8 Supersymmetry breaking and the Standard Model

In this chapter we will elaborate a little bit on how the machinery we have been constructing can be used to describe physics beyond the Standard Model, under the hypothesis that this is described by a supersymmetric theory.

The basic idea is that the Standard Model should be viewed as an effective theory, valid only up to some scale, and that Nature, at energy above such scale, is described by some suitable $\mathcal{N} = 1$ supersymmetric extension of the Standard Model itself. The most economic option we can think of would be a $\mathcal{N} = 1$ Lagrangian which just includes known particles (gauge bosons, Higgs fields, leptons and quarks) and their superpartners. In fact, strictly speaking this does not apply to the Higgs sector, which should be doubled, not to spoil the anomaly-free properties of the Standard Model, since the Higgs fermionic partner, the higgsino, would introduce $SU(2)_L$ gauge anomalies. Therefore, two Higgs multiplets are needed, having opposite $U(1)_Y$ charge. Another way to realize the need of two Higgs doublets in supersymmetric extensions of the Standard Model is to notice that the Higgs field $H$ gives mass to down quarks (and charged leptons) and $H$ to up quarks. Due to holomorphy of the superpotential, $\overline{H}$ cannot enter $W$ and so one needs a second, independent doublet to give up quarks a mass. These two chiral superfields are dubbed $H_u$ and $H_d$ and the minimal extension of the Standard Model, MSSM.

Within such minimal supersymmetric extension, one might then ask whether is it possible to break supersymmetry spontaneously and be consistent with phenomenological constraints and expectations. Addressing this question will be our main concern, in this lecture.

So far we have discussed possible supersymmetry breaking scenarios at tree level: the Lagrangian is supersymmetric but the classical potential is such that the vacuum state breaks supersymmetry (or at least there exist metastable but sufficiently long-lived supersymmetry breaking vacua, besides supersymmetric ones). Can these scenarios, i.e. either F or D-term supersymmetry breaking at tree level, occur in such a minimal extension of the Standard Model? In what follows, we will claim the answer is no: things cannot be as simple as that.
8.1 Towards dynamical supersymmetry breaking

From a purely theoretical viewpoint there is at least one point of concern as far as tree level supersymmetry breaking. As discussed in the first lecture, there are several reasons to prefer sparticle masses around the TeV scale or so. This scale is not much different from the electro-weak scale and as such much smaller than any natural UV cut-off one can think of, like the Planck mass. If supersymmetry is broken at tree level, the mass setting the scale of supersymmetry breaking would be some mass parameter entering the bare Lagrangian. For instance, in the O’Raifeartaigh model that we discussed in previous chapter this scale is $\mu$, the coefficient of the linear term in the superpotential. This way, we would have a scenario where an unnaturally small mass scale has been introduced in a theory in order to solve the unnatural hierarchy between the electro-weak scale and, say, the Planck scale, and avoid a fine-tuning problem for the Higgs mass. What we gain introducing supersymmetry would then just be that this small parameter, put by hand into the Lagrangian, would be protected against quantum corrections. Is this a satisfactory solution of the hierarchy problem?

It would be much more natural for this small mass parameter to be explained in some dynamical way. This is possible, in fact. In order to understand how it comes, we should first recall two facts.

First, recall that due to non-renormalization theorems, in a supersymmetric theory the superpotential is tree-level exact in perturbation theory, meaning that its full structure looks like

$$W_{\text{eff}} = W_{\text{tree}} + W_{\text{np}},$$

(8.1)

where the subscript $\text{np}$ stands for non-perturbative. As we already observed, this implies that if supersymmetry is unbroken at tree level, then it cannot be broken at any order in perturbation theory, but only non-perturbatively.

The second piece of knowledge we need comes from a well-known property that many gauge theories share, i.e. dimensional transmutation. Due to the running of the gauge coupling, which becomes bigger and bigger towards the IR, any UV-free gauge theory possesses an intrinsic scale, $\Lambda$, which governs the strong-coupling IR dynamics of the theory, is RG-invariant and is exponentially suppressed with respect to the scale $M_X$ at which the theory is weakly coupled. Its one-loop expression reads

$$\Lambda \sim M_X e^{-\frac{\theta}{g^2(M_X)}} \ll M_X,$$

(8.2)
where \( \# \) is a number which depends on the details of the specific theory, but is roughly of order 1.

Suppose now we have some complicated supersymmetric gauge theory which does not break supersymmetry at tree level (so all F-terms coming from \( W_{\text{tree}} \) are zero), but whose strong coupling dynamics generates a contribution to the superpotential \( W_{\text{np}} \) which does provide a non-vanishing F-term. This F-term will be order the dynamical scale \( \Lambda \), since it should vanish in the classical limit \( \Lambda \to 0 \), and so will the scale of supersymmetry breaking. This would imply

\[
M_s \sim \Lambda << M_X, \tag{8.3}
\]

hence giving a natural hierarchy between \( M_s \) and the UV scale \( M_X \) (which can be the GUT scale or any other scale of the UV-free theory under consideration). This idea is known as Dynamical Supersymmetry Breaking (DSB) and can be regarded as the most natural way we can think of supersymmetry breaking in a fully satisfactory way.

We will discuss several DSB models in a subsequent chapter. For now, let me just anticipate that what we learned about tree-level supersymmetry breaking in previous chapter will be of great help also as far as DSB is concerned. As we will see, in DSB models the effective superpotential has typically an O’Raifeartaigh-like structure: at low enough energy gauge degrees of freedom typically disappear from the low energy spectrum (because of confinement, higgsing and alike) and the effective theory ends-up being a theory of chiral superfields, only. The analysis will then follow the one of the previous chapter, but with the great advantage that the mass parameter setting the scale of supersymmetry breaking and sparticle masses has been dynamically generated (and with the complication that in general the Kähler potential will be non-canonical, of course).

### 8.2 The Supertrace mass formula

There is yet another reason making tree-level supersymmetry breaking not welcome in the MSSM. This is more phenomenological in nature, and related to the so-called supertrace mass formula.

Let us consider the most general \( \mathcal{N} = 1 \) renormalizable Lagrangian (5.79) and suppose that supersymmetry is spontaneously broken at tree level. Suppose we want to compute the trace over all bosonic and fermionic fields of the mass matrix squared
in an arbitrary supersymmetry breaking vacuum. To this aim, let us suppose that, generically, all $F$ and $D$ auxiliary fields have some non-vanishing vacuum expectation value.

- **Vectors**

If $F$ and $D$-fields are non vanishing, it means that some scalar fields $\phi^i$ have acquired a non-vanishing VEV. If such fields are charged under the gauge group, a mass for some vector fields will be induced since $D_\mu \phi^i D^\mu \phi^i \supset g^2 \bar{\phi} T^a T^b \phi v_a v_b^\mu$. Hence, we have for the mass matrix squared of vector bosons

$$[(\mathcal{M}_1)^2]^{ab} = 2g^2 \langle \bar{\phi} T^a \rangle \langle T^b \phi \rangle = 2 \langle D_\mu^a \rangle \langle D^b \rangle,$$

where the lower index on the mass matrix refers to the spin, which is one for vectors and we have defined $D_\alpha^a = \partial D^a / \partial \phi^i$, $D^{ai} = \partial D^a / \partial \bar{\phi}_i$.

- **Fermions**

The fermion mass matrix can be easily read from the Lagrangian (5.79) to be

$$\mathcal{M}_{1/2} = \left( \begin{array}{cc} \langle F_{ij} \rangle & \sqrt{2}i \langle D_i^a \rangle \\ \sqrt{2}i \langle D_j^a \rangle & 0 \end{array} \right),$$

where, as usual, $F_{ij} = \partial^2 W / \partial \phi^i \partial \phi^j$. The matrix squared reads

$$\mathcal{M}_{1/2}^2 \mathcal{M}_{1/2}^\dagger = \left( \begin{array}{cc} \langle F_{ij} \rangle + 2 \langle D_i^a \rangle \langle D_j^b \rangle & -\sqrt{2}i \langle F_{ij} \rangle \langle D_i^a \rangle \\ \sqrt{2}i \langle D_j^a \rangle \langle F_{ij} \rangle & 2 \langle D_i^a \rangle \langle D_j^a \rangle \end{array} \right),$$

with $F_{ij} = \partial^2 W / \partial \bar{\phi}_j \partial \bar{\phi}_i$.

- **Scalars**

The scalar mass matrix squared is instead

$$\langle \mathcal{M}_0 \rangle^2 = \left( \begin{array}{cc} \langle \frac{\partial^2 V}{\partial \phi^i \partial \phi^j} \rangle & \langle \frac{\partial^2 V}{\partial \phi^i \partial \bar{\phi}^j} \rangle \\ \langle \frac{\partial^2 V}{\partial \bar{\phi}^i \partial \phi^j} \rangle & \langle \frac{\partial^2 V}{\partial \bar{\phi}^i \partial \bar{\phi}^j} \rangle \end{array} \right).$$

Recalling that $V = \bar{F}_i F^i + \frac{1}{2} D^a D_a$, one can write it as

$$\langle \mathcal{M}_0 \rangle^2 = \left( \begin{array}{cc} \langle \bar{F}_{ip} \rangle \langle F^{kp} \rangle + \langle D^a \rangle \langle D_i^a \rangle + \langle D_i^a \rangle D^{ak} & \langle F^p \rangle \langle \bar{F}_{ip} \rangle + \langle D_i^a \rangle \langle D_i^a \rangle + \langle D_i^a \rangle D^{aj} \\ \langle \bar{F}_{ip} \rangle \langle F^{jk} \rangle + \langle D^a \rangle \langle D_i^a \rangle & \langle \bar{F}_{ip} \rangle \langle F^{jk} \rangle + \langle D^{aj} \rangle \langle D_i^a \rangle + \langle D^a \rangle D^{aj} \end{array} \right),$$

where $D_i^{aj} = -g T_i^a$, $\bar{F}_{ijk} = \partial^3 W / \partial \bar{\phi}_i \partial \bar{\phi}_j \partial \bar{\phi}_k$ and $F^{ijk} = \partial^3 W / \partial \phi_i \partial \phi_j \partial \phi_k$.  

4
Taking the trace over gauge and flavor indexes of the three matrices (8.4), (8.6) and (8.8) we get

\[
\text{Tr} (M_1^2) = 2h_a D^a_i h_D i D^a_i + 4h_D^i h_D i D^i_a
\]

\[
\text{Tr} (M_0^2) = 2\langle F_{ai} \rangle \langle F^{ai} \rangle + 2\langle D^{ai} \rangle \langle D^i_a \rangle - 2g\langle D^a \rangle \text{Tr} T^a
\]

and finally for the supertrace

\[
\text{STr} M^2 = 3\text{Tr} (M_1)^2 + \text{Tr} (M_0)^2 - 2\text{Tr} (M_{1/2})^2 = -2g\langle D^a \rangle \text{Tr} T^a .
\] (8.9)

This formula puts severe phenomenological constraints. First notice that, because of the trace on gauge generators, the rhs is non vanishing only in presence of U(1) factors. If this is the case, then one needs non trivial FI terms to let it being non-vanishing, since we know that if \( \xi = 0 \) then also \( \langle D^a \rangle = 0 \). Now, suppose supersymmetry were broken spontaneously at tree level in the MSSM. We have only two U(1) factors we can play with, the hypercharge generator \( U(1)_Y \) and, eventually, \( U(1)_{em} \). The latter cannot be of any use since if the corresponding FI parameter \( \xi \) were non-vanishing, some squarks or sleptons would get a VEV and hence would Higgs \( U(1)_{em} \) (comparing with the FI model we discussed in the previous lecture, being all MSSM scalars massless at tree level, we will be in the mixed F and D-term phase, and hence the potential would have a minimum at non-vanishing value of some scalar field VEV). As for the hypercharge, this again cannot work, since the trace of \( U(1)_Y \) taken over all chiral superfields vanishes in the Standard Model (this is just telling us that the Standard Model is free of gravitational anomalies). The upshot is that within the MSSM, formula (8.9) reduces to

\[
\text{STr} M^2 = 0 .
\] (8.10)

It is easy to see that this formula is hardly compatible with observations. Since supersymmetry commutes with internal quantum numbers, the vanishing of the supertrace would imply that for any given Standard Model set of fields with equal charge we should observe at least a real component of a sparticle with a mass smaller than all particles with the same charge. Take for instance a charged SU(3) sector. Gluons are massless, since SU(3) is unbroken. From (8.4) it then follows that \( \langle D^a_i \rangle = \langle D^{bi} \rangle = 0 \), which, by (8.5), implies that the corresponding gluinos are also massless. Then, in such charged sector, only quarks and squarks can contribute non-trivially to eq. (8.10). Since they contribute with opposite sign, the squarks cannot
all be heavier than the heaviest quark, and some must be substantially lighter. Take, for instance, the color-triplet sector with electric charge $e = -1/3$, to which down, strange and bottom quarks belong. We get $m_d^2 + m_s^2 + m_b^2 \simeq (5 \text{GeV})^2$. In order to satisfy eq. (8.10) we need scalar partners to satisfy $\sum_i m_{\phi_i}^2 \simeq 2(5 \text{GeV})^2$, which implies that a charged scalar with mass smaller than 7 GeV should exist. This is clearly excluded experimentally.

The upshot of this discussion is that we should give up with the idea that the whole story is as simple as just tree level supersymmetry breaking in the MSSM.

### 8.3 Beyond the MSSM

The supertrace mass formula derived above comes from at tree-level analysis and, for one thing, we know that masses get modified by loop effects. So one might hope that at quantum level things could sensibly change. However, within the MSSM such modifications are small since the Standard Model is a weakly interacting theory at the electro-weak scale, so this would not help much.

A way to avoid the supertrace mass formula severe constraints, while still keeping the MSSM, would be to allow for supersymmetry breaking beyond tree-level, that is dynamical supersymmetry breaking. We do have a dynamical scale we can play with in the Standard Model, the $SU(3)$ strong coupling scale $\Lambda_{\text{QCD}}$. This would provide violations of eq. (8.10) of order $\Lambda_{\text{QCD}}$. These violations would then be of order 300 MeV, which is by far too low for accommodating any sensible phenomenology. Hence, this option could not work either.

The punchline is that in order to describe beyond the Standard Model physics we need something more than just the MSSM. We might need new particles and fields and/or new strong interactions. The options one can play with are many, and understanding the correct path of supersymmetry breaking beyond the Standard Model has been, and still is, a matter of concern and great challenge for theoretical physicists. There are, however, at least two basic properties a competitive model should have. Supersymmetry should be broken dynamically, so to generate the low scale we need in a natural way. Second, in order to avoid the unpleasant constraints coming from the supertrace mass formula, we should better rely on non-renormalizable couplings, or loop effects, to transmit this breaking to the MSSM. Indeed, as we will see shortly, in both cases this would provide corrections to MSSM kinetic terms which could provide, in turn, large violations of eq. (8.10),
hence allowing for phenomenologically meaningful sparticle spectra. Moreover, be-
sides invalidating formula (8.10), such an option would also have the free-bonus of
providing an extra suppression between the supersymmetry breaking scale of the un-
derlying UV theory and the scale of MSSM sparticle masses. Hence, the primordial
supersymmetry breaking scale would not need to be comparable with electro-weak
scale. It could be sensibly higher.

8.4 Spurions, soft terms and the messenger paradigm

Let us deviate, for a while, from what we have been saying so far, and come back to
what we said at the very beginning of previous lecture about possible mechanisms
for supersymmetry breaking. We have a second option we have not yet considered:
explicit supersymmetry breaking by soft terms. Let us suppose we add explicit su-
persymmetry breaking terms to the MSSM Lagrangian. In order to save the nice UV
properties of supersymmetry, these terms should be UV irrelevant. For instance, if
we were to add non-supersymmetric dimensionless couplings, like Yukawa couplings
and scalar quartic couplings, we would certainly destroy the pattern of UV cancel-
lations which makes supersymmetry solving, e.g. the hierarchy problem. We can
instead add mass terms, and more generally, positive dimension couplings, like cubic
scalar couplings. These would simply tell us below which scale UV cancellations will
stop working. Such soft supersymmetry breaking Lagrangian will schematically be
of the form

$$L_{\text{soft}} = m_{\lambda} \lambda \lambda - m^2 \phi \phi + b \phi \phi + a \phi^3 + h.c. ,$$

where $\lambda$ represents gauginos and $\phi$ any possible scalar of the MSSM. The first two
terms provide masses for gauginos (wino, zino, photino, gluino) and scalar particles
(squarks, sleptons and Higgs particle), respectively. The third term, known as B-
term, may arise in the Higgs sector and couples the up and down scalar Higgs $H_u$
and $H_d$. Finally the fourth, known as A-term, corresponds to cubic gauge and flavor
singlet combinations of MSSM scalars, e.g. Higgs and left and right squark com-
ponents. A-terms are in one-to-one correspondence with Yukawa couplings (which
belong to the supersymmetric part of the MSSM Lagrangian): each quark and lepton
is just substituted by its scalar partner.

All terms appearing in eq. (8.11) are UV irrelevant and renormalizable, and it
was indeed shown time ago that the full Lagrangian

$$L = L_{\text{MSSM}} + L_{\text{soft}}$$

(8.12)
is free of quadratic divergences to all orders in perturbation theory. Notice that such a Lagrangian would automatically solve the supertrace mass formula problem. A Lagrangian like the one above would violate eq. (8.10) precisely by terms of order the sparticle masses, see the expression (8.11), which is, by construction, compatible with observations.

There is a number of very important issues one should discuss regarding the Lagrangian (8.12), including a number of potential problems some of the soft terms could pose, like the so-called supersymmetry flavor, CP and fine-tuning problems, to name a few. These put severe constraints on the precise form of $\mathcal{L}_{\text{soft}}$, but we will not discuss any such issues here. What we want to do instead, is to reconnect to our previous discussion and show how such a rather ad hoc Lagrangian as (8.12), where supersymmetry is broken explicitly, can actually be generated by spontaneous supersymmetry breaking in a larger theory, which includes fields and interactions beyond the MSSM ones.

First, let us recall the idea of spurion fields. In a supersymmetric theory any constant, non-zero value for the lowest component of a superfield (a VEV) does not break supersymmetry. Hence, in a supersymmetric Lagrangian each coupling constant can be promoted to a background superfield, a spurion, with non-vanishing such lowest component VEV. Let us take, for instance, the WZ model

$$\mathcal{L} = \int d^2\theta d^2\bar{\theta} Z\Phi + \int d^2\theta \left( 1/2 M\phi^2 + 1/6 y\phi^3 \right) + \text{h.c.} ,$$

and think of $Z$ as a real background superfield while $M$ and $y$ as chiral background superfields. If only their lowest components have a non-vanishing VEV, this is nothing but the WZ model itself.

Interestingly, one can include supersymmetry breaking terms in the above Lagrangian by allowing these superfields having higher (scalar) component VEVs. We can set in general

$$\langle Z \rangle = 1 + \theta^2 B + \text{h.c.} + c \theta^2 \bar{\theta}^2$$

$$\langle M \rangle = \mu - \theta^2 F_M$$

$$\langle y \rangle = y - \theta^2 F_y .$$

Plugging these expressions into the Lagrangian (8.13), after integrating out the auxiliary field $F_\Phi$ we find for the potential

$$V = V_{\text{SUSY}} - \left( c - |B|^2 \right) \bar{\phi}\phi + \left[ (F_M + B\mu) \phi^2 + \left( 1/3 F_y + 1/2 By \right) \phi^3 + \text{h.c.} \right] ,$$

(8.14)
where \( V_{\text{SUSY}} = \left| \mu \phi + \frac{1}{2} y \phi^2 \right|^2 \). We see that the non-supersymmetric contribution to the potential exactly reproduces the second, third and fourth soft terms of the Lagrangian (8.11), upon the trivial identifications

\[
m^2 = c - |B|^2 \\
b = F_M + B\mu \\
a = \frac{1}{3} F_y + \frac{1}{2} B y .
\]

Following the same logic for the SYM action

\[
\mathcal{L} = \int d^2 \theta \, \tau W_\alpha^a W_\alpha^a ,
\]  

one can seemingly reproduce gaugino masses by promoting the complexified gauge coupling \( \tau \) to a chiral superfield and provide a non-vanishing VEV for its F-term

\[
\langle \tau \rangle = \tau + \theta^2 m_\lambda .
\]

Applying this logic to the Lagrangian (8.12), one can actually write all soft terms by means of spurion couplings, and rewrite (8.12) using a pure supersymmetric formalism. This turns out to be a very convenient thing to do when it comes to compute the divergence structure of the theory and prove, e.g. the absence of quadratic divergences.

Although phenomenologically viable and logically consistent, this picture is still not completely satisfactory. The Lagrangian (8.12) has more than 100 free parameters (masses, phases, mixing angles, etc...), meaning that there are few unambiguous predictions one can really make. One might want to find some organizing principle, where these many parameters could be naturally explained in terms of some simpler underlying theory.

Here is where we can close the gap between soft term breaking and spontaneous supersymmetry breaking. It is enough to promote spurions to fully fledged superfields with their own Lagrangian and kinetic terms. By some suitable and for the time being unspecified mechanism, they acquire non-vanishing F and D-terms spontaneously, and then generate soft terms by their interactions with the MSSM fields via couplings of the kind (8.13)/(8.15). This is the basic idea of the so-called \textit{messenger paradigm}: one imagines a fully renormalizable theory where supersymmetry is broken spontaneously in some hidden sector and then communicated to
the MSSM fields by non-renormalizable interactions and/or loop effects. After integrating out heavy fields, this will generate effective couplings precisely as those in the Lagrangian (8.13) and (8.15), with non-vanishing F and D-components for some fields. These F and D-terms will then give rise to soft terms through a procedure like the one above. This way, all specific properties that MSSM supersymmetric breaking soft terms should have, will be ultimately generated (and explained) by a larger theory in which supersymmetry breaking occurs spontaneously.

![Diagram](image)

Figure 8.1: The messenger paradigm: supersymmetry is broken in a hidden sector and then communicated to the visible MSSM sector (or any viable supersymmetric extension of the Standard Model) via interactions felt by the MSSM particles.

### 8.5 Mediating the breaking

What are the possible ways in which a scenario as the one outlined above can actually be realized?

An obvious candidate as messenger of supersymmetry breaking is gravity, since any sort of particle couples universally to it. Gravity is inherently non-renormalizable, at least as it manifests itself at energies lower than the Planck scale. Hence, couplings like those appearing in eqs. (8.13) and (8.15) are precisely what one expects, in this scenario.

Another possibility is that supersymmetry breaking is mediated by gauge interactions. We can imagine that supersymmetry is broken in the hidden sector and that some fields, known as messenger fields, feeling (or directly participating in, this is a model-dependent property) supersymmetry breaking are also charged under Standard Model gauge interactions. Gauginos will directly couple to such messenger fields and get a mass at one-loop. Scalar sparticles, instead, would get mass at two
loops, interacting with messenger fields via intermediate MSSM vector superfields, to which gauginos belong to. In this scenario, soft terms will be generated after integrating out heavy fields, ending-up again with effective couplings of the kind (8.13) and (8.15). Obviously, the main source of mediation can be gauge interactions only in a regime where the always present gravity mediation is suppressed.

In what follows, we are not going to discuss these two mediation mechanisms in detail, nor any of their diverse phenomenological benchmarks, neither the many variants of the basic models which have appeared in the literature, with their pros and cons. What we want to do is just to give a rough idea on how these two mediation mechanisms work and show, in particular, how they can naturally generate, at low energy, spurion-like couplings with MSSM fields and, eventually, give rise to soft terms.

### 8.5.1 Gravity mediation

From a low energy point of view, one can parameterize the effect of unknown physics at the Planck scale $M_{Pl}$ by higher order operators, suppressed by $M_{Pl}$. Suppose that some hidden sector field $X$ gets a non-vanishing $F$-term, that is

$$
\langle X \rangle = 0, \quad \langle F_X \rangle \neq 0.
$$

(8.17)

The most general form of the Lagrangian describing the gravitational interaction between $X$ and the visible sector fields will be something like

$$
\mathcal{L}_{\text{int}} = \int d^2 \theta d^2 \bar{\theta} \left( \frac{c}{M_{Pl}^2} X \overline{Q}_i Q_i + \frac{b}{M_{Pl}^2} X H_u H_d + \frac{b}{M_{Pl}} \overline{X H_u H_d} + h.c. \right) \\
+ \int d^2 \theta \left( \frac{s}{M_{Pl}} X W^a W_a + \frac{a}{M_{Pl}} X Q^i H_u e H_d + h.c. \right)
$$

(8.18)

plus, possibly, higher order operators. In the above expression $Q_i$’s represent all matter superfields as well as the two Higgs doublets, while $H_u$ and $H_d$ obviously refer to the up and down Higgs only. For the sake of simplicity, we have taken all order one dimensionless coefficients in each term to be the same, that is $i$-independent.

Plugging the values (8.17) into the above Lagrangian one gets all possible MSSM soft terms! The first term on the rhs of eq. (8.18) gives rise to non-supersymmetric masses for all sfermions (squarks, sleptons and scalar Higgs particles), while the second and third terms provide mass terms for the scalar Higgs only (more below). The first term of the second line provides gaugino masses. Finally, the last term
generates all A-terms. We see that we get a rather simple pattern of soft terms. Up to order one coefficients, they share one and the same mass scale

\[ m_{\text{soft}} \sim \frac{\langle F_X \rangle}{M_{\text{Pl}}} , \]  

(8.19)

Imposing \( m_{\text{soft}} \) to be order the TeV scale we see that in a gravity mediated scenario the primordial supersymmetry breaking scale, the so-called intermediate scale, is order

\[ M_s = \sqrt{\langle F_X \rangle} \sim \sqrt{m_{\text{soft}} M_{\text{Pl}}} \sim 10^{11} \text{ GeV} , \]  

(8.20)

somewhat in between the electro-weak scale and the Planck scale (as anticipated, sensibly higher than \( m_{\text{soft}} \), the scale of MSSM sparticle masses).

In the general framework outlined in section 8.4, a gravity mediation scenario can be visualized as the following steps, from a theory of quantum gravity (let us assume, for definiteness, this theory to be string theory whose field theory limit is reached by sending the string scale \( \alpha' \) to 0, but this choice does not make a difference in the general logic) down to a (M)SSM low energy effective Lagrangian

\[
\mathcal{L}_{\text{QG}} \xrightarrow{\alpha' \to 0} \mathcal{L}_{\text{non-ren}}^{\text{SUSY}} \xrightarrow{\text{integrate out heavy fields}} \mathcal{L}_{\text{non-ren}}^{\text{spurion (SUSY breaking) VEVs}} \mathcal{L}_{\text{MSSM}} + \mathcal{L}_{\text{soft}} ,
\]  

(8.21)

where the mass scale in \( \mathcal{L}_{\text{soft}} \) is given by (8.19).

Let us spend a few more words on Higgs mass terms. From the Lagrangian (8.18) we see three contributions to scalar Higgs mass. The first gives rise to mass terms for the up and down Higgs, respectively (they are proportional to \( \overline{H}_u H_u \) and \( \overline{H}_d H_d \)). The second term is a B-term, which gives rise to a quadratic term mixing \( H_u \) and \( H_d \). Finally, as for the third term, notice that it can be re-written as

\[
\int d^2 \theta d^2 \overline{\theta} \frac{b}{M_{\text{Pl}}} \overline{X} H_u H_d = b \frac{\langle F_X \rangle}{M_{\text{Pl}}} \int d^2 \theta H_u H_d .
\]  

(8.22)

This contribution is a so-called \( \mu \) term contribution and upon integration in chiral superspace it gives a quadratic contribution similar in structure to the first term.

Notice that all these three couplings are needed in order to trigger electro-weak symmetry breaking (and should all be of the same order of magnitude). The first such terms gives masses to scalar Higgs particle, and it can actually give a negative mass square to some of them, something we certainly need to trigger spontaneous symmetry breaking. The second one is also necessary. One can show the B term to be proportional to \( \sin 2\beta \) where \( \tan \beta \) is the ratio between the VEVs of the up and
down Higgs, $\tan \beta = \langle H_u \rangle / \langle H_d \rangle$. Clearly, if $B = 0$, either the up or the down Higgs do not get a VEV, and therefore one cannot provide masses to all Standard Model particles. Finally, the $\mu$-term is the only possible contribution which can provide higgsino a mass, and therefore should certainly be there.

The way we have re-written the $\mu$-term makes it clear that it can also be (and generically is) generated from a perfectly supersymmetric superpotential coupling in the MSSM Lagrangian

$$W = \mu_{\text{SUSY}} H_u H_d.$$  

(8.23)

There is no a priori reason why the above term, which comes from a supersymmetric contribution and is then not related to the dynamics driving the breaking of supersymmetry, should come to be the same scale of the soft terms, as it should. In principle, it could be any scale between $m_{\text{soft}}$ and $M_{\text{Pl}}$. This is the famous $\mu$ (or $\mu/B\mu$) problem: how to avoid large $\mu$-terms and at the same time have them the same order of magnitude of $B$-terms. Gravity mediation provides an elegant and simple way to solve this problem. First, one can impose some PQ-like discrete symmetry on the MSSM Lagrangian which forbids a tree level $\mu$ term in the superpotential, $\mu_{\text{SUSY}} = 0$. This is achieved giving charge 1 under this symmetry to $H_u$ and $H_d$ and charge -1/2 to all other chiral superfields (quarks and leptons). This way, both the $\mu$- and the $B$-terms are generated radiatively. The non-trivial thing is to make them be the same order of magnitude. However, as we have seen above, this is exactly what happens in a gravity mediation scenario: up to coefficients of order unity, all soft terms, including $B$- and $\mu$-terms, are of the same order, eq. (8.19)!

A typical problem of gravity mediation scenarios is instead the so-called supersymmetry flavor problem. In order not to spoil the excellent agreement between flavor changing neutral current (FCNC) effects predicted by the Standard Model and known experimental bounds, any sort of new physics should not induce extra (relevant) FCNC contributions. In order for this to be the case the interactions mediating supersymmetry breaking better be flavor-blind. Gravity as seen at low energy is certainly flavor blind. However, general relativity is just an effective theory and there is no guaranty that a UV completed quantum theory of gravity is flavor blind (actually quite the opposite, given that global symmetries are believed not to be possible in quantum gravity). Therefore, in general, in gravity mediation scenarios one has to confront with the flavor problem. We will not discuss this further here and refer to the references at then end of the chapter for details. Let us just remark that there exist different proposals on how to overcome this problem, the
most compelling and natural one being the so-called anomaly mediation scenario.

8.5.2 Gauge mediation

Any gauge mediation model is characterized by the assumption that there exist messenger fields. The latter, by definition, are those hidden sector fields which are charged under the Standard Model gauge group. The basic idea of gauge mediation is as follows.

Messengers couple (in a model-dependent way) to hidden sector supersymmetry breaking dynamics and this affects their mass matrix which, besides a supersymmetric contribution (which is supposed to be large enough not to make messengers appear at energies of order the electro-weak scale), receives a non-supersymmetric contribution. By coupling radiatively with MSSM fields, supersymmetry breaking is communicated to MSSM fields and provides all desired soft terms, as we are going to show next. For instance, gaugini get a mass at one-loop while squarks, sleptons and Higgs fields feel supersymmetry breaking at two loops through ordinary $SU(3)\times SU(2)\times U(1)_Y$ gauge boson and gauginos interactions. One of the beauties of gauge mediation as opposed to gravity mediation, is that gauge mediation supersymmetry breaking can be understood entirely in terms of loop effects in a renormalizable framework. Hence, it has a high level of reliability and calculability.

There are different schemes for gauge mediation, e.g. minimal, direct and semi-direct gauge mediation, which differ, ultimately, by the way the messenger mass matrix is affected by the hidden sector supersymmetry breaking dynamics. This provides different patterns for the MSSM soft terms texture. As an exemplification, in what follows we will briefly discuss minimal gauge mediation (MGM) which is simple, still rich enough scenario to let one get a feeling on how things work. In MGM all complicated hidden sector dynamics is parameterized in terms of a single chiral superfield $X$ which couples to the messenger sector through a tree-level superpotential coupling. The messenger sector is made of two set of chiral superfields $\Phi$ and $\bar{\Phi}$ transforming in complex conjugate representation of the SM gauge group, so not to generate gauge anomalies. The interaction term is as simple as

$$W = X\bar{\Phi}\Phi$$

(8.24)

A rough scheme of MGM is depicted in Figure 8.2.

The spurion-like field $X$ inherits non-vanishing F and lower component term
Figure 8.2: Minimal gauge mediation. Messengers feel supersymmetry breaking via a cubic coupling with a spurion-like chiral superfield $X$ which has a non-vanishing F-term VEV inherited from the hidden sector non-supersymmetric dynamics.

VEVs from the hidden sector,

$$\langle X \rangle = M + \theta^2 \langle F_X \rangle .$$  \hspace{1cm} (8.25)

Once plugged into the messenger Lagrangian, this gives a splitted messenger mass spectrum

$$m^2, \tilde{m}^2 = M^2 \pm \langle F_X \rangle, \quad m_{\psi,\tilde{\psi}} = M .$$  \hspace{1cm} (8.26)

While fermions receive only the supersymmetric contribution, scalars receive both supersymmetric and non-supersymmetric contributions. Recalling that messenger fields are charged under the SM gauge group we see there is a stability bound which forces us to take $M^2 > \langle F_X \rangle$ (if not, some messenger scalars would get a non-vanishing VEV and would break part of the SM gauge group). If $M$ is large enough we can then integrate the messengers out and the effective low energy theory at scale lower than $M$ breaks supersymmetry.

The net low energy effect boils down to radiative corrections to gaugino propagator, which get a mass at one loop, while gauge bosons remain massless since they are protected by gauge invariance. Via intermediate Standard Model gauge coupling interactions, also MSSM scalar fields get a non-supersymmetric mass contribution, though at two-loop order. Feynman diagrams contributing to gaugino and scalar masses are reported in Figures 8.3 and 8.4, respectively.

The gaugino mass computation is rather easy, since only one type of diagram contributes. In the limit of small $\langle F_X \rangle / M^2$ the end result can be organized in a
series expansion and reads, to leading order in $\langle F_X \rangle / M^2$

$$m_\lambda \sim \frac{g^2}{16\pi^2} \frac{\langle F_X \rangle}{M} \left[ 1 + \mathcal{O}\left(\frac{\langle F_X \rangle^2}{M^2}\right) \right]. \quad (8.27)$$

Summing-up all two-loop contributions renormalizing scalar masses is instead quite laborious even if conceptually straightforward. However, the end result is surprisingly simple and, again in the limit of small $\langle F_X \rangle / M^2$, reads

$$m_{sf}^2 \sim \left( \frac{g^2}{16\pi^2} \right)^2 \frac{\langle F_X \rangle^2}{M^2} \left[ 1 + \mathcal{O}\left(\frac{\langle F_X \rangle^2}{M^2}\right) \right]. \quad (8.28)$$

In principle, there is also a one-loop contribution to sfermion masses originating from the quartic scalar coupling involving two sfermions and two scalar messengers, the same vertex which gives raise to the two-loop contribution in the last diagram of figure 8.4. Due to contraction on gauge indexes, this can be non-vanishing for abelian factors only, like e.g. $U(1)_Y$. This contribution would be proportional to the hypercharge of the corresponding sfermions and therefore may induce tachyonic mass contributions which would be problematic, phenomenologically. Therefore, a symmetry in the messenger sector is usually imposed in order to avoid such dangerous one-loop contributions.

A-terms are also generated radiatively, via the insertion of the renormalized gaugino propagator of Figure 8.3 inside a fermion-higgsino-gaugino loop to which a Higgs field and two sfermions can be attached as external legs, as shown in figure 8.5. Overall, this is again a two-loop effect.

Note, finally, that B- and \( \mu \)-terms cannot be generated by any of the diagrams in Figure 8.4 and require a separate discussion, as we will see shortly.

In agreement with the general philosophy advocated in section 8.4, one can get these same results working within the effective low energy theory valid at scales
Figure 8.4: Two-loops diagrams providing sfermion masses. There are four different class of diagrams, the first three originating from a specific MSSM scalar field one-loop diagram by inserting messenger loop corrections, as indicated. The last is a two-loop diagram which comes from D-terms and mixes MSSM and messenger scalars. Conventions are as in Figure 8.3.

smaller than $M$, which is obtained by integrating messenger fields out. At $E < M$ the effect of the messengers is taken care of in the wave function renormalization of gauge and matter kinetic terms of the MSSM fields. Soft terms arise from derivatives in the $X$-field of the renormalized gauge and matter kinetic functions, $Z_V(X, \mu)$ and $Z_Q(X, \overline{X}, \mu)$, which can be evaluated at a scale $\mu$ by solving the RG equations. For example, soft masses read

$$m_\lambda \sim \left. \frac{\partial \ln Z_V(X, \mu)}{\partial \ln X} \right|_{X=M} \langle F_X \rangle M$$

$$m_{sjf}^2 \sim \left. \frac{\partial^2 \ln Z_Q(X, \overline{X}, \mu)}{\partial \ln X \partial \ln \overline{X}} \right|_{X=M} \langle F_X \rangle^2 M$$ (8.29) (8.30)

and similar formulae hold for the A-terms. This powerful method, originally proposed by Giudice and Rattazzi, is not specific to gauge mediation but works whenever supersymmetry breaking is communicated by renormalizable perturbative interactions. We refer to the bibliography at the end of the chapter for more details.
We see from eqs. (8.27)-(8.28) that in MGM all soft terms come naturally of the same order of magnitude
\[ m_{\text{soft}} \sim \frac{g^2}{16\pi^2} \frac{\langle F_X \rangle}{M} . \] (8.31)

Imposing again that soft masses are order the TeV scale and setting \( g^2/16\pi^2 \sim 10^{-2} \) one then gets
\[ \frac{\langle F_X \rangle}{M} \sim 10^5 \text{GeV} , \] (8.32)

which implies that in MGM the primordial supersymmetry breaking scale \( M_s \) is bounded from below as
\[ M_s = \sqrt{\langle F_X \rangle} \sim 10\sqrt{m_{\text{soft}} M} \geq 10^5 \text{ GeV} , \] (8.33)

where in the last inequality we used that \( M^2 \geq \langle F_X \rangle \) and eq. (8.32) (the lower bound for \( M_s \) is reached for \( M^2 \sim \langle F_X \rangle \)).

The analogous of the structure (8.21) is now
\[ L_{\text{ren}}^{\text{SUSY}} \overset{\text{integrate out heavy fields}}{\rightarrow} L_{\text{non-ren spurion}} \overset{\text{spurion (SUSY breaking) VEVs}}{\rightarrow} L_{\text{MSSM}} + L_{\text{soft}} , \] (8.34)

where the mass scale in \( L_{\text{soft}} \) is given in this case by (8.31).

As we have already observed, gravity mediation is an always present contribution to supersymmetry breaking mediation mechanisms (\textit{i.e.} the field \( X \) would also interact gravitationally with the visible sector via a Lagrangian like (8.18), in general). Hence, it is only when its contribution is suppressed with respect to that of gauge mediation that the latter can play a role. In order for gravity effects to be negligible, say to contribute no more than 1/1000 to soft mass squared, one gets an upper bound for the scale \( M \)
\[ \frac{g^2}{16\pi^2} \frac{\langle F_X \rangle}{M} \geq 10^{3/2} \frac{\langle F_X \rangle}{M_{\text{Pl}}} \rightarrow M \leq \frac{g^2}{16\pi^2} 10^{-3/2} M_{\text{Pl}} \sim 10^{15} \text{ GeV} . \] (8.35)
Using the relation $M_s \sim 10\sqrt{m_{\text{soft}}M}$ this gives an upper bound for $M_s$ of order $10^{10}$ GeV. Together with the lower bound (8.33) this implies that the supersymmetry breaking scale $M_s$ can range from $10^5$ to up to $10^{10}$ GeV, in gauge mediation scenarios.

Let us close this brief overview on gauge mediation saying a few words about flavor and $\mu$ problems. We are in a sort of reversed situation with respect to gravity mediation. Gauge interactions are intrinsically flavor-blind. Hence, gauge mediation does not provide any further FCNC contribution to the Standard Model and the flavor problem is then automatically solved in this framework. On the contrary, the $\mu$ problem is much harder. One can again avoid a supersymmetric $\mu$ term by means of some discrete symmetry to be imposed on the Higgs sector supersymmetric Lagrangian. What is problematic, though, is to generate radiatively $\mu$ and B terms of the same order of magnitude. The two-loop diagrams in Figure 8.4 do not provide B and $\mu$ terms and one should then argue for a direct coupling between the Higgs and the messenger sectors. The simplest possible model one can think of, does not work. Allowing a cubic coupling between $H_u, H_d$ and the field $X$

$$W_\mu = \lambda_H X H_u H_d ,$$  \hspace{1cm} (8.36)

one could in principle generate both a $\mu$- and a B-term from supersymmetry breaking dynamics but they do not come of the same order of magnitude. In order for the $\mu$-term being of the order of other soft masses, as it should be, we need

$$\mu = \lambda_H M \sim 1 \text{ Tev} .$$  \hspace{1cm} (8.37)

This implies that $\lambda_H$ is order $10^{-2}$ or smaller. This enhances the B-term. Indeed, recalling that $\langle F_X \rangle \leq M^2$, the non-supersymmetric to supersymmetric mass ratio contribution coming from the superpotential coupling (8.36) is

$$\frac{B}{\mu^2} \sim \frac{\lambda_H \langle F_X \rangle}{\lambda_H^2 M^2} = \frac{\langle F_X \rangle}{\mu M} \sim 10^2 ,$$  \hspace{1cm} (8.38)

where in the last step we used the fact that $\langle F_X \rangle/M \sim 10^5$GeV. This gives an unacceptably large B-term. This problem is not specific to MGM nor to the actual way we have generated $\mu$ and B-terms here. It is a problem which generically plagues any gauge mediation scenario. Even though several proposals has been put forward to solve the $\mu$-problem in gauge mediation, it is fair to say that a fully satisfactory and natural framework to solve this problem is not yet available.
It is finally worth stressing that the simple mass pattern (8.27)-(8.28) is not a generic feature of gauge mediation but specific to MGM only. Indeed, in another popular scheme, direct gauge mediation, the soft spectrum tends to be split, that is gauginos are typically suppressed with respect to scalar particles.

A generic, model-independent prediction of gauge mediation scenarios, instead, is that the gravitino is the lightest supersymmetric particle. Gravitinos interact only gravitationally and get a mass due to higgsing of order \( m_{3/2} \sim \langle F_X \rangle / M_{\text{Pl}} \). Therefore, while in a gravity mediation scenario gravitinos have a mass of the same order of magnitude of all other soft terms, in gauge mediation they are suppressed, since

\[
m_{3/2} \sim \frac{\langle F_X \rangle}{M_{\text{Pl}}} = \frac{\langle F_X \rangle}{M} \frac{M}{M_{\text{Pl}}} << \frac{\langle F_X \rangle}{M} = m_{\text{soft}},
\]  

and they can as light as few eV.

Let me conclude this brief overview stressing again what is the main point of this all business. What all these mediation models are about is to provide a theory of the soft terms, a predictive pattern for these extra terms that one can (and has to) add to the MSSM Lagrangian or any desired supersymmetric extension of the Standard Model. We have been trying to give an idea on how things might work, and reviewed few aspects of the most basic mediation mechanisms. A thorough analysis of the phenomenology of these schemes and their variants is not our goal here and we refer to the bibliography at the end of the chapter for a more detailed analysis. In the remainder of these lectures we will instead focus on the hidden sector dynamics, trying to deepen our understanding of supersymmetric dynamics at strong coupling. Besides its intrinsic interest (and the far reaching consequences in our understanding of strong coupling regimes of gauge theories in general), this will also allow us to study concrete models of dynamical supersymmetry breaking.

8.6 Exercises

1. Derive the gaugino mass formula (8.27) from the Feynman diagram of Figure 8.3.

2. Compute the contribution of two diagrams arbitrarily chosen out of those depicted in Figure 8.4 to the sfermion mass formula (8.28).
References


