Christmas Workshop on Quivers, Moduli Spaces and Integrable Systems

Genoa, December 19-21, 2016

Speakers and abstracts

Ada Boralevi, Moduli spaces of framed sheaves on (p,q)-toric singularities

I will report on a joint work in progress with Francesco Sala. Our goal is to achieve a description of the moduli space of framed sheaves on (p,q)-toric singularities and on their desingularizations via ADHM data, and thus via quiver varieties. The key point consists in studying the Beilinson spectral sequence for a semi-orthogonal decomposition associated with a coherent sheaf, and determining from such spectral sequence a quiver data description in the case of a framed sheaf. I will present some results holding for toric singularities and hopes and conjectures for their desingularizations.

Mattia Cafasso, Some analytical aspects of the Kontsevich matrix model.

Following my two last works with M. Bertola, I will consider two different aspects concerning the Kontsevich matrix models: its universality and the convergence of its partition function to a particular tau function of the Painlevé I hierarchy.

Alberto Celotto, Topology of the moduli spaces of framed sheaves on A_k orbifolds

We introduce the problem of computing the Poincaré polynomial of moduli spaces of framed torsion free sheaves on (possibly orbifold) surfaces X. We sketch the computation in the case X is an A_k orbifold, i.e., a certain stacky compactification of the resolution of a cyclic quotient singularity

Alessandro Chiodo, Hodge integrals and r-spin curves

Buryak, Dubrovin, Guéré and Rossi have recently found a new approach to integrable hierarchies involving r-spin curves and the Hodge classes. The standard virtual r-spin class is well-defined but difficult to compute beyond genus zero. I will try to explain why their approach can be systematically solved via Givental formalism and Grothendieck-Riemann-Roch.

Giordano Cotti, Monodromy local moduli for Frobenius coalescent structures and Dubrovin's conjecture.

Based on joint work with B. Dubrovin and D. Guzzetti. In occasion of the 1998 ICM in Berlin, B. Dubrovin conjectured an intriguing connection between the enumerative geometry of a Fano manifold X with algebro-geometric properties of exceptional collections in the derived category $D^b(X)$. Under the assumption of semisimplicity of the quantum cohomology of X, the conjecture prescribes an explicit form for local invariants of $QH^*(X)$, the so-called “monodromy data”, in terms of Gram matrices and characteristic classes of objects of exceptional collections. Frobenius manifolds appearing in the study of the conjectural relations mentioned above typically show a coalescence phenomenon at points where the Frobenius algebra is semisimple, but the operator of multiplication by the Euler vector field has not simple spectrum. On the one hand, the definition of monodromy data is based on the analytic theory of isomonodromy deformations, which a priori cannot be applied at coalescence semisimple points of $QH^*(X)$. On the other hand, it turns out that the Frobenius structure may be known only at coalescence points, which are thus the only locus where the monodromy data can actually be computed. This is the case of the small quantum cohomology of complex Grassmannians, for which the occurrence and frequency of the coalescence phenomenon is surprisingly subordinate to the distribution of prime numbers. In this talk I will firstly show how under minimal conditions the classical theory of M. Jimbo, T. Miwa and K. Ueno (1981) can be extended to describe isomonodromy deformations at a coalescing irregular singularity; I will also show how to locally describe the Frobenius structure near coalescing semisimple points, and finally, what is the “mirror counterpart” of our description in terms of exceptional collections in the derived category.

Peter Dalakov, G_2 Hitchin systems and cubics

I will discuss some work in progress (with U. Bruzzo) on the generalised G_2 Hitchin system. In particular, I will address the invariance of the Donagi-Markman cubic under the Langlands involution of the Hitchin base, as well as some examples in low genus.

Valeriano Lanza, Moduli spaces of framed sheaves on Hirzebruch surfaces: the minimal case

In 2015 Bartocci, Bruzzo, and Rava provided a necessary and sufficient condition for the moduli space of framed sheaves on Hirzebruch surfaces in order to be nonempty. This condition is expressed by means of an inequality involving the Chern invariants. Whenever this inequality is satisfied as an equality we say that we are considering a "minimal case." After introducing the topic, we shall prove that, in the minimal case, this moduli space reduces to a Grassmannian or a vector bundle over a Grassmannian.

Paolo Lorenzoni, Bi-flat F-manifolds, Painlevé transcendents and complex reflection groups

We study F-manifolds equipped with a pair of flat connections (and a pair of F-products), that are required to be compatible in a suitable sense. In the first part of the talk I will discuss some examples related to complex reflection groups. In the second part I will show that bi-flat F-manifolds in dimension 3 are locally parameterized by solutions of the full Painlevé IV,V and VI equations, according to the Jordan normal form of the operator of multiplication by the Euler vector field. Based on joint works with Alessandro Arsie.

Emanuele Macrì, Derived categories of cubic fourfolds and non-commutative K3 surfaces

The derived category of coherent sheaves on a cubic fourfold has a subcategory which can be thought of as the derived category of a non-commutative K3 surface. This subcategory was studied recently in the work of Kuznetsov and Addington-Thomas, among others. In this talk, I will present joint work in progress with Bayer, Lahoz, and Stellari on how to construct Bridgeland stability conditions on this subcategory. This proves a conjecture by Huybrechts, and it allows one to start developing the moduli theory of semistable objects in these categories, in an analogue way as for the classical Mukai theory for (commutative) K3 surfaces. I will also discuss a few applications of this result.

Andrea Maiorana, Semistable sheaves on a quadric as quiver representations

Using a well-known equivalence between the derived category of $\mathbf{P}^1$x$\mathbf{P}^1$ and that of a certain quiver, we show that Gieseker-semistable sheaves on $\mathbf{P}^1$x$\mathbf{P}^1$ are in a natural correspondence with quiver representations that are semistable in an appropriate sense.

Davide Masoero, Affine Opers and Bethe Ansatz

The ODE/IM correspondence is a conjectural and surprising link between nonlocal observables of integrable quantum field theories and monodromy data of linear analytic ODEs. In this talk g is a simple Lie algebra over the complex field, (g,1) the corresponding untwisted Kac-Moody algebra and Lan(g,1) the Langlands dual of (g,1). We prove the following conjecture of Feigin and Frenkel ['11]: If L is an Lan(g,1)-affine oper (of a particular type), its monodromy data satisfy the Bethe Ansatz equations of the Quantum g-KdV model.

The talk is based on a joint work with A. Raimondo and D. Valeri: Bethe Ansatz and the Spectral Theory of affine Lie algebra-valued connections I. The simply-laced case. Comm Math Phys, 344 (2016), no. 3; Bethe Ansatz and the Spectral Theory of affine Lie algebra-valued connections II: The non simply-laced case. Comm Math Phys (2016).w

Giovanni Ortenzi, Boundary effects in stratified Euler fluids

In this talk we shall discuss some results on the dynamical effects of the stratification-boundary interplay in Euler fluids. In particular, we will speak about quasilinear models in different density-difference regimes. This is based on joint works with R. Camassa, G. Falqui, B. Konopelchenko and M. Pedroni.

Andrea Raimondo, A brief history of the ODE/IM correspondence.

In this talk I will outline some of the historical aspects regarding the ODE/IM correspondence, an intriguing relation between ordinary differential equations and (quantum) integrable models. Starting from the original discovery by Dorey and Tateo that the spectral determinant of certain Schrödinger operators satisfies the Bethe Ansatz for the quantum Korteweg-de Vries model, I will gradually move towards more elaborated examples, involving arbitrary simple Lie algebras. More recent aspects of the ODE/IM correspondence, including the relations with affine opers and the Geometric Langlands correspondence, will be addressed in the complementary talk by Davide Masoero.

Paolo Rossi, Quantum integrable systems of double ramification type

In a recent paper with Buryak, Dubrovin and Guéré, studying the double ramification hierarchy (a quantum integrable systems one can associate to a cohomological field theory on the moduli space of curves), we realized that a certain geometric recursion for the intersection numbers involved in the construction could be abstracted from its geometric context. The outcome is a strikingly simple master equation for a vast class of (quantum or classical) integrable systems with many interesting properties. We called them “integrable systems of double ramification type.” I will present their properties and our results on their classification.

Volodya Rubtsov, Elliptic Algebras, Heisenberg group, unimodularity and Cremona transformations

We shall discuss some aspects of polynomial Poisson structures on $\mathbf{C}^n$ and $\mathbf{P}^n$ with n = 3, 4. The quasi-classical limit of the famous elliptic Sklyanin algebra is a particular important example of such structures. We use the Heisenberg group invariance and describe a unimodularity property of elliptic Poisson algebras. The case of n=5 is of a special interest because of presence of two non-isomorphic families of Sklyanin elliptic algebras (Odesskii-Feigin).

Francesco Sala, K-HA/CoHA of the stack of Higgs sheaves on a curve

K-HA/CoHA of preprojective algebras play a preeminent role in algebraic geometry, representation theory and mathematical physics. For example, if the preprojective algebra is the one of the Jordan quiver, the corresponding CoHA is the Maulik-Okounkov Yangian associated with the Jordan quiver. It acts on the equivariant cohomology of Hilbert schemes of points on the complex affine plane ("extending" the previous results of Nakajima, Grojnowski, Vasserot, etc for actions of Heisenberg algebras) and of moduli spaces of framed sheaves on the complex projective plane. The latter action yields an action of W-algebras and hence provides a proof of the Alday-Gaiotto-Tachikawa conjecture for pure supersymmetric gauge theories on the real four-dimensional space. In this talk, I will discuss K-HA/CoHA associated with the (derived) stack of (nilpotent) Higgs sheaves on a smooth projective complex curve. Their representations can be realized by using the equivariant K-theory/cohomology of moduli spaces of (framed) torsion free sheaves on minimal resolutions of (p, p-1)-toric singularities (discussed in Ada's talk). Also, I will state a list of conjectures about the interpretation of these algebras in terms of quantum groups and their actions on the moduli spaces of stable pairs on the total space of the direct sum of the canonical line bundle of the curve with the trivial line bundle. If time permits, I will discuss also the twisted case: the construction of representations, in this case, should involve moduli spaces of (framed) torsion free sheaves on minimal resolutions of arbitrary (p,q)-toric singularities, that are discussed in Ada's talk.

Tom Sutherland, Poincaré quivers

I will associate a quiver to each of the six Painlevé equations whose underlying graph is that of a finite, affine or elliptic root system according to the (rational, trigonometric, elliptic) trichotomy of Painlevé equations. I will consider the corresponding Painlevé integrable system in the context of spaces of stability conditions and cluster varieties of the associated quivers.

Alberto Tacchella, Integrable systems on quivers

Every quiver determines an associative algebra on which one can define geometric objects such as symplectic forms and Poisson brackets. This opens up the possibility of considering abstract Hamiltonian systems on such algebras, inducing (a family of) ordinary Hamiltonian systems on the representation spaces of the corresponding quiver. These systems may be "integrable" in various senses of the word. The goal of this talk is to show some examples of the different situations which may arise.