

7 Supersymmetry breaking

If supersymmetry is at all realized in Nature, it must be broken at low enough energy: we do not see any mass degeneracy in the elementary particle spectrum, at least at energies of order 10^2 GeV or lower. The idea is then that supersymmetry is broken at some scale M_s , such that at energies $E > M_s$ the theory behaves in a supersymmetric way, while at energies $E < M_s$ it does not. On general ground, there are two ways supersymmetry can be broken, either spontaneously or explicitly.

- *Spontaneous supersymmetry breaking*: the theory is supersymmetric but has a scalar potential admitting (stable, or metastable but sufficiently long-lived) supersymmetry breaking vacua. In such vacua one or more scalar fields acquire a VEV of order M_s , which then sets the scale of supersymmetry breaking.
- *Explicit supersymmetry breaking*: the Lagrangian contains terms which do not preserve supersymmetry by themselves. In order for them not to ruin the nice and welcome UV properties of supersymmetric theories, these terms should have positive mass dimension, in other words they should be irrelevant in the far UV. In this case we speak of *soft* supersymmetry breaking. In such scenario, the scale M_s enters explicitly in the Lagrangian.

As we will later show, soft supersymmetry breaking models can (and typically do) actually arise as low energy effective descriptions of models where supersymmetry is broken spontaneously. Therefore, we will start focusing on spontaneous supersymmetry breaking. Only later we will discuss supersymmetry breaking induced by soft terms.

7.1 Vacua in supersymmetric theories

We have already seen that supersymmetric vacua are in one-to-one correspondence with the zero's of the scalar potential. In other words, the vacuum energy is zero if and only if the vacuum preserves supersymmetry. Hence, non-supersymmetric vacua correspond to minima of the potential which are not zero's. In this case supersymmetry is broken in the perturbative theory based on these positive energy vacua (aka spontaneous supersymmetry breaking).

Notice how different is spontaneous supersymmetry breaking with respect to spontaneous breaking of e.g. gauge symmetries. There, what matters is the location

of the minima of the potential in field space, while here it is the absolute *value* of the potential at such minima. This implies that in a supersymmetric gauge theory there can be minima which preserve both gauge symmetry and supersymmetry, others which break both, and others which preserve gauge symmetry and break supersymmetry, or viceversa. A schematic picture of these different situations is reported in figure 7.1.

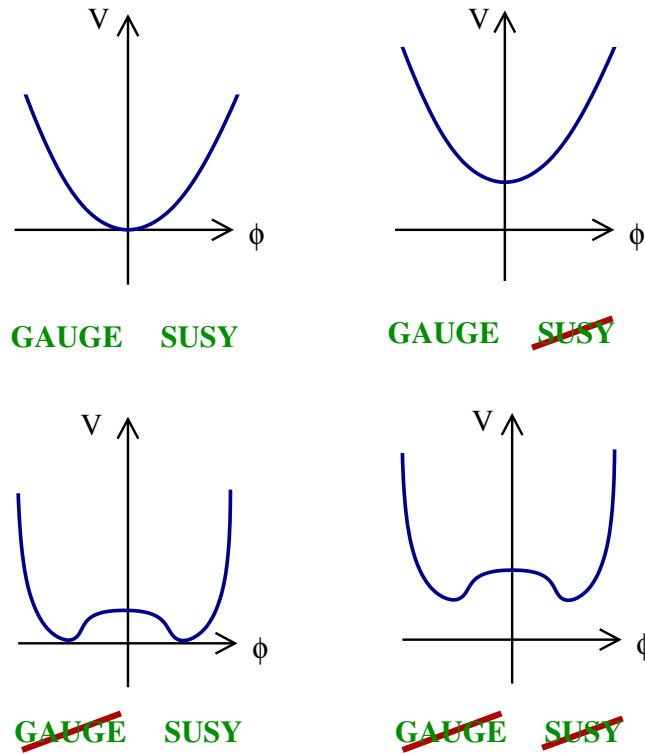


Figure 7.1: A schematic picture of possible patterns of spontaneous gauge symmetry and supersymmetry breakings. The potential on the upper left does not admit any symmetry breaking vacuum. The one on the lower right, instead, admits two vacua breaking *both* gauge symmetry and supersymmetry. The other two represent mixed situations where either supersymmetry or gauge symmetry are broken.

While non-supersymmetric vacua can be either global or local minima of the potential (corresponding to stable or metastable vacua, respectively), supersymmetric vacua, if present, are obviously global minima of the potential.

Recall the expression (5.81), that is

$$V(\phi, \bar{\phi}) = \bar{F}F + \frac{1}{2}D^2 \quad (7.1)$$

(we focus here on models with canonical Kähler potential; later we will also discuss situations where the Kähler potential is not canonical) where

$$\bar{F}_i = \frac{\partial W}{\partial \phi^i} \quad , \quad D^a = -g (\bar{\phi}_i (T^a)^i_j \phi^j + \xi^a) \quad . \quad (7.2)$$

Supersymmetric vacua are described by all possible set of scalar field VEVs satisfying the D and F-term equations

$$\bar{F}^i(\phi) = 0 \quad , \quad D^a(\bar{\phi}, \phi) = 0 \quad . \quad (7.3)$$

If there exist more than one possible solution, then we have more supersymmetric vacua, generically a moduli space of vacua, if these are not isolated. If there does not exist a set of scalar field VEVs for which eqs. (7.3) are satisfied, then supersymmetry is broken, and the minima of the potential are all necessarily positive, $V_{min} > 0$.

Notice, on the contrary, that on any vacuum, supersymmetric or not, global or local, the following equations always hold

$$\frac{\partial V(\phi, \bar{\phi})}{\partial \phi^i} = 0 \quad , \quad \frac{\partial V(\phi, \bar{\phi})}{\partial \bar{\phi}_i} = 0 \quad . \quad (7.4)$$

An equivalent statement about supersymmetric vacua is that on supersymmetric vacua the supersymmetric variations of the fermion fields vanish. This can be seen as follows. Due to Lorentz invariance, on a vacuum any field's VEV or its derivative should vanish, but scalar fields. Recalling how each field component of a chiral or vector superfield transforms under supersymmetry transformations, it then follows that on a vacuum state we have

$$\begin{aligned} \delta \langle \phi^i \rangle &= 0 \quad , \quad \delta \langle F^i \rangle = 0 \quad , \quad \delta \langle \psi_\alpha^i \rangle \sim \epsilon_\alpha \langle F^i \rangle \\ \delta \langle F_{\mu\nu}^a \rangle &= 0 \quad , \quad \delta \langle D^a \rangle = 0 \quad , \quad \delta \langle \lambda_\alpha^a \rangle \sim \epsilon_\alpha \langle D^a \rangle \quad . \end{aligned}$$

Therefore, in a generic vacuum the supersymmetric variations of the fermions is not zero: it is actually proportional to F and D-terms. A supersymmetric vacuum state is by definition supersymmetric invariant (!). Hence, from above equations it follows that on a supersymmetric vacuum also the supersymmetric variations of the fermions should be zero, the latter being equivalent to the D and F-term equations (7.3), as anticipated.

7.2 Goldstone theorem and the goldstino

When a global symmetry is spontaneously broken, Goldstone theorem says that there is a massless mode in the spectrum, the Goldstone field, whose quantum num-

bers should be related to the broken symmetry. We should expect this theorem to work also for spontaneously broken supersymmetry. In fact, given that supersymmetry is a fermionic symmetry, the Goldstone field should be in this case a Majorana spin 1/2 fermion, the so-called *goldstino*.

Consider the most general supersymmetric Lagrangian with gauge and matter fields, eq. (5.80), and suppose it admits some vacuum where supersymmetry is broken. In this vacuum eqs. (7.4) hold, while (some of) eqs. (7.3) do not. Recalling eqs. (5.79) and (5.81) we have in this vacuum that

$$\frac{\partial V(\phi, \bar{\phi})}{\partial \phi^i} = F^j(\bar{\phi}) \frac{\partial^2 W}{\partial \phi^i \partial \phi^j} - g D^a \bar{\phi}_j (T^a)_i^j = 0. \quad (7.5)$$

On the other hand, since the superpotential is gauge invariant, we have that

$$\delta^a W = \frac{\partial W}{\partial \phi^i} \delta^a \phi^i = \bar{F}_i (T^a)_j^i \phi^j = 0. \quad (7.6)$$

Combining the former equation with the complex conjugate of the latter evaluated in the vacuum, we easily get a matrix equation

$$M \begin{pmatrix} \langle F^j \rangle \\ \langle D^a \rangle \end{pmatrix} = 0 \quad \text{where} \quad M = \begin{pmatrix} \langle \frac{\partial^2 W}{\partial \phi^i \partial \phi^j} \rangle & -g \langle \bar{\phi}_l \rangle (T^a)_i^l \\ -g \langle \bar{\phi}_l \rangle (T^b)_j^l & 0 \end{pmatrix}. \quad (7.7)$$

This implies that the matrix M has an eigenvector with zero eigenvalue. Now, this matrix is nothing but the fermion mass matrix of the Lagrangian (5.80)! This can be seen looking at the non-derivative fermion bilinears of (5.80), which on the vacuum get contributions also from the cubic coupling between scalar fields, their superpartners and gauginos, and which can be written as

$$\dots - \frac{1}{2} \left(\psi^i, \sqrt{2}i\lambda^b \right) M \begin{pmatrix} \psi^j \\ \sqrt{2}i\lambda^a \end{pmatrix} + h.c. + \dots \quad (7.8)$$

Hence, on the supersymmetry breaking vacuum the spectrum necessarily admits a massless fermion, the goldstino. In terms of spin 1/2 particles belonging to the different multiplets, the goldstino ψ_α^G corresponds to the following linear combination

$$\psi_\alpha^G \sim \langle F^i \rangle \psi_\alpha^i + \langle D^a \rangle \lambda_\alpha^a. \quad (7.9)$$

Our proof of the goldstino theorem has been based on the Lagrangian (5.80). In fact, one can provide a similar proof using properties of the supercurrent and Ward

identities, which does not rely on the existence of an explicit classical Lagrangian. The supersymmetry Ward identity reads

$$\langle \partial^\mu S_{\mu\alpha}(x) \bar{S}_{\nu\dot{\beta}}(0) \rangle = -\delta^4(x) \langle \delta_\alpha S_{\nu\dot{\beta}} \rangle = -2 \sigma_{\alpha\dot{\beta}}^\mu \langle T_{\mu\nu} \rangle \delta^4(x) , \quad (7.10)$$

where the last equality follows from the current algebra, see eq. (4.71). This implies, for the two-point function of the supercurrent, that

$$\langle S_{\mu\alpha}(x) \bar{S}_{\nu\dot{\beta}}(0) \rangle = \dots + (\sigma_\mu \bar{\sigma}^\rho \sigma_\nu)_{\alpha\dot{\beta}} \frac{x_\rho}{x^2} \langle T \rangle , \quad (7.11)$$

where $\langle T \rangle = \eta^{\mu\nu} \langle T_{\mu\nu} \rangle$ and the \dots are terms which are not relevant to the present discussion. Upon Fourier transforming eq. (7.11) we get

$$\langle S_{\mu\alpha}(k) \bar{S}_{\nu\dot{\beta}}(-k) \rangle = \dots + (\sigma_\mu \bar{\sigma}^\rho \sigma_\nu)_{\alpha\dot{\beta}} k_\rho \frac{\langle T \rangle}{k^2} , \quad (7.12)$$

which shows the presence of a massless pole (the goldstino), proportional to the vacuum energy density, in the supercurrent two-point function. The above equation shows that the goldstino is the lowest energy excitation of the supercurrent, and it is so if and only if the vacuum energy is non-vanishing. This shows, as anticipated, that the goldstino theorem holds universally, *i.e.*, also vacua where supersymmetry is broken in a strongly coupled phase of the theory, where classical arguments may not apply.

From our discussion it should be clear that given a generic Lagrangian, there are two a priori independent ways we can break supersymmetry: either giving a non vanishing expectation value to (some) F-terms or to (some) D-terms. In what follows, we will consider the two possibilities in turn.

7.3 F-term breaking

Since we are focusing here on F-term breaking, let us assume for the time being to deal with a theory with chiral superfields, only. The most general renormalizable Lagrangian of this sort reads

$$\mathcal{L} = \int d^2\theta d^2\bar{\theta} \bar{\Phi}_i \Phi^i + \int d^2\theta W(\Phi^i) + \int d^2\bar{\theta} \bar{W}(\bar{\Phi}_i) , \quad (7.13)$$

where

$$W(\Phi^i) = a_i \Phi^i + \frac{1}{2} m_{ij} \Phi^i \Phi^j + \frac{1}{3} g_{ijk} \Phi^i \Phi^j \Phi^k . \quad (7.14)$$

The equations of motions for the auxiliary fields read

$$\bar{F}_i(\phi) = \frac{\partial W}{\partial \phi^i} = a_i + m_{ij}\phi^j + g_{ijk}\phi^j\phi^k, \quad (7.15)$$

and the potential is

$$V(\phi, \bar{\phi}) = \sum_i |a_i + m_{ij}\phi^j + g_{ijk}\phi^j\phi^k|^2. \quad (7.16)$$

Supersymmetry is broken if and only if there does not exist a set of scalar field VEVs such that all F-terms vanish, $\langle F^i \rangle = 0$. This implies that in order for supersymmetry to be broken, it is necessary some a_i to be different from zero. If not, the trivial solution $\langle \phi^i \rangle = 0$ solves all F-equations. So, any model of F-term supersymmetry breaking needs a superpotential admitting linear terms (notice that this conclusion applies also for a superpotential with higher non-renormalizable couplings).

This result holds also in presence of a non-canonical Kähler potential. This can be easily seen recalling the expression of the scalar potential when a non-canonical Kähler metric is present

$$V(\phi, \bar{\phi}) = (K^{-1})^i_j \frac{\partial W}{\partial \phi^i} \frac{\partial \bar{W}}{\partial \bar{\phi}_j}. \quad (7.17)$$

From this expression it is clear that unless it were singular (something signalling, as already discussed, an inconsistency of the effective theory analysis), a non-trivial Kähler metric could not influence the existence/non existence of supersymmetric vacua, which is still dictated by the possibility/impossibility to make the first derivatives of the superpotential vanish. The only (and important) properties which get modified by a non-trivial Kähler potential would be the value of the vacuum energy (for non-supersymmetric vacua, only!) and the particle spectrum around a given vacuum.

Given this minimal necessary requirement, in what follows we will consider several examples of F-term breaking.

Example 1 : The Polonyi model.

Let us consider the theory of a single chiral superfield with canonical Kähler potential and a linear superpotential

$$K = \bar{X}X, \quad W = \lambda X. \quad (7.18)$$

This is the most minimal set-up one can imagine for a F-term supersymmetry breaking model. The potential reads

$$V = \left| \frac{\partial W}{\partial X} \right|^2 = |\lambda|^2, \quad (7.19)$$

Supersymmetry is clearly broken for any $|X|$, and the latter is in fact a flat direction. The supersymmetry breaking scale is set by the modulus of λ itself, $|\lambda| = M_s^2$. In Figure 7.2 we report the (trivial) shape of the scalar potential.

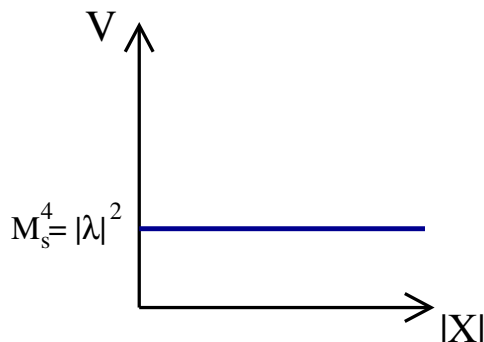


Figure 7.2: The potential of the Polonyi model.

A few comments are in order. First notice that the theory possesses an R-symmetry, the R-charge of X being $R(X) = 2$. At a generic point of the moduli space, then, both supersymmetry and R-symmetry are broken. Second, notice that although supersymmetry is broken, the spectrum is degenerate in mass: $|X|$, its phase α , and ψ_X are all massless. The fermion field has a good reason to be massless: it is the goldstino predicted by Goldstone theorem. Seemingly, the phase of X is expected to be massless: it is nothing but the goldstone boson associated to the broken R-symmetry. Finally, the modulus of the scalar field $|X|$ is massless since it parametrizes the (non-supersymmetric) moduli space. As already noticed, however, such moduli space is not protected, in principle, against quantum corrections, since supersymmetry is broken. Hence, generically, one would expect it to be lifted at the quantum level and $|X|$ to get a mass. This is not the case in this simple theory, since it is a non-interacting theory, and there are no quantum corrections whatsoever. In general, however, things are different: a non-supersymmetric moduli space gets typically lifted at one or higher loops, and the putative moduli get a mass. For this reason, non-supersymmetric moduli spaces are dubbed *pseudo-moduli* spaces, and the moduli parametrizing them, pseudo-moduli. We will see examples of this sort soon.

Let us now consider the following innocent-looking modification of the model above. Let's add a mass term to X ,

$$\Delta W = \frac{1}{2}mX^2 . \quad (7.20)$$

Things drastically change, since supersymmetry is now restored. Indeed, the F-term equation now reads

$$\bar{F}(X) = mX + \lambda = 0 , \quad (7.21)$$

which admits the solution $\langle X \rangle_{SUSY} = -\frac{\lambda}{m}$. Hence, there is a choice of scalar field VEV which makes the potential $V = |\lambda + mX|^2$, depicted in figure 7.3, vanish.

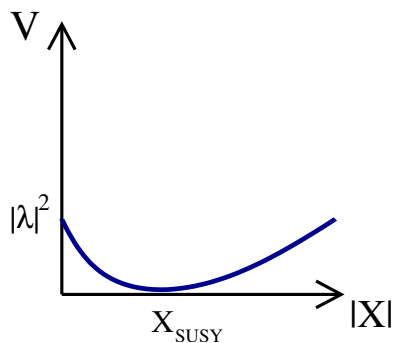


Figure 7.3: The potential of the massive Polonyi model.

The spectrum is supersymmetric *and* massive: all fields have mass m . This agrees with physical expectations: ψ_X is no more the goldstino, since this is not expected to be there now; $|X|$ is no more a (pseudo)modulus since the supersymmetric vacuum is isolated (the VEV of $|X|$ is not a flat direction); finally, α is not anymore the goldstone boson associated to the broken R-symmetry since the superpotential term ΔW breaks the R-symmetry explicitly (more precisely, $W = \lambda X + \frac{1}{2}mX^2$ does not admit any R-charge assignment for X such that $R(W) = 2$).

Things might also change (both qualitatively and quantitatively) if one allows the Kähler potential not being canonical. Suppose we keep $W = \lambda X$ but we let the Kähler metric be non-trivial, that is

$$V = (K_{X\bar{X}})^{-1} |\lambda|^2 \quad \text{with} \quad K_{X\bar{X}} = \frac{\partial^2 K}{\partial X \partial \bar{X}} \neq 1 . \quad (7.22)$$

A non-trivial Kähler metric can deform sensibly the pseudo-moduli space of figure 7.2. A sample of possible different behaviors, which depend on the asymptotic properties (or singularities) of the Kähler metric, is reported in Figure 7.4.

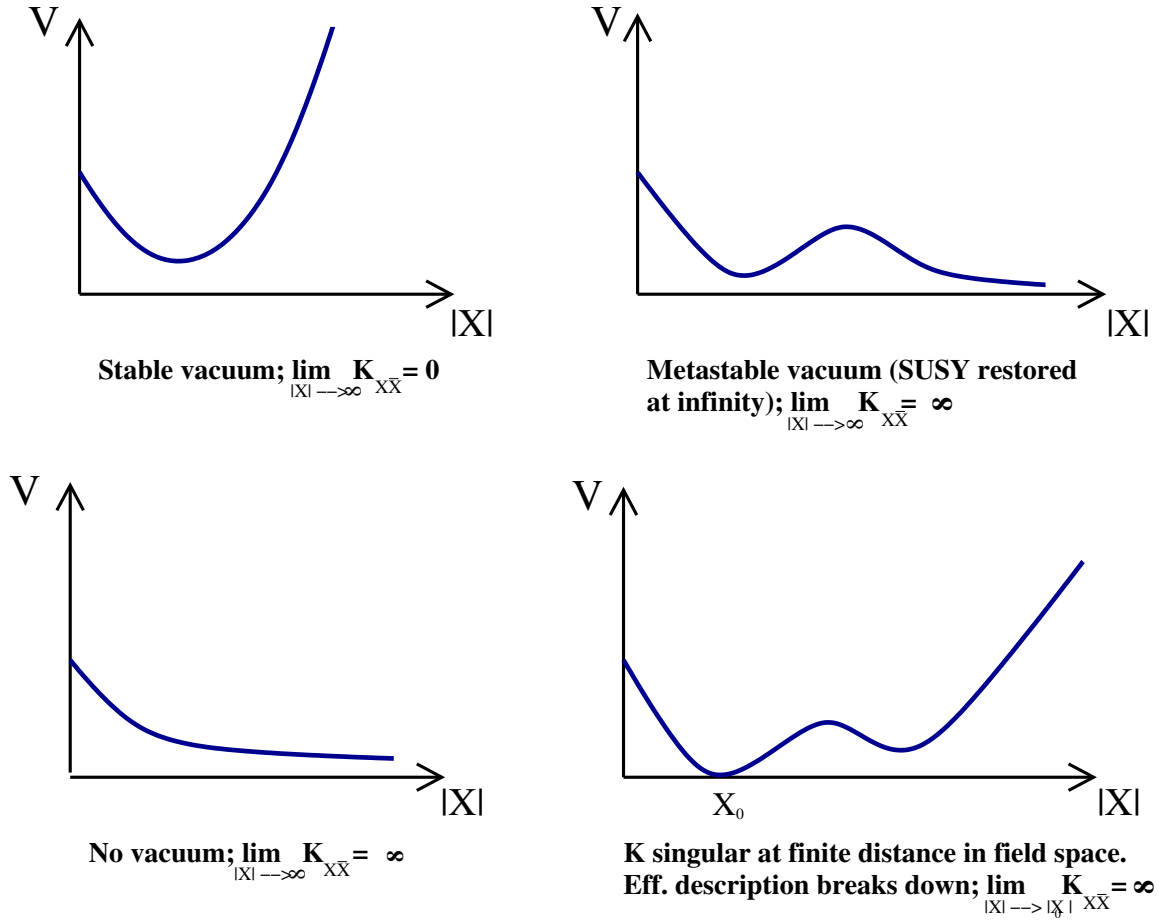


Figure 7.4: Different potentials of non-canonical Polonyi-like models.

Physically, the different behaviors reported in figure 7.4 should be understood as follows. In presence of a classical pseudomoduli space like the one in figure 7.2, the lifting of the pseudomoduli at quantum level occurs because the massless particles which bring from a vacuum to another get a mass at one loop. Sometime, such effect can be mimicked by a non-canonical Kähler potential, like the one above. In fact, these seemingly ad-hoc theories can and sometime do arise at low energies as *effective theories* of more complicated UV-renormalizable ones: the mass scale entering the Kähler potential is nothing but the UV cut-off of these low energy effective theories. Let us try to make the above discussion more concrete by considering an explicit example.

Example 2: A Polonyi model with quartic Kähler potential.

Let us consider the following model

$$K = \bar{X}X - \frac{c}{\Lambda^2} (\bar{X}X)^2 \quad , \quad W = \lambda X \quad , \quad (7.23)$$

where $c > 0$. Notice that the R-symmetry is not broken by the non-canonical Kähler potential, which is R-symmetry invariant, as any D-term. So the $U(1)_R$ symmetry is a symmetry of the theory. The Kähler metric and the scalar potential now read

$$K_{X\bar{X}} = 1 - \frac{4c}{\Lambda^2} \bar{X}X \quad , \quad V = K^{X\bar{X}} |\lambda|^2 = \frac{|\lambda|^2}{1 - \frac{4c}{\Lambda^2} X\bar{X}} \quad . \quad (7.24)$$

The Kähler potential is an instance of models like those in the upper-left diagram of Figure 7.4: indeed the Kähler metric vanishes for large enough $|X|$, $|X| \rightarrow |\Lambda|/2\sqrt{c}$, which is order the natural cut-off of the theory. The potential, which is depicted in figure 7.5, admits a (unique) minimum at $\langle X \rangle = 0$. There is an *isolated* vacuum

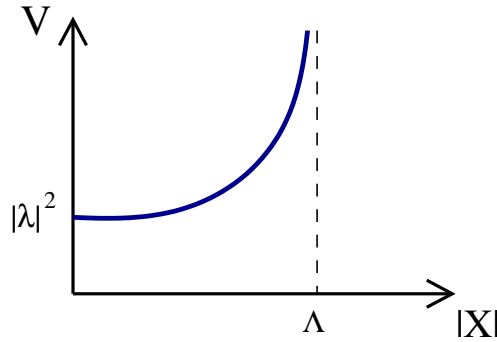


Figure 7.5: The potential of a Polonyi model with quartic Kähler potential (with $c = 1/4$).

now, and it is a supersymmetry breaking one. One can compute the spectrum around such vacuum and find that ψ_X is consistently massless (it is the goldstino), while now the scalar field is massive, $m_X^2 \sim c|\lambda|^2/\Lambda^2$.

Example 3: Supersymmetry restoration by new degrees of freedom.

Let us now deform the basic Polonyi model by adding a new superfield, Y , while keeping the Kähler potential canonical. The superpotential reads

$$W = \lambda X + \frac{1}{2} hXY^2 \quad . \quad (7.25)$$

Notice that this model has an R-symmetry, with R-charge assignment $R(X) = 2$, $R(Y) = 0$. From the F-equations one can compute the potential which reads

$$V = |hXY|^2 + \left| \frac{1}{2} hY^2 + \lambda \right|^2 \quad , \quad (7.26)$$

implying that there are two supersymmetric vacua at

$$\langle X \rangle_{SUSY} = 0 \quad , \quad \langle Y \rangle_{SUSY} = \pm \sqrt{-2\frac{\lambda}{h}} . \quad (7.27)$$

So we see that the additional degrees of freedom have restored supersymmetry. Interestingly, there are other *local* minima of the potential, a pseudo-moduli space in fact, where supersymmetry is broken

$$\langle X \rangle_{SB} = \text{any} \quad , \quad \langle Y \rangle_{SB} = 0 \quad \text{where} \quad V = |\lambda|^2 . \quad (7.28)$$

The physical interpretation is as follows. For large $\langle X \rangle$, the superfield Y gets a large mass and affects the low energy theory lesser and lesser. The theory reduces effectively to the original Polonyi model, which breaks supersymmetry and whose vacuum energy is indeed $V = |\lambda|^2$. It is a simple but instructive exercise to compute the mass spectrum around the non-supersymmetric minima. The chiral superfield X is obviously massless while Y gets a mass. There is a first (obvious) contribution to both the scalar and the fermion components of Y from $h\langle X \rangle$, and a second contribution which affects only the scalar component of Y coming from F_X , which is non-vanishing. The end result is

$$m_Y^2 = |h\langle X \rangle|^2 \pm |h\lambda| \quad , \quad m_{\psi_Y} = h\langle X \rangle . \quad (7.29)$$

From the above expressions, we see that the supersymmetry breaking pseudomoduli space has a tachyonic mode which develops (and destabilizes the vacuum) for

$$|X|^2 < |\lambda/h| \equiv |X_c|^2 . \quad (7.30)$$

In such region the potential decreases along the $\langle Y \rangle$ direction towards the supersymmetry vacua. A qualitative picture of the potential is reported in Figure 7.6.

Example 4 : Runaway behavior.

A minimal modification of the above theory gives a completely different dynamics. Let us suppose the cubic term of the superpotential (7.25) has the squared shifted from Y to X . We get

$$W = \lambda X + \frac{1}{2} h X^2 Y . \quad (7.31)$$

The potential reads

$$V = \left| \frac{1}{2} h X^2 \right|^2 + |hXY + \lambda|^2 , \quad (7.32)$$

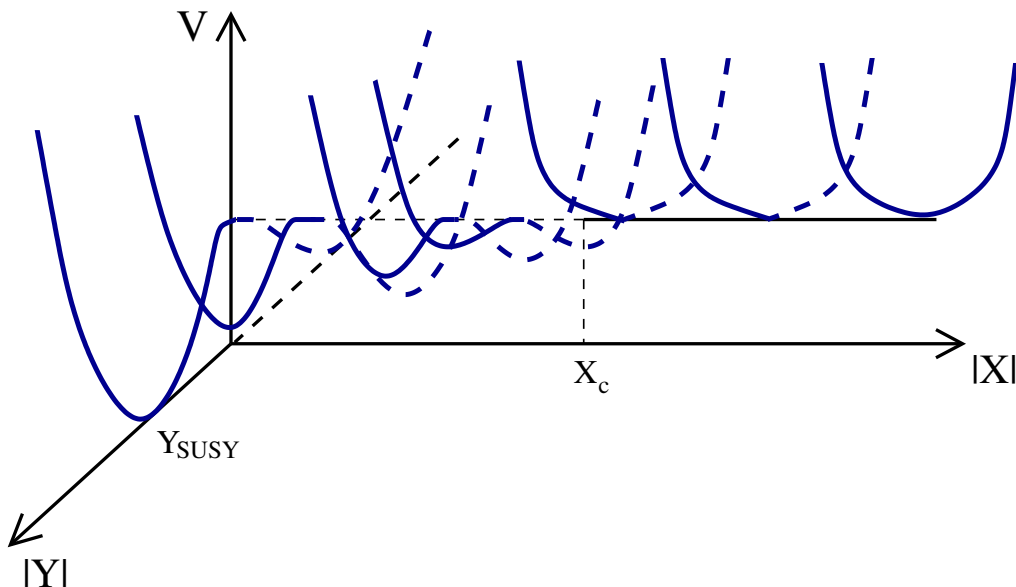


Figure 7.6: The potential of the model of Example 3: supersymmetry restoration.

and there are no supersymmetric ground states! Notice again that R-symmetry is preserved by the superpotential, with charge assignment $R(X) = 2$ and $R(Y) = -2$. Now the question is: where is the minimum of the potential? An analysis of V shows that the minimum is reached for $Y \rightarrow \infty$. This can be seen by setting $X = -\frac{\lambda}{hY}$, which kills the second contribution to the potential. By plugging this back into V one gets

$$V = \left| \frac{\lambda^2}{2hY^2} \right|^2 \xrightarrow{Y \rightarrow \infty} 0. \quad (7.33)$$

In other words, there is no stable vacuum but actually a *runaway* behavior and supersymmetry is restored at infinity in field space.

Notice that for large $|Y|$ the amount of supersymmetry breaking gets smaller and smaller and X mass larger and larger. Hence the theory can be described by a theory where X is integrated out. Doing so, the superpotential becomes

$$W_{\text{eff}} = -\frac{\lambda^2}{2hY}, \quad (7.34)$$

which gives the runaway behavior described by the potential (7.33).

Of all models we have been considering so far, the only renormalizable one which breaks supersymmetry in a stable vacuum is the original Polonyi model (in fact there is a all pseudo-moduli space). This model is however rather uninteresting per sé, since it describes a non-interacting theory. One might wonder whether there exist

reasonably simple models which are renormalizable, interacting and break supersymmetry in stable vacua. The simplest such model is the re-known O’Raifeartaigh model, which we now describe.

Example 5: The O’Raifeartaigh model.

Let us consider the theory of three chiral superfields with canonical Kähler potential and a superpotential given by

$$W = \frac{1}{2}hX\Phi_1^2 + m\Phi_1\Phi_2 - \mu^2X . \quad (7.35)$$

The superpotential respects the R-symmetry, the R-charge assignment being $R(X) = 2, R(\Phi_1) = 0$ and $R(\Phi_2) = 2$. The F-term equations read

$$\begin{cases} \bar{F}_X = \frac{1}{2}h\phi_1^2 - \mu^2 \\ \bar{F}_1 = hX\phi_1 + m\phi_2 \\ \bar{F}_2 = m\phi_1 \end{cases} \quad (7.36)$$

Clearly the first and the third equations cannot be solved simultaneously. Hence supersymmetry is broken. Let us try to analyze the theory a bit further. There are two dimensionful scales, μ and m . Let us choose in what follows $|\mu| < |m|$ (nothing crucial of the following analysis would change choosing a different regime). In this regime the minimum of the potential is at

$$\phi_1 = \phi_2 = 0 \quad , \quad X = \text{any} \quad (7.37)$$

and the vacuum energy is $V = |\mu^2|^2$. Again, we find a pseudo-moduli space of vacua since X is not fixed by the minimal energy condition. In Figure 7.7 we depict the potential as a function of the scalar fields.

Let us compute the (classical) spectrum around the supersymmetry breaking vacua. The full chiral superfield X is massless, right in the same way as for the Polonyi model (notice that for larger and larger $|X|$ the model gets closer and closer to the Polonyi model since all other fields get heavier and heavier). The massless fermion mode ψ_X is nothing but the goldstino (notice that the only non vanishing F-term in the vacuum is indeed F_X). The phase α of the scalar field $X = |X|e^{i\alpha}$ is the Goldstone boson associated to R-symmetry, which is spontaneously broken in the vacuum. Finally, $|X|$ is massless since it is a modulus (at least at classical level). One can easily compute the ($|X|$ -dependent) mass spectrum of all other fields and

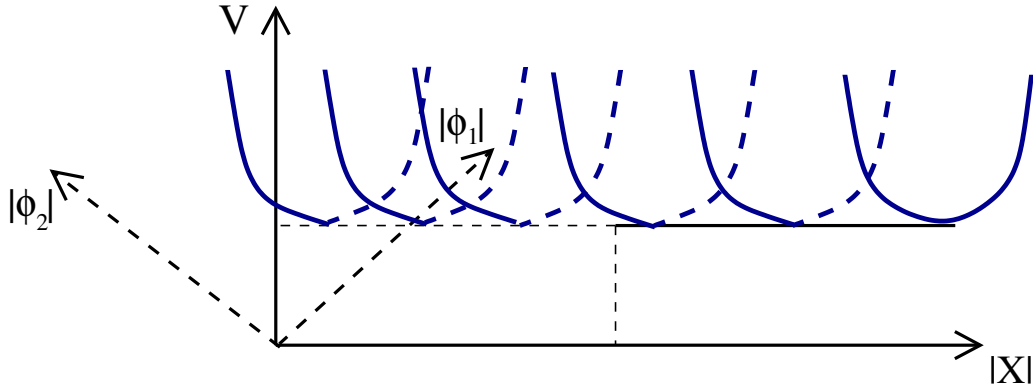


Figure 7.7: The potential of the O’Raifeartaigh model.

get

$$\begin{aligned}
m_0^2(|X|) &= |m|^2 + \frac{1}{2}\eta|h\mu^2| + \frac{1}{2}|hX|^2 \\
&\quad \pm \frac{1}{2}\sqrt{|h\mu^2|^2 + 2\eta|h\mu^2||hX|^2 + 4|m|^2|hX|^2 + |hX|^4} \\
m_{1/2}^2(|X|) &= \frac{1}{4}\left(|hX| \pm \sqrt{|hX|^2 + 4|m|^2}\right)^2 .
\end{aligned} \tag{7.38}$$

where $\eta = \pm 1$, giving different masses to the four real scalar modes belonging to Φ_1 and Φ_2 . As expected, the spectrum is manifestly non-supersymmetric. Notice that $m_{1/2}^2(|X|) = m_0^2(|X|)|_{\mu^2=0}$. Recall finally, that these infinitely many vacua are in fact physically inequivalent, since the mass spectrum depends on $|X|$.

Example 6: A modified O’Raifeartaigh model.

Let us end this overview of supersymmetry breaking models by considering a modification of the previous model. Let us add a (small) mass perturbation for Φ_2

$$\Delta W = \frac{1}{2}\epsilon m\Phi_2^2 \quad \text{with} \quad \epsilon \ll 1 . \tag{7.39}$$

Notice that this term breaks the R-symmetry enjoyed by the original O’Raifeartaigh model. The only F-equation which gets modified is the one for Φ_2 which now reads

$$\bar{F}_2 = m\phi_1 + \epsilon m\phi_2 . \tag{7.40}$$

The presence of the second term removes the conflict we had before between this equation and the F-equation for X . Hence we can solve all F-term equations simultaneously and supersymmetry is not broken anymore. The (two) supersymmetric

vacua are at

$$X = \frac{m}{h\epsilon} \quad , \quad \phi_1 = \pm \sqrt{\frac{2\mu^2}{h}} \quad , \quad \phi_2 = \mp \frac{1}{\epsilon} \sqrt{\frac{2\mu^2}{h}} \quad . \quad (7.41)$$

Notice that for $\epsilon \ll 1$ these vacua are far away, in field space, from where the supersymmetry breaking vacua of the O’Raifeartaigh model sit (the VEV of ϕ_2 becomes larger and larger and hence very far from $\phi_2 = 0$, the value of ϕ_2 in O’Raifeartaigh model supersymmetry breaking vacua). In fact, near the origin of field space the potential of the present model is practically identical to the one of the original O’Raifeartaigh model. Therefore, a classically marginal pseudo-moduli space of supersymmetry breaking minima is present.

Computing the mass spectrum near the origin one gets now

$$\begin{aligned} m_0^2(|X|) &= \frac{1}{2} \left\{ |hX|^2 + |m|^2 (2 + |\epsilon|^2) + \eta|h\mu^2| \right. \\ &\quad \left. \pm \sqrt{[|hX|^2 + |m|^2 (2 + |\epsilon|^2) + \eta|h\mu^2|]^2 - 4|m|^2 [|hX\epsilon - m|^2 + \eta|h\mu^2| (1 + |\epsilon|^2)]} \right\} \\ m_{1/2}^2(|X|) &= m_0^2(|X|)|_{\mu^2=0} \quad . \end{aligned} \quad (7.42)$$

A close look to the above spectrum shows that in order for mass eigenvalues being all positive, the following inequality should be satisfied

$$\left| 1 - \frac{\epsilon hX}{m} \right|^2 > (1 + |\epsilon|^2) \left| \frac{h\mu^2}{m^2} \right| \quad . \quad (7.43)$$

For small ϵ and μ/m , the marginally stable region described by the above inequality includes a large neighborhood around the origin, and the tachyonic mode develops only for $|X|$ (parametrically) larger than a critical value $|X_c|$ (notice that this is quite the opposite of what we got in Example 4, where the marginally stable region was *above* a critical value; as we will see, this difference has crucial consequences at the quantum level). For $\epsilon \rightarrow 0$ one gets that $X_c, X_{SUSY} \rightarrow \infty$ and the supersymmetric vacua are pushed all the way to infinity. This is consistent with the fact that for $\epsilon = 0$ one recovers the original O’Raifeartaigh model potential where supersymmetric vacua are not there. A rough picture of the potential is given in Figure 7.8.

For future reference, let me notice the following interesting fact. In all models we have been considering so far, the existence of an R-symmetry was a necessary condition for the existence of (stable) supersymmetry breaking vacua (think of the original Polonyi model in Example 1, the model in Example 2, the O’Raifeartaigh model of Example 5, and to some extent the model of Example 4). It is not a sufficient condition, however: think of the counterexample of the model of Example 3. On the

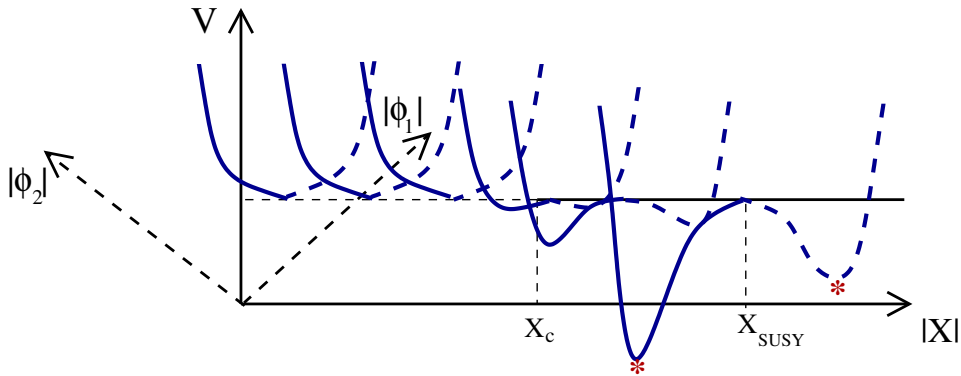


Figure 7.8: The (classical) potential of the *modified* O’Raifeartaigh model.

contrary, whenever superpotential terms explicitly breaking the R-symmetry were introduced (the massive Polonyi model of Example 1, and Example 6), supersymmetric vacua were found. Finally, every time we found metastable supersymmetry breaking vacua (Example 6 again), in the vicinity of such vacua an approximate R-symmetry, which the theory does not possess as an exact symmetry, was recovered (essentially, in Examples 6 the superpotential perturbation responsible for the explicit breaking of the R-symmetry becomes negligible near the metastable vacua). All this suggests some sort of relation between R-symmetry and supersymmetry breaking. We will discuss this issue later in this lecture, and put such apparent connection on a firm ground.

7.4 Pseudomoduli space: quantum corrections

In most supersymmetry breaking models we discussed, we found a pseudo-moduli space of non supersymmetric vacua. Associated to this, we found a massless scalar mode $|X|$. While the masslessness of the goldstino and of the Goldstone boson associated to R-symmetry breaking are protected by symmetries, there are no symmetries protecting the pseudo-modulus from getting a mass. There isn’t any symmetry relating the (degenerate in energy) non supersymmetric vacua. Therefore, by computing quantum corrections, one might expect this field to get a mass, somehow. Let us stress the difference with respect to a moduli space of supersymmetric vacua. Think about the harmonic oscillator. When we quantize the bosonic harmonic oscillator, the energy of the ground state gets a $\frac{1}{2}\hbar\omega$ contribution. On the contrary, if the ground state is fermionic, the contribution is the same in modulus but with opposite sign (fermions tend to push the energy down). In a supersymmet-

ric situation, the mass degeneracy between bosonic and fermionic degrees of freedom provides equal but opposite contribution to the vacuum energy and the total energy hence remains zero. In a non supersymmetric vacuum the degeneracy is not there anymore (think about the spectrum we computed in the O’Raifeartaigh model) so one expects things to change. In what follows we will try to make this intuition concrete by computing the one-loop Coleman-Weinberg effective potential for both the O’Raifeartaigh and the modified O’Raifeartaigh models. In practice, what we have to do is to compute corrections in the coupling h at one loop in the background where the pseudo-modulus $|X|$ has a non-vanishing VEV.

For a supersymmetric theory the Coleman-Weinberg potential reads

$$V_{\text{eff}} = \frac{1}{64\pi^2} \text{STr } \mathcal{M}^4 \log \frac{\mathcal{M}^2}{\Lambda^2} = \frac{1}{64\pi^2} \left(\text{Tr } m_B^4 \log \frac{m_B^2}{\Lambda^2} - \text{Tr } m_F^4 \log \frac{m_F^2}{\Lambda^2} \right) , \quad (7.44)$$

where $\mathcal{M} = \mathcal{M}(|X|)$ is the full tree level mass matrix, m_B and m_F correspond to boson and fermion masses respectively, and Λ is a UV cut-off.

There are a few terms missing in the expression (7.44) of the effective potential, if compared to a generic non supersymmetric theory. Let us consider them in turn. First, we miss the cosmological constant term

$$\sim \Lambda^4 . \quad (7.45)$$

This term is missing since it only depends on the spectrum, and not on the masses of the different modes. In a supersymmetric theory the spectrum admits an equal number of bosonic and fermionic degrees of freedom at any non-vanishing energy level, no matter whether one is in a supersymmetric or non supersymmetric vacuum. Since bosons and fermions contribute opposite to this term, this degeneracy ensures this term to be vanishing. A second term which is missing is the one proportional to

$$\sim \Lambda^2 \text{STr } \mathcal{M}^2 . \quad (7.46)$$

This is not expected to vanish in our supersymmetry breaking vacuum since particles have different masses. In other words, the mass spectrum is not supersymmetric along the pseudo-moduli space, recall for instance eqs. (7.38). In fact, an explicit computation shows that also this term is vanishing. This is not specific to this model. As we will show later, every time supersymmetry is broken spontaneously at tree level, provided the Kähler potential is canonical and in the absence of FI terms, cancellations occur so to give $\text{STr } \mathcal{M}^2 = 0$. Finally, there is another divergent term

$$\sim \log \Lambda^2 \text{STr } \mathcal{M}^4 , \quad (7.47)$$

which however does not depend on $|X|$ and, as we will see momentarily, can be reabsorbed in the renormalization of the tree level vacuum energy $|\mu^2|^2$. So the only non-trivial $|X|$ -dependent term is the finite term

$$\sim \text{STr } \mathcal{M}^4 \log \mathcal{M}^2 . \quad (7.48)$$

Let us first focus on the O’Raifeartaigh model. We should simply plug the tree level masses (7.38) into formula (7.44). A lengthy but straightforward computation shows that V_{eff} is a monotonic increasing function of $|X|$ and can hence be expanded in a power series in $|X|^2$. For small $|X|$ we get

$$V_{\text{eff}}(|X|) = V_0 + m_X^2 |X|^2 + \mathcal{O}(|X|^4) \quad (7.49)$$

where

$$V_0 = |\mu^2|^2 \left[1 + \frac{|h|^2}{32\pi^2} \left(\log \frac{|m|^2}{\Lambda^2} + v(y) + \frac{3}{2} \right) + \mathcal{O}(h^4) \right] , \quad (7.50)$$

with

$$y = \left| \frac{h\mu^2}{m^2} \right| < 1 \quad \text{and} \quad v(y) = -\frac{y^2}{12} + \mathcal{O}(y^4) , \quad (7.51)$$

and

$$m_X^2 = \frac{1}{32\pi^2} \left| \frac{h^4 \mu^4}{m^2} \right| z(y) \quad \text{where} \quad z(y) = \frac{2}{3} + \mathcal{O}(y^2) . \quad (7.52)$$

The minimum of the potential is at $|X| = 0$, and besides the tree level contribution $|\mu^2|^2$ it gets a contribution proportional $\sim |h^2|$ which is just a constant, $|X|$ -independent shift. As anticipated, the UV cut-off dependence can be reabsorbed in a renormalization of the vacuum energy. Indeed, we can define a running coupling

$$\mu^2(E) \equiv \mu_{\text{bare}}^2 \left[1 + \frac{|h|^2}{64\pi^2} \left(\log \frac{E^2}{\Lambda^2} + \frac{3}{2} \right) + \mathcal{O}(h^4) \right] \quad (7.53)$$

getting

$$V_0 = |\mu^2(E = m)|^2 \left(1 + \frac{|h|^2}{32\pi^2} v(y) + \mathcal{O}(h^4) \right) \quad (7.54)$$

and the Λ -dependence has disappeared from the potential.

The upshot of this analysis is that loop corrections have lifted the classical pseudo-moduli space, leaving just one isolated non supersymmetric vacuum. In this vacuum the scalar field X gets a (one-loop) mass while ψ_X , which is in fact the goldstino, remains massless (notice that in the unique supersymmetry breaking vacuum R-symmetry is preserved, as in Example 2). The shape of the potential in the X -direction becomes at all similar to that of Figure 7.5.

Let us now see what quantum corrections say about the marginally stable supersymmetry breaking vacua of the modified O’Raifeartaigh model of Example 6, the one including the superpotential perturbation (7.39). We will just briefly sketch the main results. The interested reader could try to work out all data in detail. One should again evaluate the Coleman-Weinberg potential, using the tree level spectrum computed near the origin of field space, where the putative marginally stable vacua live, eqs. (7.42). Plugging the latter into formula (7.44), what one finds is that, again, the vacuum degeneracy is lifted and a (meta)stable non supersymmetric vacuum survives at $|X| = |X_{min}|$ where in our regime, $\epsilon \ll 1$, X_{min} is near the origin and very far, in field space, from the two supersymmetric vacua sitting at $|X| = |X_{SUSY}|$. More precisely we get

$$V_{\text{eff}}(|X|) = V_0 + m_X^2 |X - X_{min}|^2 + \mathcal{O}(\epsilon^2, |X - X_{min}|^4), \quad (7.55)$$

where $X_{min} \sim \frac{\epsilon m}{h} f(y) + \mathcal{O}(\epsilon^3)$. The spectrum in the supersymmetry breaking vacuum enjoys a massless fermion, ψ_X , the goldstino, while the X -field gets a (ϵ -independent) mass, as in the original O’Raifeartaigh model. The effective potential, once projected into X -direction, looks roughly like that in Figure 7.9 below.

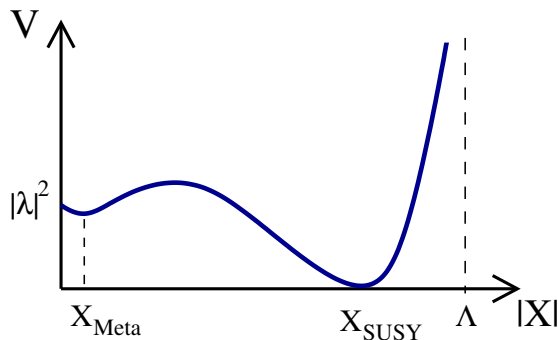


Figure 7.9: The effective potential of the modified O’ Raifeartaigh model project onto the $|X|$ direction.

One might ask whether such metastable supersymmetry breaking minimum is of any physical relevance. An estimate of its lifetime τ can be given looking at the decay rate

$$\Gamma \sim e^{-S_B} \quad (7.56)$$

(recall that $\tau \sim 1/\Gamma$) where S_B is the so-called bunch action, the difference between the Euclidean action of the tunneling configuration and that of remaining in the metastable vacuum. Its exact form depends on the details of the potential, but

a simple estimate can be given in the so-called thin wall approximation, which is justified when $|X_{SUSY} - X_{Max}|^4 \gg V_{Meta}$, and which is the case here. In this approximation the bunch action reads

$$S_B \sim \frac{\langle \Delta X \rangle^4}{\Delta V} \quad \text{where} \quad \langle \Delta X \rangle = \langle X \rangle_{SUSY} - \langle X \rangle_{Meta} \quad , \quad \Delta V = V_{Meta} - V_{SUSY} = V_{Meta} . \quad (7.57)$$

An explicit computation shows that $S_B \sim \epsilon^{-\alpha}$ where $\alpha > 0$. Hence, the metastable vacuum is parametrically long-lived. The upshot of this analysis is that at the quantum level the classically marginal pseudo-moduli space of the modified O’Raifeartaigh model is lifted but a local, parametrically long-lived supersymmetry breaking vacuum survives. It is an instructive exercise, which is left to the reader, to repeat this quantum analysis for the classically marginal pseudo-moduli space of Example 3. In this case, the pseudo-moduli space gets completely lifted, and no locally stable supersymmetric minimum survives quantum corrections.

Let us close this section stressing again that nothing we said (and computed) about quantum corrections, pseudomoduli lifting, etc... affects the supersymmetry breaking mechanism itself. All models we have been discussing so far, if breaking supersymmetry, were doing it at tree level. We have not encountered examples where supersymmetry was unbroken at tree level and one-loop quantum corrections induced supersymmetry breaking. Everything coming from the one-loop potential can and does modify the classical supersymmetry breaking vacua (which are not protected by supersymmetry), while it leaves completely unaffected the supersymmetric ones, if any. This agrees with non-renormalization theorems and our claims about the robustness of supersymmetric moduli spaces against (perturbative) quantum corrections.

We will have more to say about F-term breaking in due time. Let us pause a bit now, and consider the other possibility we have alluded to, *i.e.*, spontaneous supersymmetry breaking induced at tree level by D-terms.

7.5 D-term breaking

In a generic theory, where chiral and vector superfields are present, in absence of FI terms it is F-term dynamics which governs supersymmetry breaking. This is because whenever one can set all F-terms to zero, using (global) gauge invariance acting on the scalar fields one can set to zero all D-terms, too. So, if one wants to consider genuine D-term breaking, one should consider FI-terms, hence abelian

gauge factors. In what follows, we will review the most simple such scenario, where two massive chiral superfields with opposite charge are coupled to a single $U(1)$ factor, and a FI-term is present in the Lagrangian. The Lagrangian reads

$$\begin{aligned} \mathcal{L} &= \frac{1}{32\pi} \text{Im} \left(\tau \int d^2\theta W^\alpha W_\alpha \right) + \int d^2\theta d^2\bar{\theta} (\xi V + \bar{\Phi}_+ e^{2eV} \Phi_+ + \bar{\Phi}_- e^{-2eV} \Phi_-) \\ &+ m \int d^2\theta \Phi_+ \Phi_- + h.c. , \end{aligned} \quad (7.58)$$

where under a gauge transformation the two chiral superfields transform as $\Phi_\pm \rightarrow e^{\pm ie\Lambda} \Phi_\pm$. The equations of motion for the auxiliary fields read

$$\begin{cases} \bar{F}_\pm = m\phi_\mp \\ D = -\frac{1}{2} [\xi + 2e(|\phi_+|^2 - |\phi_-|^2)] \end{cases} \quad (7.59)$$

It is clearly impossible to satisfy all auxiliary fields equations, due to the presence of the FI-parameter ξ . Hence supersymmetry is broken, as anticipated. The scalar potential reads

$$\begin{aligned} V &= \frac{1}{8} [\xi + 2e(|\phi_+|^2 - |\phi_-|^2)]^2 + m^2 (|\phi_+|^2 + |\phi_-|^2) \\ &= \frac{1}{8} \xi^2 + \left(m^2 - \frac{1}{2} e \xi \right) |\phi_-|^2 + \left(m^2 + \frac{1}{2} e \xi \right) |\phi_+|^2 + \\ &+ \frac{1}{2} e^2 (|\phi_+|^2 - |\phi_-|^2)^2 . \end{aligned} \quad (7.60)$$

The vacuum structure and the low energy dynamics depends on the sign of $m^2 - \frac{1}{2} e \xi$. We will consider the two cases in turn.

- $m^2 > \frac{1}{2} e \xi$. All terms in the potential are positive and the minimum of V is at $\langle \phi_\pm \rangle = 0$, where $V = \frac{1}{8} \xi^2$. Supersymmetry is broken but gauge symmetry is preserved. The *only* auxiliary field which gets a VEV is D , so one speaks of *pure* D-term breaking. We are in a situation like the one depicted in the upper right diagram of Figure 7.1.

One can compute the spectrum and find agreement with expectations. The two fermions belonging to the two chiral multiplets have (supersymmetric) mass m and hence form a massive Dirac fermion. The two scalar fields ϕ_+ and ϕ_- have masses $\sqrt{m^2 + 1/2e\xi}$ and $\sqrt{m^2 - 1/2e\xi}$, respectively. Finally, both the photon A_μ and the photino λ remain massless. The former, because gauge symmetry is preserved, the latter because supersymmetry is broken and

a massless fermionic mode, the goldstino (which in this case gets contribution from the photino, only, $\psi_\alpha^G \sim \langle D \rangle \lambda_\alpha$), is expected.

- $m^2 < \frac{1}{2}e\xi$. Now the sign of the mass term for ϕ_- is negative. The minimum of the potential is at $\langle \phi_+ \rangle = 0$, $\langle \phi_- \rangle = \sqrt{\frac{\xi}{2e} - \frac{m^2}{e^2}} \equiv h$. Hence both supersymmetry and gauge symmetry are broken. Both the D-field and F_+ get a VEV: in this case we have *mixed* D-term and F-term breaking. The value of the potential at its minimum is $V = \frac{1}{8}\xi^2 - \frac{1}{2}e^2h^4$. We are in a situation of the type depicted in the lower right diagram of Figure 7.1.

In order to compute the mass spectrum one should expand the potential around $\langle \phi_+ \rangle = 0$ and $\langle \phi_- \rangle = h$. A lengthy but simple computation gives the following answer. The complex scalar field ϕ_+ has mass $m_{\phi_+} = \sqrt{2}m$. The real part of ϕ_- , ϕ_-^R , gets a mass $m_{\phi_-^R} = \sqrt{2}eh$, while the imaginary part ϕ_-^I disappears from the spectrum (in fact, it is eaten by the photon, which becomes massive, $m_{A_\mu} = \sqrt{2}eh$). The three fermions mix between themselves (there is a mixing induced from Yukawa couplings). One eigenfunction is massless, and is nothing but the goldstino $\psi_\alpha^G \sim \langle D \rangle \lambda_\alpha + \langle \bar{F}_+ \rangle \psi_{+\alpha}$. The other two get equal mass $m_{\tilde{\psi}_\pm} = \sqrt{2e^2h^2 + m^2} = \sqrt{e\xi - m^2}$.

Figure 7.10 gives a summary of the mass spectrum of the FI model as a function of $\frac{1}{2}e\xi$ which, for fixed m , is the order parameter of the supersymmetry breaking transition.

One of the most attractive features of supersymmetric fields theories is the stability of masses under quantum corrections. In models where the FI mechanism plays a role, the physical mass spectrum depends on ξ which is not protected, a priori, since it appears in a D-term. Therefore, it is important to investigate the circumstances under which the FI-term does not get renormalized. This can be done, and the upshot is that the contribution renormalizing the FI-term is proportional to the trace of the $U(1)$ generator taken over all chiral superfields present in the model. However, in order not to have gravitational anomalies, this trace must vanish. Therefore the FI-term does not renormalize for theories free of gravitational anomalies.

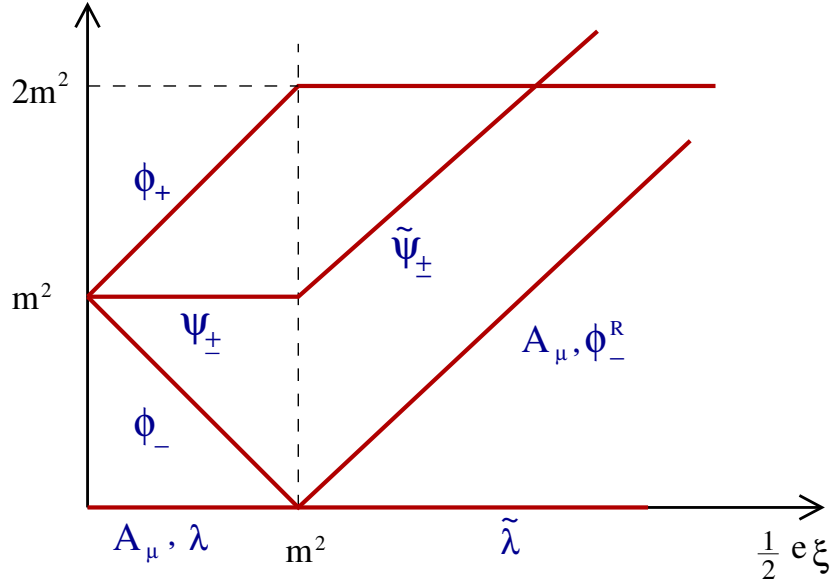


Figure 7.10: The mass spectrum of the Fayet-Iliopoulos model as a function of the FI-parameter ξ .

7.6 Indirect criteria for supersymmetry breaking

We have already alluded to some possible relation between supersymmetry breaking and R -symmetry. In what follows, we will try to make this intuition precise and, more generally, present a few general criteria one can use to understand whether a theory might or might not break supersymmetry, without having a precise knowledge of the details of the theory itself. These criteria might be useful as guiding principles when trying to construct models of supersymmetry breaking in a bottom-up approach and, at the same time, they allow to have a handle on theories which are more involved than the simple ones we analyzed in previous sections. Finally, having some general criteria, possibly being valid also beyond the realm of perturbative physics might also be useful when one has to deal with theories in strongly coupled phases, where a perturbative, semi-classical approach is not possible, and where the direct study of the zero's of the potential is not easy or even not possible.

7.6.1 Supersymmetry breaking and global symmetries

Let us consider a supersymmetric theory which has a spontaneously broken global symmetry and which does not admit (non compact) classical flat directions. This theory, generically, breaks supersymmetry. This can be easily proven as follows.

Since there is a broken global symmetry, the theory admits a goldstone boson (a massless particle with no potential). If supersymmetry were unbroken then one should expect a scalar companion of this goldstone boson, which, being in the same multiplet of the latter, would not admit a potential either. But then, the theory would admit a flat direction, contrary to one of the hypotheses. This is a *sufficient* condition for supersymmetry breaking.

In the above reasoning we have assumed that the second massless scalar corresponds to a non-compact flat direction. This is typically the case since the Goldstone boson is the phase of the order parameter, and its scalar companion corresponds to a dilation of the order parameter, and therefore represents indeed a non-compact flat direction.

Consider now a theory of F chiral superfields Φ^i with superpotential W . Supersymmetry is unbroken if

$$\bar{F}_i = \frac{\partial W}{\partial \phi^i} = 0 \quad , \quad \forall i = 1, 2, \dots, F . \quad (7.61)$$

These are F holomorphic conditions on F complex variables. Therefore, if the superpotential W is *generic*, one expects (typically distinct) solutions to exist. Hence, supersymmetry is unbroken. By the superpotential being generic we mean the following. The superpotential is a function of the Φ^i 's of degree, say, n . It is generic if all possible polynomials of degree n or lower compatible with the symmetries of the theory are present.

Suppose now that W preserves some global non-R symmetry. Hence, W is a function of singlet combinations of the Φ^i 's. It is easy to see that in terms of these reduced number of variables, eqs. (7.61) impose an equal number of independent conditions. Let us see how this goes. Suppose, for definiteness, that the global symmetry is a $U(1)$ symmetry and call q_i the corresponding charge of the i -th chiral superfield Φ^i . Hence, we can rewrite the superpotential as e.g.

$$W = W(X_i) \quad \text{where} \quad X_i = \Phi_i \Phi_1^{-q_i/q_1} \quad , \quad i = 2, 3, \dots, F . \quad (7.62)$$

If we now consider eqs. (7.61) we have

$$\begin{cases} j \neq 1 & \frac{\partial W}{\partial \phi^j} = \phi_1^{-q_j/q_1} \frac{\partial W(X_i)}{\partial X_j} = 0 \\ j = 1 & \frac{\partial W}{\partial \phi^1} = \frac{\partial W(X_i)}{\partial X_k} \frac{\partial X_k}{\partial \phi_1} = 0 . \end{cases} \quad (7.63)$$

We see that the equation for \bar{F}_1 is automatically satisfied if the others $F - 1$ are satisfied. Hence, having a system of $F - 1$ holomorphic equations in $F - 1$ variables,

generically the system allows for solutions. The same reasoning holds for a generic global symmetry. The global symmetry, under which the superpotential is a singlet, diminishes the number of independent variables, but it diminishes also the number of independent F-equations by the same amount. Hence, again, if W is generic, eqs. (7.61) can be solved and supersymmetry is unbroken.

Suppose now that the global symmetry under consideration is a R-symmetry. The crucial difference here is that the superpotential is charged under this symmetry, $R(W) = 2$. Let us call r_i the R-charge of the i -th superfield Φ^i . We can now rewrite the superpotential as

$$W = \Phi_1^{2/r_1} f(X_i) \quad \text{where} \quad X_i = \Phi_i \Phi_1^{-r_i/r_1} \quad , \quad i = 2, 3, \dots, F . \quad (7.64)$$

If we now compute eqs. (7.61) we get

$$\begin{cases} j \neq 1 & \frac{\partial W}{\partial \phi^j} = \phi_1^{\frac{2-r_j}{r_1}} \frac{\partial f(X_i)}{\partial X_j} = 0 \\ j = 1 & \frac{\partial W}{\partial \phi^1} = \frac{2}{r_1} \phi_1^{\frac{2}{r_1}-1} f(X_i) + \phi_1^{\frac{2}{r_1}} \frac{\partial f(X_i)}{\partial X_k} \frac{\partial X_k}{\partial \phi_1} = 0 . \end{cases} \quad (7.65)$$

Once the first $F - 1$ equations are satisfied, the latter reduces to $f(X_i) = 0$, which is not at all trivial. So now we have F independent equations in $F - 1$ variables and hence, generically, solutions do not exist. Therefore, supersymmetry is broken, generically. This implies that the existence of an R-symmetry is a *necessary* condition for supersymmetry breaking, if the potential is generic. And, if it is then spontaneously broken, it is a *sufficient* condition (if there are no classical flat directions).

This is known as the Nelson-Seiberg criterium. The O’Raifeartaigh model meets this criterium. It possesses an R-symmetry (which is then spontaneously broken along the pseudo-moduli space), the superpotential is generic, and it breaks supersymmetry. The modified O’Raifeartaigh model instead enjoys supersymmetry restoration. Indeed, R-symmetry is absent since the mass perturbation ΔW breaks it explicitly. So, one would expect the model not to break supersymmetry. And in fact it doesn’t. Notice however that somewhere else in the space of scalar field VEVs this model admits non supersymmetric vacua which, if the mass perturbation is small enough, we have proven to be long-lived. In this region the perturbation is negligible and an *approximate* R-symmetry (O’Raifeartaigh model’s original one) is recovered. This property is not specific to the modified O’Raifeartaigh model, but is a generic feature of supersymmetry breaking metastable vacua.

Summarizing, a rough guideline in the quest for supersymmetry breaking theories can be as follows:

No R-symmetry	→	SUSY unbroken
R-symmetry	→	SUSY (maybe) broken
Approximate R-symmetry	→	SUSY (maybe) broken locally, restored elsewhere

Since necessary conditions are quite powerful tools, let me stress again one important point. The existence of an R-symmetry is a necessary condition for supersymmetry breaking under the assumption that the superpotential is generic. If this is not the case, supersymmetry can be broken even if the R-symmetry is absent. Another possibility, which typically occurs when gauge degrees of freedom are present in the Lagrangian, is that R-symmetry is absent, but then it arises as an accidental symmetry in the low energy effective theory. Also in this case supersymmetry can be broken even if R-symmetry was absent in the UV Lagrangian. We will see examples of this sort later in this course.

7.6.2 Topological constraints: the Witten Index

Another powerful criterium exists which helps when dealing with theories with complicated vacuum structure and for which it is then difficult to determine directly whether supersymmetry is broken, i.e. to find the zero's of the potential. This criterium, which again provides a necessary condition for supersymmetry breaking, has to do with the so-called Witten index, which, for a supersymmetric theory, is a topological invariant quantity.

The Witten index, let us dub it I_W , is an *integer number* which measures the difference between the number of bosonic and fermionic states, for any given energy level. In a supersymmetric theory, for any *positive* energy level there is an equal number of bosonic and fermionic states. This is obvious if supersymmetry is unbroken, but it also holds if supersymmetry is broken: every state is degenerate with the state obtained from it by adding a zero-momentum goldstino (which is certainly there, if supersymmetry is broken), i.e. a state $|\Omega\rangle$ is paired with $|\Omega + \{p_\mu = 0 \text{ goldstino}\}\rangle$. On the contrary, zero energy states can be unpaired since, due to the supersymmetry algebra, such states are annihilated by the supercharges. Therefore, in a supersymmetric theory the Witten index can get contribution from the zero energy states only, regardless the vacuum one is considering preserves supersymmetry or it does not.

Disclaimer. Strictly speaking what's above holds only if we put the theory in a finite volume. In an infinite volume, when supersymmetry is broken one has to deal with IR singularity issues. In particular, the (broken) supercharge diverges and acting with it on a physical state gives a non-normalizable state. This can be seen as follows. From the current algebra

$$E \eta^{\mu\nu} = \langle T^{\mu\nu} \rangle = \frac{1}{4} \bar{\sigma}^{\mu\dot{\alpha}\alpha} \langle \{Q_\alpha, \bar{S}_{\dot{\alpha}}^\nu\} \rangle \quad (7.66)$$

one sees that if the vacuum energy is non vanishing, the vacuum is transformed into one goldstino states (recall that the supercurrent creates a goldstino when acting on the vacuum). In an infinite volume a non-vanishing vacuum energy density corresponds to an infinite total energy, implying that the supercharge indeed diverges and that the zero-momentum goldstino state is not defined (the corresponding state does not exist in the Hilbert space). Putting the theory in a finite volume is a way to regularize (translational invariance can be maintained imposing periodic boundary conditions on all fields). Therefore, in what follows we will start considering the theory in a finite volume V and only later take the infinite volume limit. As we will see, what is relevant for the argument we want to convey is not affected by these issues and holds true also when $V \rightarrow \infty$.

A theory in a finite volume has a discrete energy spectrum, all states in the Hilbert space are discrete and normalizable and can be counted unambiguously. First notice that in a supersymmetric theory the energy is \geq than the momentum $|\vec{P}|$, so zero energy states have $\vec{P} = 0$. In what follows we can then restrict to the zero-momentum subspace of the Hilbert space, since all what we are concerned with is to look for the existence of zero energy states. In such subspace the supersymmetry algebra simplifies. In particular, using four-component spinor notation, we have $Q_1^2 = Q_2^2 = Q_3^2 = Q_4^2 = H$, where H is the Hamiltonian of the system and Q_i are the four components of the supercharge.

Suppose to have a bosonic state $|b\rangle$ for which $Q^2|b\rangle = E|b\rangle$, where Q is one of the Q_i 's. Then the fermionic state obtained from $|b\rangle$ as

$$|f\rangle \equiv \frac{1}{\sqrt{E}} Q|b\rangle, \quad (7.67)$$

has also energy E . This does not apply to zero-energy states since they are annihilated by Q , and hence are not paired. So, as anticipated, the Witten index receives contributions only from the zero energy states. In Figure 7.11 we report the general form of the spectrum of a supersymmetric theory.

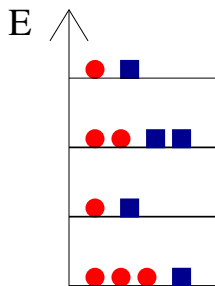


Figure 7.11: The spectrum of a supersymmetric theory in a finite volume. Circles indicate bosons, squares indicate fermions. The zero energy level is the only one where there can exist a different number of circles and squares.

In order to appreciate its topological nature, let us define the index a bit more rigorously. A supersymmetric theory is a unitary representation of the Poincaré superalgebra on some Hilbert space \mathcal{H} . Let us assume that

$$\mathcal{H} = \bigoplus_{E \geq 0} \mathcal{H}_E . \quad (7.68)$$

The Witten index is defined as

$$I_W(\beta) = \text{STr}_{\mathcal{H}} e^{\beta H} \equiv \text{Tr}_{\mathcal{H}} (-1)^F e^{\beta H} , \quad \beta \in \mathbb{R}^+ . \quad (7.69)$$

It follows that

$$\begin{aligned} I_W(\beta) &= \sum_{E \geq 0} e^{\beta E} \text{Tr}_{\mathcal{H}_E} (-1)^F = \sum_{E \geq 0} e^{\beta E} [n_B(E) - n_F(E)] = \\ &= n_B(0) - n_F(0) = \text{Tr}_{\mathcal{H}_0} (-1)^F = I_W(0) . \end{aligned} \quad (7.70)$$

We have been rewriting what we have already shown to hold. The point is that this way it is clear that the index does not depend on β : its value does not vary if we vary β . More generally, one can prove that the Witten index does not depend on *any* parameter (i.e. couplings) and can then be computed in appropriate corner of the parameter space (e.g. at weak coupling) and the result one gets is exact. In other words, the Witten index is a topological invariant.

Suppose one starts from a situation like the one depicted in Figure 7.11. Varying the parameters of the theory, like masses, couplings, etc..., it may very well be that some states move around in energy. The point is that they must do it in pairs, in a supersymmetric theory. Hence, it can happen that a pair of non-zero energy states moves down to zero energy; or, viceversa, that some zero energy states may acquire

non-zero energy. But again, this can only happen if an equal number of bosonic and fermionic zero energy states moves towards a non-zero energy level. The upshot is that the Witten index does not change. All what we have just said is summarized in Figure 7.12.

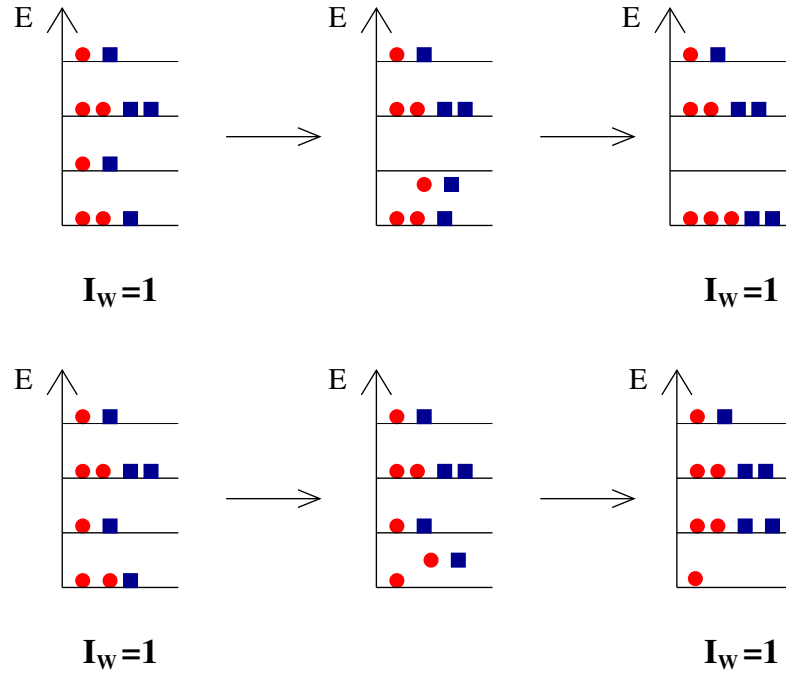


Figure 7.12: Supersymmetric theory dynamics. Upon modifications of parameters of the theory, the number of zero energy states can change; but the Witten index remains the same.

What is this useful for? The crucial point is that the Witten index measures the difference between zero-energy states only. Suppose it is different from zero, $I_W \neq 0$. This means that there exists some zero-energy state, hence supersymmetry is unbroken. But, because of the topological nature of I_W , this conclusion holds at any order in perturbation theory and even non-perturbatively! A theory with non vanishing Witten index cannot break supersymmetry. Suppose instead that $I_W = 0$. Now one cannot conclude anything, just that the number of bosonic and fermionic zero-energy state is the same; but one cannot tell whether this number is zero (broken supersymmetry) or different from zero (unbroken supersymmetry).

So we conclude that having a non-vanishing Witten index is a sufficient condition for the existence of supersymmetric vacua, which also means that having it vanishing is a *necessary* condition for supersymmetry breaking. And a robust one, since I_W is

an exact quantity.

Few comments are in order at this point.

First, the fact that we have been working in a finite volume does not question our main conclusions. If supersymmetry is unbroken in an arbitrary finite volume it means the ground state energy $E(V)$ is zero for any V . Since the large- V limit of zero is still zero, supersymmetry is unbroken also in the infinite volume limit. If one can explicitly compute the Witten index at finite volume and find that it is not vanishing, one can safely conclude that supersymmetry is not broken even in the actual theory, i.e. at infinite volume. On the contrary, the converse is not necessarily true. It might be that supersymmetry is broken at finite volume and restored in the infinite volume limit. Suppose that $I_W = 0$ and that one knows that supersymmetry is broken, that is the minimal energy states have positive energy. The energy density goes as $E(V)/V$ and it may very well be that for $V \rightarrow \infty$ the increase of E is not enough to compensate for the larger and larger volume. So the energy density can very well become zero in the infinite volume limit and supersymmetry restored. But this does not hurt much, since all what the vanishing of the Witten index provides is a necessary condition for supersymmetry breaking, not a sufficient one.

A second comment regards the relation between classical and quantum results. Suppose one can explicitly check at tree level that a given theory has non-vanishing Witten index, $I_W \neq 0$. For what we said above, this implies that supersymmetry is unbroken classically *and* that it cannot be broken whatsoever, neither perturbatively nor non-perturbatively. On the contrary, if $I_W = 0$ at tree level and we know that supersymmetry is unbroken classically, it can very well be that (non-perturbative) quantum effects may break it.

The theorem we have discussed may find very useful applications. For one thing, it turns out that pure SYM theories have non-vanishing Witten index (for SYM with gauge group G , the index equals the dual Coxeter number of G , which for SU and Sp groups is just $r + 1$, where r is the rank of G). So pure SYM theories cannot break supersymmetry. As a corollary, SYM theories with massive matter (like massive SQCD) cannot break supersymmetry either. This is because for low enough energy all massive fields can be integrated out and the theory flows to pure SYM, which has non vanishing index. More generally, non-chiral theories, for which a mass term can be given to all matter fields, are not expected to break supersymmetry.

What about chiral theories, instead? Chiral theories behave differently. In this case some chiral superfield cannot get a mass anyway, and so these theories cannot

be obtained from deformation of vector-like theories, as massive SQCD. Hence, one cannot conclude that these theories cannot break supersymmetry. As we will see, most known examples of theories breaking supersymmetry are, in fact, chiral theories.

There is a subtlety in all what we said, so far, which is sort of hidden in some of our claims. We said that the Witten index is robust against any continuous change of parameters. But it turns out that a perturbation that changes the asymptotic behavior of the potential may induce a change in I_W . This is related to the topological nature of the index, which makes it depending on boundary effects. Consider the following simple potential for a scalar field ϕ ,

$$V(\phi) = (m\phi - g\phi^2)^2 . \quad (7.71)$$

For $g = 0$ low energy states correspond to $\phi \sim 0$. For $g \neq 0$ low energy states may correspond to $\phi = 0$ but also to $\phi \sim m/g$ (no matter how small g is). So we see here that $I_W(g = 0) \neq I_W(g \neq 0)$. What is going on? The point is that switching on and off g changes the asymptotic behavior of V for large ϕ (that is, at the boundary of field space): in the large ϕ region, for $g = 0$ $V \sim \phi^2$ while for $g \neq 0$ $V \sim \phi^4$. The punchline is that the Witten index is invariant under any change in the parameters of a theory in which, in the large field regime, the potential changes by terms no bigger than the terms already present. If this is not the case, the Witten index can indeed change discontinuously. In other words, I_W is independent of numerical values of parameters as long as these are non-zero. When sending a set of parameters to zero, or switching on some couplings which were absent, one should check that the asymptotic behavior of the potential is unchanged, in order to avoid new states coming in from (or going out to) infinity. Coming back to our SQCD example, we see that for massive SQCD the potential in the large field regime is quadratic, while for massless SQCD is flat: the two theories do not have a priori the same Witten index. Therefore, while massive SQCD is expected to be in the same equivalence class of pure SYM (as far as supersymmetry breaking is concerned), this is not guaranteed for massless SQCD. In other words, no conclusions can be drawn for the massless regime by the analysis in the massive regime. We will see explicit examples of this phenomenon in later lectures.

Let us finally notice that Witten index argument limits a lot the landscape of possible supersymmetry breaking theories; for instance, non-chiral gauge theories most likely (cfr. the subtlety above) cannot break supersymmetry.

7.6.3 Genericity and metastability

Before concluding this section, there is yet another important conclusion we can draw from all what we have learned. Both the Nelson-Seiberg criterium and Witten index argument seem to favor, at least statistically, supersymmetry breaking into metastable vacua.

For one thing, thinking about R-symmetry one might have the impression to fall into a vicious circle. Having an R-symmetry (which is a necessary condition for supersymmetry breaking) forbids a mass term for the gaugino which, being a fermion in a real representation and having R-charge $R(\lambda) = 1$, would have a R-symmetry breaking mass term. But we do not see any massless gauginos around, so gauginos should be massive. If we have an R-symmetry which is spontaneously broken, we could have gaugino masses but we should also have an R-axion, which is not observed. This suggests that R-symmetry should be broken explicitly. But then, generically, we cannot break supersymmetry! The conclusion is that asking for stable vacua compatible with phenomenological observations implies one should look for non-generic theories, which are obviously much less than generic ones. If one admits that supersymmetry might be broken in metastable vacua, then R-symmetry would not be an exact symmetry but only an approximate one. In this case gaugino mass and supersymmetry breaking would be compatible, generically, at least in the metastable vacuum (it is worth noticing that in concrete models there is, not unexpectedly, some tension between the magnitude of gaugino mass and the lifetime of such supersymmetry breaking vacua).

Regardless the R-axion problem mentioned above (which can also get a mass by gravitational effects), there exists another argument favoring metastability. This is related with the computations we performed in section 7.4 when we studied one loop corrections of the O’Raifeartaigh model. We have seen that quantum corrections lift the classical pseudomoduli space, rendering one unique supersymmetry breaking vacuum. However, such vacuum is (the only) one where R-symmetry is in fact not broken! This is not a specific feature of the O’Raifeartaigh model but applies to any model where the R-charges of superfields are either 0 or 2. It has been proven that a necessary condition for having the true vacuum to break the R-symmetry is to have fields in the Lagrangian having R-charge different from 0 and 2. Models of this kind exist and have been constructed. However, in all such models supersymmetry preserving vacua also exist. Hence, supersymmetry breaking vacua where also R-symmetry is in the end broken, are actually metastable.

Also Witten index argument favors metastability, statistically. If accepting we leave in a metastable vacuum, we would allow supersymmetry preserving vacua elsewhere in field space. Hence, all theories with non-vanishing Witten index would not be anymore excluded from the landscape of possible supersymmetry breaking and phenomenologically sensible theories. For example, non-chiral theories would be back in business.

The punchline is that, generically, it may be more likely we leave in a metastable vacuum rather than in a fully stable one. Just... we need to ensure that its lifetime is long enough to be safe!

7.7 Exercises

1. Consider a theory of n chiral superfields Φ^i with superpotential as in eq. (7.14). Prove that, for an interacting theory (that is, some g_{ijk} should be non-vanishing), in order to have spontaneous supersymmetry breaking one needs at least three chiral superfields. Derive the generic form a three-superfield superpotential should have in order to break supersymmetry.
2. Compute the one-loop effective potential on the classically marginal, non supersymmetric vacua of the model in Example 3. What is the fate of these vacua after quantum corrections are taken into account? How is Figure 7.6 modified?
3. Compute the mass spectrum of the FI model both in the pure D-term as well as in the mixed D and F-terms breaking phases, and check explicitly that the spectrum satisfies the so-called supertrace mass formula, that is $\text{STr } \mathcal{M}^2 = 0$. (Notice that this identity is trivially satisfied in supersymmetric vacua).
4. Consider all models of F-term breaking of section 7.3 and discuss whether and how the Nelson-Seiberg criterium applies or not.

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