at least from a chronological point of view, is also the more phenomenological in nature. Back in 2012 the missing building block of the Standard Model, the Higgs particle, has been discovered at CERN's Large Hadron Collider (LHC). This has been an impressive achievement, one of the greatest success of the way we think Nature works at short distances and of the tool we use to describe it, *i.e.* Quantum Field Theory (QFT). On the other hand, there exist many reasons - some of which we will review in the following - which suggest that this cannot be the end of the story: new physics should show-up at energy scales higher than those we have been able to have access to, so far (but way lower than, say, the Planck scale). It turns out that of all possible options, the most compelling and motivated scenario for such beyond the Standard Model physics is supersymmetry. So, when it comes to try and understand how particles behave at high energy, equivalently at shorter and shorter distance, supersymmetry is a piece of basic knowledge any particle physicist should have. It should be said that at the time of writing no supersymmetric particles have been discovered yet, nor we have any indirect evidence for their existence. While very few high energy physicists doubt that supersymmetry is actually realized in Nature, this lack of experimental signature is putting the very idea of low energy supersymmetry into question, suggesting at least some twist in the way we think about it. Things might be slightly more involved than we imagined, supersymmetric particles might not be around the corner but actually a few more steps ahead, implying that the way supersymmetry tackles the different phenomenological problems it is expected to solve, might be more tricky than we thought. However, I do not think we are yet at a stage to declare supersymmetry phenomenology dead, and I keep on thinking there is still room for such a phenomenological motivation for supersymmetry.

An even more profound role supersymmetry is believed to play in the dynamics and ultimate structure of space-time, in the way gravity behaves at very high energy, as high as the Planck scale, via string theory. The latter is the more successful framework to describe all interactions, including gravity, in a way consistent with quantum mechanics. However, differently from an ordinary quantum field theory, string theory is inherently supersymmetric. From this point of view, no matter the scale at which it might show up, supersymmetry looks as a crucial ingredient in our understanding of the ultimate laws of Nature.

Supersymmetry is also at the core of what is probably the more amazing and far-reaching discovery in theoretical physics in the last few decades, the celebrated AdS/CFT correspondence. In short, this correspondence predicts that a (non-gravitational) QFT in d space-time dimensions can actually be dual to a theory of quantum gravity in one dimension higher. This means that the two theories are equivalent at the full quantum level and, upon using a proper dictionary, all observables agree. The best studied (and solid) examples of such remarkable duality involve supersymmetric QFTs in d-dimensional Minkowski space and (super)string theory in $d+1$-dimensional anti-de Sitter space. This is why supersymmetric quantum field theories have now also become a tool to study quantum gravity.

Supersymmetry turns out to be relevant also outside the realm of particle physics, like in some condensed matter systems, and it has also be at the core of what is probably the more amazing and far-reaching discovery in theoretical physics in the last decades, namely the celebrated AdS/CFT correspondence.

One other thing we, theoretical physicists, want to understand is the behavior of quantum field theories at strong coupling. This is a regime where usual perturbative techniques fail and we lack analytical tools. However, many phenomena we observe in Nature are described by the behavior of quantum field theories in such a regime, the most notable example being the way phenomena like confinement, dynamical mass generation and chiral symmetry breaking are realized in Nature. One spectacular property of supersymmetry is that it makes these phenomena more accessible: supersymmetric quantum field theories turn out to have a much more constrained dynamics with respect to non-supersymmetric ones, so constrained that it is often possible to understand their strong coupling regime analytically. In this regard, supersymmetry is seen (and is being used) as a theoretical laboratory to study quantum field theories at strong coupling and get some intuition on how phenomena like those mentioned above are realized in non-supersymmetric field theories (as QCD). Remarkably, several ideas that had been proposed to account for such phenomena and which could only be conjectural as far as ordinary quantum field theories, have been analytically proven in the supersymmetric context, notable examples being that confinement is due to monopole condensation, or that at strong coupling fermion bilinears condense. From this point of view, even setting aside its phenomenological or formal applications, supersymmetry is useful in that is a way in which we can deepen our understanding of QFT in general, seeing all of its features at work in a well-controlled setting.

I won't be able to discuss all these aspects in detail. The aim of this course is just to provide the minimum foundation you need to get into this fascinating subject and to give you some taste of some advanced topics. What to do with it... will be your choice.

In this first lecture I will give a brief overview on *what* is supersymmetry and *why* it is interesting to study it. In the rest of the course I will try to provide (much) more detailed answers to these two basic questions. I hope you will enjoy the journey!

1.1 What is supersymmetry?

Supersymmetry (SUSY) is a *space-time symmetry* mapping particles and fields of integer spin (bosons) into particles and fields of half integer spin (fermions), and viceversa. The generators Q act as

$$Q\,|Fermion\rangle \;=\; |Boson\rangle \quad \text{and viceversa} \tag{1.1}$$

From its very definition, this operator has two obvious but far-reaching properties that can be summarized as follows:

- It changes the spin of a particle (meaning that Q transforms as a spin-1/2 particle) and hence its space-time properties. This is why supersymmetry is not an internal symmetry but a space-time symmetry.
- In a theory where supersymmetry is realized, each one-particle state has at least a superpartner. Therefore, in a SUSY world, instead of single particle states, one has to deal with (super)multiplets of particle states.

Supersymmetry generators have specific commutation properties with other generators. In particular:

- Q commutes with translations and internal quantum numbers (e.g. gauge and global symmetries), but it does not commute with Lorentz generators

$$[Q, P_\mu] = 0 \quad , \quad [Q, G] = 0 \quad , \quad [Q, M_{\mu\nu}] \neq 0 \; . \tag{1.2}$$

  This implies that particles belonging to the same supermultiplet have different spin but same mass and same quantum numbers.

A supersymmetric field theory is a set of fields and a Lagrangian which exhibit such a symmetry. As ordinary field theories, supersymmetric theories describe particles and interactions between them: SUSY manifests itself in the specific particle spectrum a theory enjoys and in the way particles interact between themselves.

A supersymmetric model which is covariant under *general coordinate transformations* is called supergravity (SUGRA) model. In this respect, a non-trivial fact, which again comes from the algebra, in particular from the (anti)commutation relation

$$\{Q,\overline{Q}\} \sim P_\mu \ , \tag{1.3}$$

is that having general coordinate transformations is equivalent to have *local* SUSY, the gauge mediator being a spin 3/2 particle, the gravitino. Hence local supersymmetry and General Relativity are intimately tied together.

One can have theories with different number of SUSY generators Q: $Q^I \quad I = 1,...,N$. The number of supersymmetry generators, however, cannot be arbitrarily large. The reason is that *any supermultiplet contains particles with spin at least as large as* $\frac{1}{4}N$. Therefore, N can be at most as large as 4 for theories with maximal spin 1 (gauge theories) and as large as 8 for theories with maximal spin 2 (gravity). Thus stated, this statement is true in four space-time dimensions. Equivalent statements can be made in higher/lower dimensions, where the dimension of the spinor representation of the Lorentz group is larger/smaller (for instance, in 10 dimensions, which is the natural dimension where superstring theory lives, the maximum allowed N is 2). What really matters is the number of single state supersymmetry generators, which is a dimension-independent statement.

Finally, notice that since supersymmetric theories automatically accomodate both bosons and fermions, SUSY looks like the most natural framework where to formulate a theory able to describe matter and interactions in a unified way.

1.2 What is supersymmetry useful for?

Let us briefly outline a number of reasons why it might be meaningful (and useful) to have such a bizarre and unconventional symmetry actually realized in Nature.

i. Theoretical reasons.

- A central role in quantum field theory is played by the S-matrix, which encodes the (exact) information about physical processes between asymptotic states. A natural question one might ask is what are the more general allowed continuous symmetries of the S-matrix for a theory defined, say, in Minkowski space. More precisely, what are the possible symmetry generators that commute with the S-matrix, that take single-particle states into single-particle states, and whose action on multiparticle states is the direct sum of their action on single-particle states. In 1967 Coleman and Mandula proved a theorem which says that in a generic quantum field theory, under a number of (very reasonable and physical) assumptions, like *locality*, *causality*, *positivity of energy* and *finiteness of number of particles*, the only possible continuous such symmetries are those generated by Poincaré group generators, P_μ and $M_{\mu\nu}$, plus some internal symmetry group generators $\mathbb{G}$ commuting with them

$$[\mathbb{G}, P_\mu] = [\mathbb{G}, M_{\mu\nu}] = 0 \ , \tag{1.4}$$

where the group G is a semi-simple group times abelian factors.

In other words, the most general symmetry group enjoyed by the S-matrix is

Poincaré × Internal Symmetries

The Coleman-Mandula theorem can be evaded by weakening one or more of its assumptions. One such assumptions is that the symmetry algebra only involves *commutators*, all generators being *bosonic* generators. This assumption does not have any particular physical reason not to be relaxed. Allowing for *fermionic* generators, which satisfy anti-commutation relations, it turns out that the set of allowed symmetries can be enlarged. More specifically, in 1975 Haag, Lopuszanski and Sohnius showed that supersymmetry (which, as we will see, is a very specific way to add fermionic generators to a symmetry algebra) is the only possible such option. This makes the Poincaré group becoming SuperPoincaré. Therefore, the most general symmetry group the S-matrix can enjoy turns out to be

SuperPoincaré × Internal Symmetries

From a purely theoretical view point, one could then well expect that Nature might have realized all possible kind of allowed symmetries, given that we already know this is indeed the case for all known symmetries, but supersymmetry (i.e., the Standard Model.

- The history of our understanding of physical laws is an history of unification. A famous example is Newton's law of universal gravitation, which says that one and the same equation describes the attraction a planet exert on another planet and on... an apple! Maxwell equations unify electromagnetism with special relativity. Quantumelectrodynamics unifies electrodynamics with quantum mechanics. And so on and so forth, till the formulation of the Standard Model which describes in an unified way all known non-gravitational interactions. Supersymmetry (and its local version, supergravity), is the most natural candidate to complete this long journey. It is a way not just to describe in a unified way all known interactions, but in fact to describe matter *and* radiation all together. This sounds compelling, and from this view point it sounds natural studying supersymmetry and its consequences.

- There is one more reason as to why one could expect that supersymmetry is out there, after all. As already emphasized, as of today string theory stands up as the most satisfactory theory where to describe quantum gravity in a consistent way and, also, to describe all known interactions in a unified framework. So, it might very well be that Nature, at high enough energy, is described by string theory. Unlike a theory of fields, a theory of strings can only be made consistent if it is supersymmetric. So, in this sense, supersymmetry is predicted to be realized in Nature, if string theory is correct. Supersymmetry is in fact one of the two more striking predictions of string theory (the other being the existence of extra-dimensions).

Note: all above arguments suggest that supersymmetry maybe realized in Nature. However, none of such arguments give any obvious indication on the energy scale at which supersymmetry might show-up. In principle, this scale can be very high, as high as the Planck scale. Below, we will present few arguments, more phenomenological in nature, which suggest that low energy supersymmetry (as low as TeV scale or slightly higher) would be the preferred option.

ii. Phenomenological reasons.

- *Naturalness and the hierarchy problem.* Three out of four of the fundamental interactions among elementary particles (strong, weak and electromagnetic) are described by the Standard Model (SM). The typical scale of the SM, the electroweak scale, is

$$M_{\rm ew} \sim 250 \text{ GeV} \iff L_{\rm ew} \sim 10^{-16} \text{ mm} . \qquad (1.5)$$

The SM is very well tested up to such energies. This cannot be the end of the story, though: for one thing, at high enough energies, as high as the Planck scale $M_{\rm pl}$, gravity becomes comparable with other forces and cannot be neglected in elementary particle interactions. At some point, we need a quantum theory of gravity. Actually, the fact that $M_{\rm ew}/M_{\rm pl} << 1$ calls for new physics at a much *lower* scale. One way to see this, is as follows. The Higgs potential reads

$$V(H) \sim \mu^2|H|^2 + \lambda|H|^4 \quad \text{where} \quad \mu^2 < 0\ . \tag{1.6}$$

Experimentally, the minimum of such potential, $\langle H\rangle = \sqrt{-\mu^2/2\lambda}$, is at around 174GeV. This implies that the bare mass of the Higgs particle is roughly around 100 GeV or so, $m_H^2 = -\mu^2 \sim (100\text{GeV})^2$. What about radiative corrections? Scalar masses are subject to quadratic divergences in perturbation theory. The SM fermion coupling $-\lambda_f H\overline{f}f$ induces a one-loop correction to the Higgs mass as

$$\Delta m_H^2 \sim -\ 2\ \lambda_f^2\ \Lambda^2 \tag{1.7}$$

due to diagrams as the one in Fig. 1.1. A natural physical UV cut-off Λ

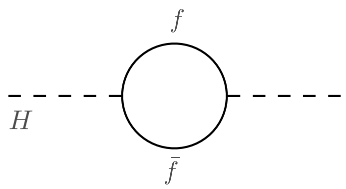

Figure 1.1: One-loop radiative correction to the Higgs mass due to fermion couplings.

should then be at around the TeV scale in order to protect the Higgs mass, and the SM should then be seen as an effective theory valid up to energies $E \leq M_{\rm eff} \sim$ TeV, well below the Planck scale.

What can be the new physics beyond such scale and how can such new physics protect the otherwise perturbative divergent Higgs mass? New physics, if any, may include new fermionic and bosonic fields, possibly coupling to the SM Higgs. Each of these fields will give radiative contributions to the Higgs mass of the kind above, hence, no matter what new physics will show-up at high energy, the natural mass for the the Higgs field would always be of order the UV cut-off of the theory, generically around $\sim M_{\rm pl}$. We would need a huge fine-tuning to get it stabilized at $\sim$ 100GeV (we now know that the physical

Higgs mass is at 125 GeV, in fact)! This is known as the *hierarchy problem*: the experimental value of the Higgs mass is unnaturally smaller than its natural theoretical value.

In principle, there is a very simple way out of this. This resides in the fact that (as you should know from your QFT course!) scalar couplings provide one-loop radiative contributions which are opposite in sign with respect to fermions. Suppose there exist some new scalar, S, with Higgs coupling $-\lambda_S|H|^2|S|^2$. Such coupling would also induce corrections to the Higgs mass via the one-loop diagram in Figure 1.2.

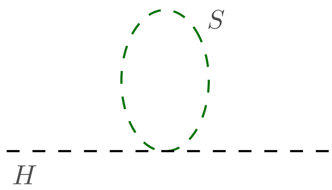

Figure 1.2: One-loop radiative correction to the Higgs mass due to scalar couplings.

Such corrections would have *opposite* sign with respect to those coming from fermion couplings, that is

$$\Delta m_H^2 \sim \lambda_S \, \Lambda^2 \ . \tag{1.8}$$

Therefore, if the new physics would be such that each quark and lepton of the SM were accompanied by two complex scalars having the same Higgs couplings of the quark and lepton, *i.e.* $\lambda_S = |\lambda_f|^2$, then all Λ^2 contributions would automatically cancel, and the Higgs mass would be stabilized at its tree level value! Such conspiracy, however, would be quite ad hoc, and not really solving the fine-tuning problem mentioned above; rather, just rephrasing it. A natural thing to invoke to have such magic cancellations would be to have a symmetry protecting m_H, right in the same way as gauge symmetry protects the masslessness of spin-1 particles. A symmetry imposing to the theory the correct matter content (and couplings) for such cancellations to occur. This is exactly what supersymmetry is: in a supersymmetric theory there are fermions and bosons (and couplings) just in the right way to provide *exact* cancellation between diagrams like the ones above. In summary, supersymmetry is a very natural and economic way (though not the only possible one) to solve the hierarchy problem.

Known fermions and bosons cannot be partners of each other. For one thing, we do not observe any degeneracy in mass in elementary particles that we know. Moreover, and this is possibly a stronger reason, quantum numbers do not match: gauge bosons transform in the adjoint representations of the SM gauge group while quarks and leptons in the fundamental or singlet representations. Hence, in a supersymmetric world, each SM particle should have a (yet not observed!) supersymmetric partner, usually dubbed *sparticle*. Roughly, the spectrum of such supersymmetric Standard Model (SSM) should be as follows

SM particles	**SUSY partners**
gauge bosons	gauginos
quarks, leptons	scalars
Higgs	higgsino

Notice: the (down) Higgs has the same quantum numbers as the scalar partner of neutrino and leptons, sneutrino and sleptons respectively, $(H_d^0, H_d^-) \leftrightarrow (\tilde{\nu}, \tilde{e}_L)$. Hence, one can imagine that the Higgs is in fact a sparticle. This cannot be. In such scenario, there would be phenomenological problems, *e.g.* lepton number violation and (at least one) neutrino mass in gross violation of experimental bounds.

In summary, the world we already had direct experimental access to, is not supersymmetric. If at all realized, supersymmetry should be a (spontaneously) broken symmetry in the vacuum state chosen by Nature. However, in order to solve the hierarchy problem without too much fine-tuning this scale should be not much higher than 1 TeV. Including lower bounds from present day experiments, it turns out that the SUSY breaking scale should be in the following energy range

$$10^2 \text{ GeV} \leq \text{SUSY breaking scale} \leq 10^3 - 10^4 \text{ GeV} .$$

Let us emphasize that these bounds are just a crude and rough estimate, as they depend very much on the specific SSM one is actually considering. In particular, the upper bound can be made higher by enriching the structure of the SSM in various ways, while keeping naturalness as a guiding principle. In any event, these bounds are the basic reason why it was believed SUSY to show-up at the LHC.

It is worth stressing that, as of today, no signal of supersymmetry has been found at LHC or elsewhere and this has made the above upper bounds more and more in tension with experimental data, and in turn the very idea of naturalness being reconsidered, at least in this context. There are ongoing discussions on these aspects, including the idea that the resolution of the hierarchy problem should not use naturalness as a guiding principle and that it should be explained by something different, as for instance anthropic arguments or something we do not yet fully understand.

- *Gauge coupling unification.* There is another reason to believe in (low energy) supersymmetry; possibly stronger, from a phenomenological point of view, than that provided by the hierarchy problem. Forget about supersymmetry for a while, and consider the $SU(3)\times SU(2)_L\times U(1)_Y$ SM as it stands. Interesting enough, besides the EW scale, the SM contains in itself a new scale of order 10^{15} GeV. The three SM gauge couplings run according to RG equations like

$$\frac{4\pi}{g_i^2(\mu)} = \frac{b_i}{2\pi}\ln\frac{\mu}{\Lambda_i} \qquad i = 1, 2, 3 \ . \tag{1.9}$$

At the EW scale, $\mu = M_Z$, there is a hierarchy between them, $g_1(M_Z) < g_2(M_Z) < g_3(M_Z)$. But RG equations make this hierarchy changing with the energy scale. In fact, supposing there are no particles other than the SM ones, at a much higher scale, $M_{GUT} \sim 10^{15}$GeV, the three couplings tend to meet! This naturally calls for a Grand Unified Theory (GUT), where the three interactions are unified in a single one, two possible GUT gauge groups being $SU(5)$ and $SO(10)$. The symmetry breaking pattern one should have in mind would then be as follows

$$\begin{array}{ccc} SU(5) & \rightarrow & SU(3)\times SU(2)_L\times U(1)_Y \rightarrow SU(3)\times U(1)_{\rm em} \\ & \phi & \qquad\qquad\qquad\qquad H \end{array}$$

where ϕ is an *heavy* Higgs inducing spontaneous symmetry breaking at energies $M_{GUT} \sim 10^{15}$GeV, and H the SM *light* Higgs, inducing EW spontaneous symmetry breaking around the TeV scale. This idea makes a lot of sense but poses several problems. First, there is a new hierarchy problem (generically, the SM Higgs mass is expected to get corrections from the heavy Higgs ϕ). Second, there is a proton decay problem: some of the additional gauge bosons predicted by the GUT group mediate baryon number violating transitions, allowing processes as $p \to e^+ + \pi_0$. This makes the proton not fully stable and it turns out that its expected lifetime in such GUT framework is violated by present experimental bounds. Finally, on a more theoretical side, if we do not allow for new particles besides the SM ones to be there at some intermediate scale, the three gauge couplings only *approximately* meet and it turns out that this cannot be taken care of just by experimental uncertainties. The latter is an unpleasant feature: small numbers are unnatural from a theoretical view point, unless there are specific reasons (as symmetries) justifying their otherwise unnatural smallness.

Remarkably, making the GUT supersymmetric (SGUT) solves all of these problems in a glance! As already emphasized, with supersymmetry, the Higgs mass is automatically protected. Moreover, just allowing for the minimal supersymmetric extension of the SM spectrum, known as MSSM, the three gauge couplings do meet (more precisely, they miss but now well within experimental uncertainties). Finally, the GUT scale is raised enough, up to around 10^{16} GeV, so to let proton decay rate being compatible with experimental bounds. So, supersymmetry makes the very natural idea of gauge coupling unification via a GUT free of any apparent drawbacks.

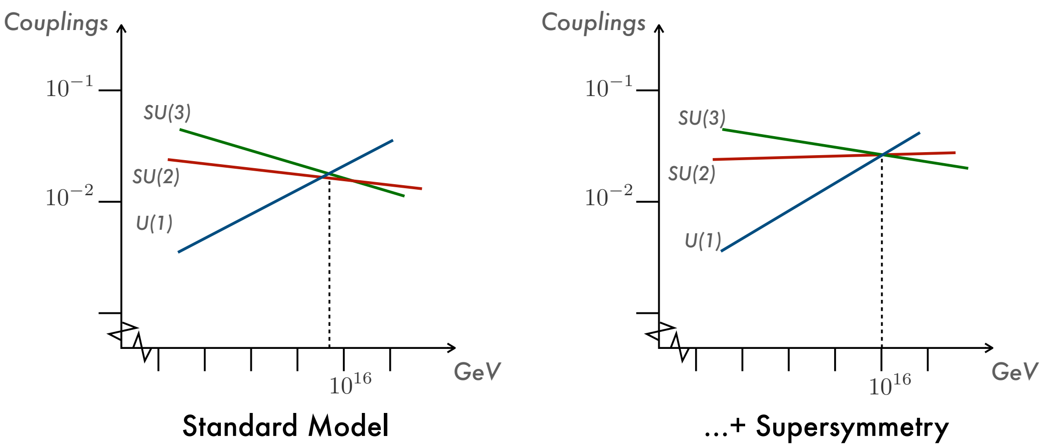

Figure 1.3: On the left a qualitative picture describing the running of the three SM couplings, which approximately meet at a scale of order 10^{15} GeV. On the right, the same picture in a minimal supersymmetric extension of the SM, where the couplings exactly meet (within experimental uncertainties) at a scale of order 10^{16} GeV.

<u>*Disclaimer*</u>: the MSSM is not the only possible option for supersymmetry beyond the SM, just the most economic one. In the MSSM one just adds a superpartner to each SM particle, therefore introducing the higgsino, the wino, the zino, together with all squarks and sleptons, and no more. [There is in fact an exception. To have a meaningful model one has to double the Higgs sector, and have two Higgs doublets. One reason for that is gauge anomaly cancellation: the higgsinos are fermions in the fundamental representation of $SU(2)_L$ hence two of them are needed, with opposite hypercharge, not to spoil the anomaly-free properties of the SM. A second reason is that in the SM the field H gives mass to down quarks and charged leptons while its charge conjugate, $H^c(\sim \overline{H})$ gives mass to up quarks. As we will see, in a SUSY model $\overline{H}$ cannot enter in the potential, which is a function of H, only. Therefore, in a supersymmetric scenario, to give mass to up quarks one needs a second, independent Higgs doublet.] There exist many non-minimal supersymmetric extensions of the Standard Model (which, in fact, are in better shape against experimental constraints with respect to the MSSM). One can in principle construct any SSM one likes. In doing so, however, several constraints are to be taken into account. For example, it is not so easy to make such non-minimal extensions keeping the nice exact gauge coupling unification enjoyed by the MSSM.

It is worth stressing that gauge coupling unification and the hierarchy problem are independent issues. Indeed, for the former to hold one does not need a full supersymmetric spectrum at low energy. Only light fermionic partners are needed. Scalar partners of SM fermions sit in full GUT families so they do not contribute to gauge coupling unification; they just shift all couplings by one and the same constant. At the price of forgetting about naturalness, this observation opened-up the idea that the SUSY spectrum can be split - with light fermions and heavy scalars - with supersymmetry being realized only at high energy. This scenario goes under the name of Split Supersymmetry.

- *Supersymmetry and dark matter.* Another context where supersymmetry might play an important role is cosmology. There are various evidences which indicate that around 26% of the energy density in the Universe should be made of *dark matter*, *i.e.* non-luminous and non-baryonic matter. The only SM candidates for dark matter are neutrinos, but they are disfavored by available experimental data (basically, neutrinos are too light to account for such an enormous energy density). Supersymmetry provides instead a valuable and very natural dark matter candidate: the neutralino. Neutralinos are mass eigenstates of a linear superposition of the supersymmetric partners of the neutral Higgs and of the SU(2) and U(1) neutral gauge bosons

$$\chi_i = \alpha_{i1}\widetilde{B}^0 + \alpha_{i2}\widetilde{W}^0 + \alpha_{i3}\widetilde{H}_u^0 + \alpha_{i4}\widetilde{H}_d^0 \ . \tag{1.10}$$

In most SUSY frameworks the neutralino is the lightest supersymmetric particle (LSP), and fully stable, as a dark matter candidate should be.

iii. Supersymmetry as a theoretical laboratory for strongly coupled gauge dynamics.

- What if supersymmetry will turn out not to be the correct theory to describe beyond the Standard Model physics? Or, worse, what if supersymmetry will turn out not to be realized at all, in Nature (something we could hardly ever being able to prove, in fact)? Interestingly, there is yet another reason which makes it worth studying supersymmetric theories, independently from the role supersymmetry might or might not play as a theory describing high energy physics.

  Let us consider non-abelian gauge theories, which strong interactions are an example of. Every time a non-abelian gauge group remains unbroken at low energy, we have to deal with strong coupling. The typical questions one should try and answer (in QCD or similar theories) are:

  - The bare Lagrangian is described in terms of quark and gluons, which are UV degrees of freedom. Which are the IR (light) degrees of freedom of QCD? What is the effective Lagrangian in terms of such degrees of freedom?
  - Strong coupling physics is very rich. Typically, one has to deal with phenomena like confinement, charge screening, the generation of a mass gap, etc.... Is there any theoretical understanding of such phenomena?
  - It is believed that the QCD vacuum is populated by *vacuum condensates* of fermion bilinears, $\langle\Omega|\overline{\psi}\psi|\Omega\rangle \neq 0$, which induce chiral symmetry breaking. What is the microscopic mechanism behind this phenomenon?

  Most of the IR properties of QCD have eluded so far a clear understanding, since we lack analytical tools to deal with strong coupling dynamics. Most results come from lattice computations, but these do not furnish a first principle understanding of the above phenomena. Moreover, they are formulated in Euclidean space and are not suited to discuss, *e.g.* transport properties.

Because of their nice renormalization properties, supersymmetric theories are more constrained than ordinary field theories and let one have a better control on strong coupling regimes, sometime. Therefore, one might hope to use them as toy models where to study properties of more realistic theories, such as QCD, in a more controlled way. Indeed, as we shall see, supersymmetric theories do provide examples where some of the above strong coupling effects can be studied exactly! This is possible due to powerful non-renormalizations theorems supersymmetric theories enjoy, and because of a very special property of supersymmetry, known as *holomorphy*, which in certain circumstances lets one compute several non-perturbative contributions to the Lagrangian exactly. We will spend a sizeable amount of time discussing these issues in the second part of this course.

This is all we wanted to say in this introductory chapter, which should be regarded just as an invitation to supersymmetry and its fascinating world. Let us end by just adding a curious historical remark. Supersymmetry did not first appear in ordinary four-dimensional quantum field theories but in string theory, at the very beginning of the seventies. Only later it was shown to be possible to have supersymmetry in ordinary quantum field theories.

1.3 Some useful references

The list of references in the literature is endless. Below I list some old and more recent books plus some reviews which are available on the Archive dialing at

https://arxiv.org/multi?group=grp

Some of these references may be better than others, depending on the specific topic one is interested in (and on personal taste). In preparing these lectures I have used most of them, some more, some less. A collection of references is also given at the end of each chapter, where I refer to some original papers, review articles or textbooks that I found useful for preparing the material presented there. This will help the reader to be guided if she/he wants to deepen any specific topics and have access to the original font... and it also let me give proper credit to authors.

1. **Historical references**

   - S. J. Gates, M. T. Grisaru, M. Rocek and W. Siegel
     Superspace Or One Thousand and One Lessons in Supersymmetry
     Front. Phys. **58** (1983) 548 p.
   - J. Wess and J. Bagger
     Supersymmetry and supergravity
     Princeton, USA: Univ. Pr. (1992) 259 p.
   - P. C. West
     Introduction to supersymmetry and supergravity
     Singapore: World Scientific (1990) 425 p.
   - M. F. Sohnius
     Introducing Supersymmetry
     Phys. Rep. 128 (1985)

2. **Some more recent books**

   - S. Weinberg
     The quantum theory of fields. Vol. 3: Supersymmetry
     Cambridge, UK: Univ. Pr. (2000) 419 p.
   - J. Terning
     Modern supersymmetry: Dynamics and duality
     Oxford University Press (2006) 324 p.
   - M. Dine
     Supersymmetry and string theory: Beyond the standard model
     Cambridge University Press (2007) 515 p.
   - H.J. Müller-Kirsten and A. Wiedemann
     Introduction to Supersymmetry
     Singapore: World Scientific (2010) 439 p.
   - S. Cecotti
     Supersymmetric field theories
     Cambridge University Press (2015) 424 p.

3. **On-line reviews: bases**

   - J. D. Lykken
     Introduction to Supersymmetry

TASI 96
arXiv:hep-th/9612114

- S. P. Martin
  A Supersymmetry Primer
  Adv. Ser. Direct. High Energy Phys. 21 (2010) [arXiv:hep-ph/9709356]

- A. Bilal
  Introduction to supersymmetry
  arXiv:hep-th/0101055

- J. Figueroa-O'Farrill
  BUSSTEPP Lectures on Supersymmetry
  arXiv:hep-th/0109172

- M. J. Strassler
  An Unorthodox Introduction to Supersymmetric Gauge Theory
  TASI 2001
  arXiv:hep-th/0309149

- R. Argurio, G. Ferretti and R. Heise
  An introduction to supersymmetric gauge theories and matrix models
  Int. J. Mod. Phys. A **19** (2004) 2015 [arXiv:hep-th/0311066]

4. **On-line reviews: advanced topics**

- K. A. Intriligator and N. Seiberg
  Lectures on supersymmetric gauge theories and electric-magnetic duality
  Nucl. Phys. Proc. Suppl. 45BC (1996) [arXiv:hep-th/9509066]

- A. Bilal
  Duality in N=2 SUSY SU(2) Yang-Mills Theory: A pedagogical introduction to the work of Seiberg and Witten
  arXiv:hep-th/9601007

- L. Alvarez-Gaume and S. F. Hassan
  Introduction to S duality in N=2 supersymmetric gauge theories: A pedagogical review of the work of Seiberg and Witten
  Fortsch. Phys. 45 (1997) [arXiv:hep-th/9701069]

- M. E. Peskin
  Duality in Supersymmetric Yang-Mills Theory

TASI 96
arXiv:hep-th/9702094

- M. Shifman
Non-Perturbative Dynamics in Supersymmetric Gauge Theories
Prog. Part. Nucl. Phys. 39 (1997) [arXiv:hep-th/9704114]

- P. Di Vecchia
Duality in supersymmetric $N = 2, 4$ gauge theories
Les Houches Summer School 1997
arXiv:hep-th/9803026

- E. D'Hoker and D. H. Phong
Lectures on supersymmetric Yang-Mills theory and integrable systems
CRM Summer School 1999
arXiv:hep-th/9912271

- P. Argyres
Lectures on Supersymmetry
available at `http://www.physics.uc.edu/~argyres/661/index.html`

- Y. Tachikawa
N=2 supersymmetric dynamics for pedestrians
Lect.Notes Phys. 890 (2014) [arXiv:1312.2684 [hep-th]]

5. **On-line reviews: supersymmetry breaking**

- G. F. Giudice and R. Rattazzi
Theories with Gauge-Mediated Supersymmetry Breaking
Phys. Rept. 322 (1999) [arXiv:hep-ph/9801271]

- E. Poppitz and S. P. Trivedi
Dynamical Supersymmetry Breaking
Ann. Rev. Nucl. Part. Sci. 48 (1998) [arXiv:hep-th/9803107]

- Y. Shadmi and Y. Shirman
Dynamical Supersymmetry Breaking
Rev. Mod. Phys. 72 (2000) [arXiv:hep-th/9907225]

- M. A. Luty
2004 TASI Lectures on Supersymmetry Breaking
arXiv:hep-th/0509029

- Y. Shadmi
  Supersymmetry breaking
  arXiv:hep-th/0601076
- K. A. Intriligator and N. Seiberg
  Lectures on Supersymmetry Breaking
  Class. Quant. Grav. **24** (2007) [arXiv:hep-ph/0702069]