

SISSA - Università di Trieste
 Corso di Laurea Magistrale in Matematica
 A. A. 2010/2011
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Problem 1

Let $u(x, t)$ be the solution to the mixed problem

$$\begin{aligned} u_{tt} &= u_{xx}, \quad x > 0, \quad t > 0, \\ u_x(0, t) &= 0, \\ u_t(x, 0) &= 0, \end{aligned}$$

with

$$u(x, 0) = \begin{cases} 2x - 2 & \text{if } x \in [1, 2] \\ -2x + 6 & \text{if } x \in [2, 3] \\ -2x + 10 & \text{if } x \in [5, 6] \\ 2x - 14 & \text{if } x \in [6, 7] \\ 0 & \text{elsewhere} \end{cases}.$$

Draw the graph of the solution for $t = 0, t = 1, t = 2.5, t = 3, t = 5.5$.

Problem 2

Find a function u harmonic for $x^2 + y^2 < R^2$ such that

$$\left(\frac{\partial u}{\partial r}\right)_{|r=R} = \cos^2 \phi - \frac{1}{2}.$$

Here

$$x = r \cos \phi, \quad y = r \sin \phi,$$

are the polar coordinates on the plane.

Problem 3

Find solution $u(x, t)$ to the Cauchy problem

$$\begin{aligned} u_t &= u_{xx}, \quad x \in (-\infty, \infty), \quad t > 0, \\ u(x, 0) &= e^{-\frac{1}{2}x^2} + e^{-2x^2}. \end{aligned}$$