Problem 1

Let $u(x,t)$ be the solution to the mixed problem

$$
\begin{aligned}
u_{tt} &= u_{xx}, \quad x > 0, \quad t > 0, \\
u_x(0,t) &= 0, \\
u_t(x,0) &= 0,
\end{aligned}
$$

with

$$
u(x,0) = \begin{cases}
2x - 2 & \text{if } x \in [1,2] \\
-2x + 6 & \text{if } x \in [2,3] \\
-2x + 10 & \text{if } x \in [5,6] \\
2x - 14 & \text{if } x \in [6,7] \\
0 & \text{elsewhere}
\end{cases} .
$$

Draw the graph of the solution for $t = 0$, $t = 1$, $t = 2.5$, $t = 3$, $t = 5.5$.

Problem 2

Find a function u harmonic for $x^2 + y^2 < R^2$ such that

$$
\left(\frac{\partial u}{\partial r} \right)_{|r=R} = \cos^2 \phi - \frac{1}{2}.
$$

Here

$$
x = r \cos \phi, \quad y = r \sin \phi,
$$

are the polar coordinates on the plane.

Problem 3

Find solution $u(x,t)$ to the Cauchy problem

$$
\begin{aligned}
u_t &= u_{xx}, \quad x \in (-\infty, \infty), \quad t > 0, \\
u(x,0) &= e^{-\frac{1}{2}x^2} + e^{-2x^2}.
\end{aligned}
$$