

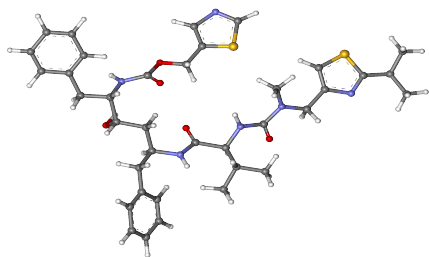
# Neural networks & Crystal structure prediction

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Electronic Structure Theory - SISSA

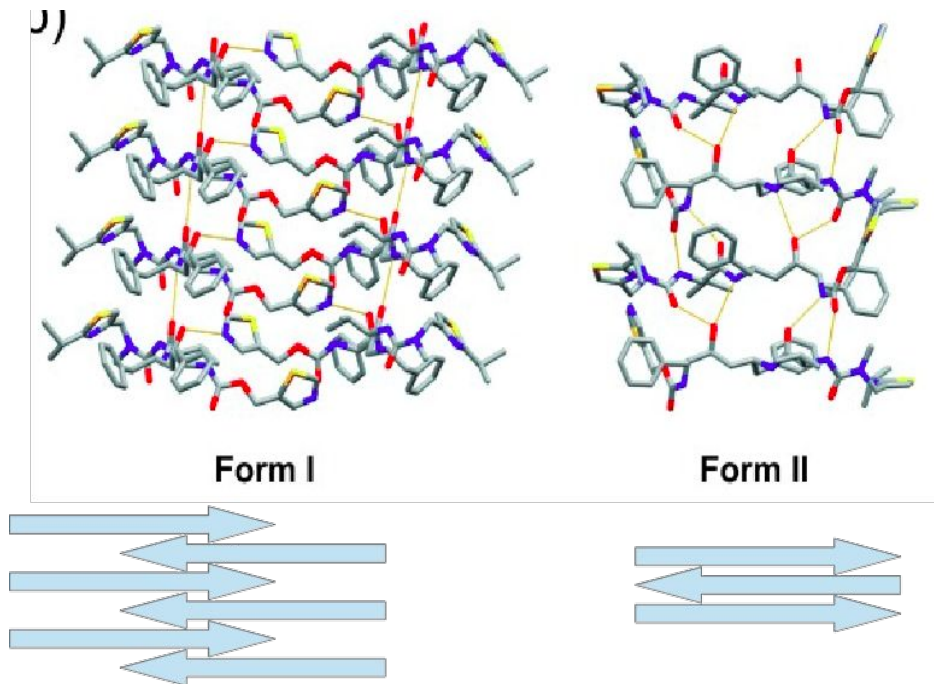
Emine Kucukbenli

# Polymorphism in real life



## Famous example: Ritonavir

- HIV/AIDS & HepC med.
- 1996 - Entered in market in “Form I” (non-ref. caps)
- 1998 - Removed from the market in “Form II”

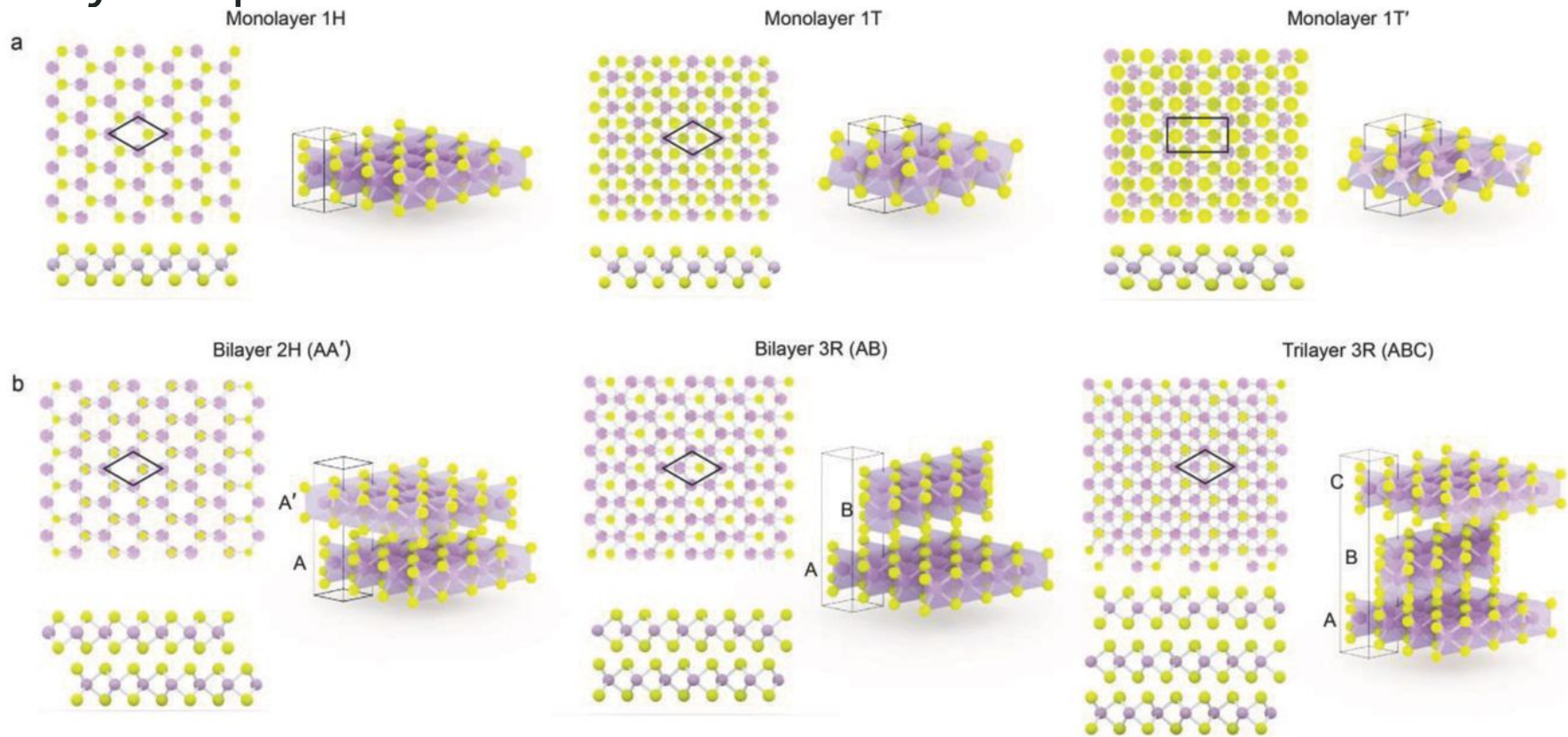


Bauer et al., *Pharmaceutical Research* 18 (2001)  
Datta et al., *Nature Reviews Drug Disc. Rev.* 3 (2004)  
Bucar et. al., *Angew. Chem.* 54 (2015) “disappearing polymorphs”

# Polymorphism

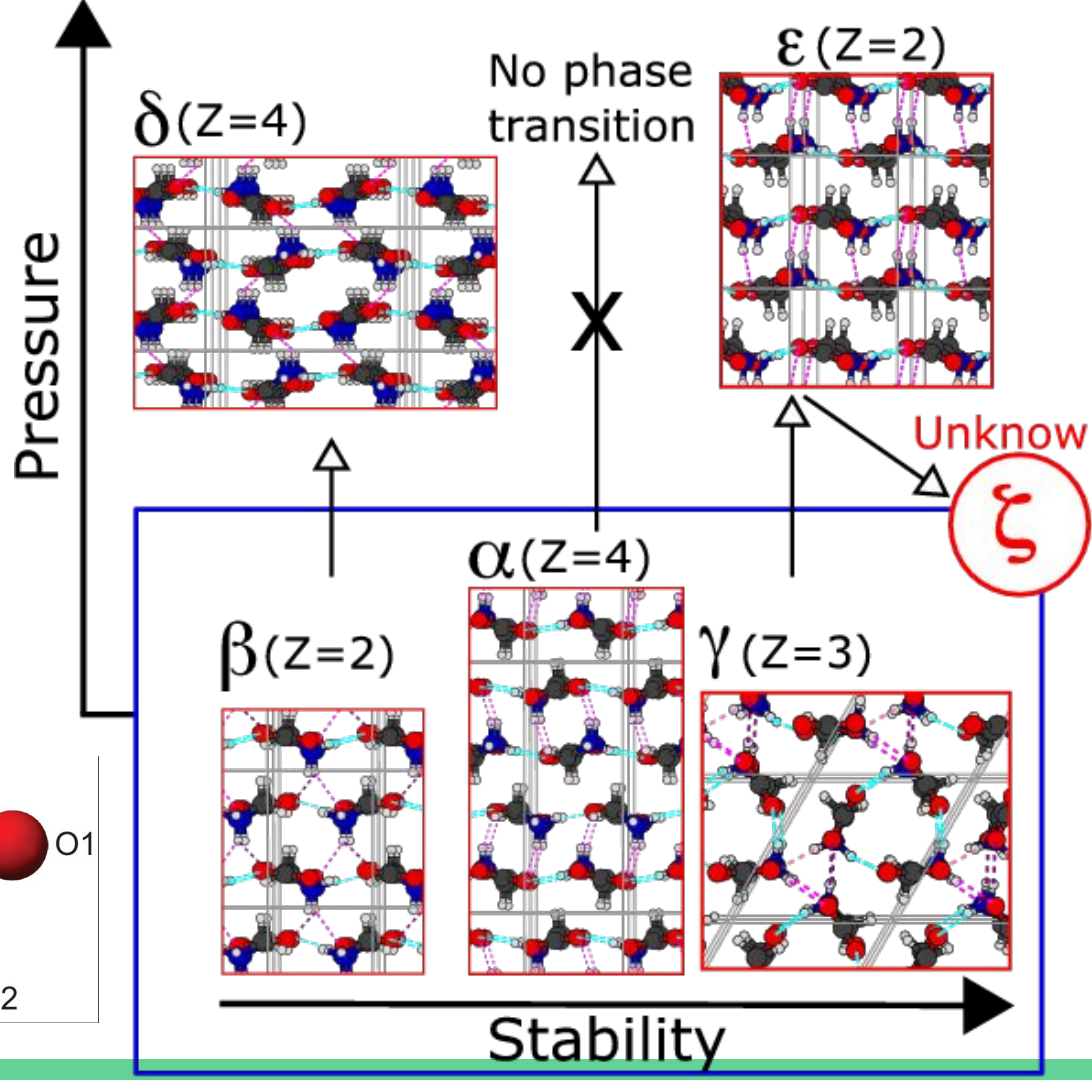
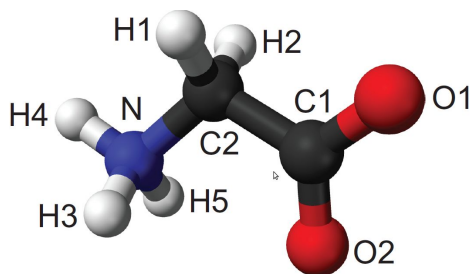
- Aspirin, paracetamol & others
- Al<sub>2</sub>O<sub>3</sub> - catalysis
- Mn-cubane - catalysis
- Coronene & magnetism
- “Jumping crystals”
- 2D-heterostructures layer polymorphism and polytypes, e.g. CO<sub>2</sub> reduction race

# Polymorphism



# Polymorphism

- Aspirin, paracetamol & others
- Al<sub>2</sub>O<sub>3</sub> - catalysis
- Mn-cubane - catalysis
- Coronene & magnetism
- “Jumping crystals”
- 2D-heterostructures layer polymorphism and polytypes, e.g. CO<sub>2</sub> reduction rate



# Given a molecule/stoichiometry:

## 1. CSP questions:

- What is the crystal structure?
- How stable/soluble/active is it?
- Structure with optimum properties?

## 3. CSP challenges:

- Many possibilities to explore
- Big unit cells ( $Z > 1$ , incomm. layers)
- Several energy & length scales
- Predictive power
- Structure-property - it's complicated

## 2. CSP Strategies:

- Explore the phase space
- Characterize what is found
- Build descriptors for desired properties

## 4. Where ML can help:

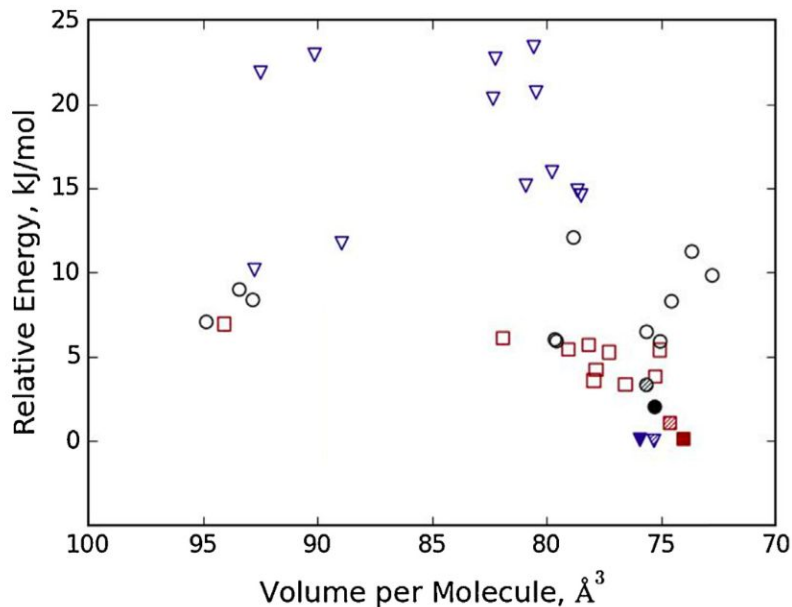
- Accelerate exploration
- Accelerate characterization
- Recognize descriptors, propose new structures with desired properties  
(Reinf. Learning, Gen. Models, Autoencoders etc.)

# Exploring polymorphism with Genetic Algorithm

1. A population of random structures
  2. Energy ordering - select champions
  3. Champions reproduce: Make offsprings
  4. Decide which children will survive
  5. Repeat 2-4 as long as you can;
  6. Repeat 1-5 for different Z
- Heredity operations
  - Random members to increase diversity
  - Similarity measurement via fingerprinting
  - Remembering family history

# Too big to explore with DFT:

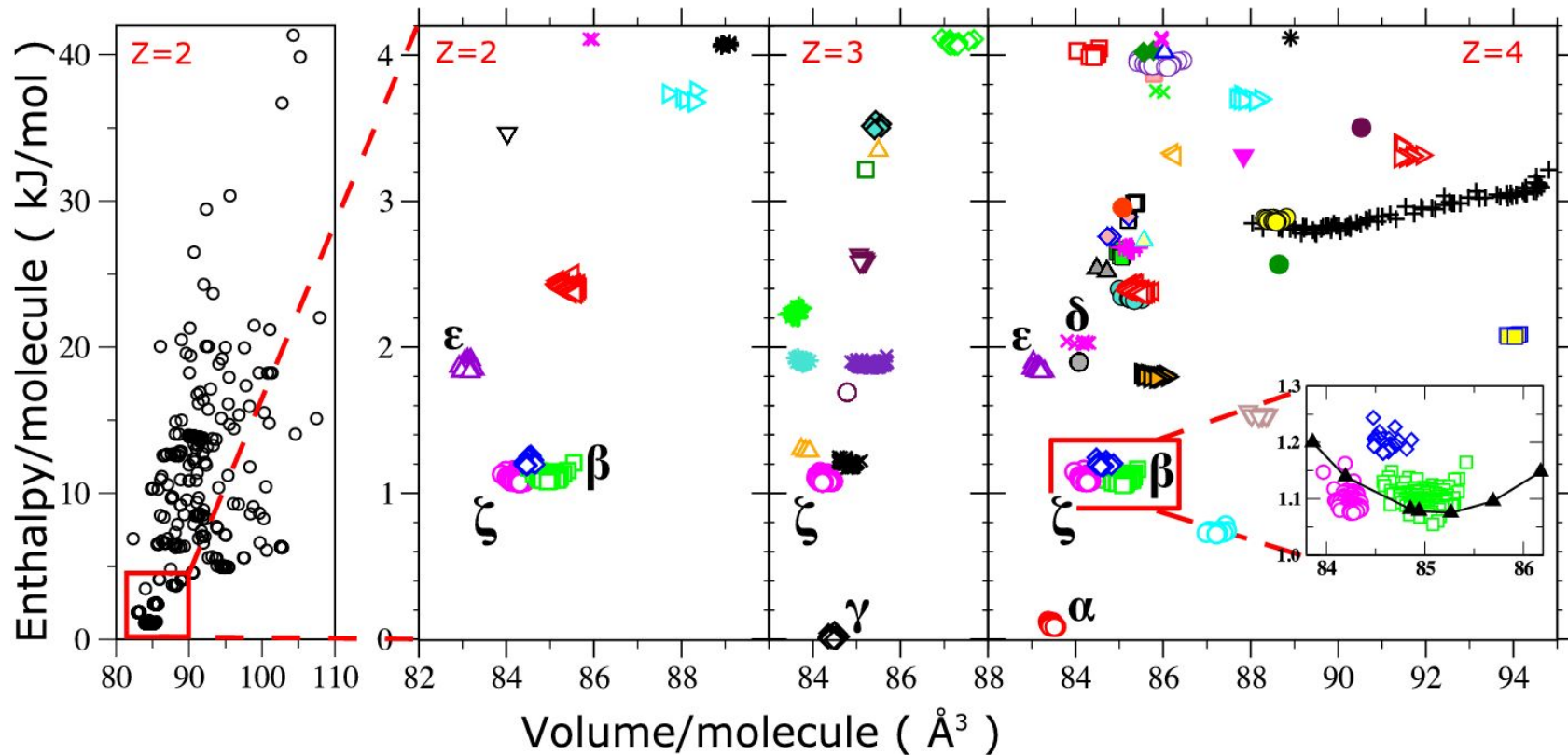
- Many more structures, shorter time than published work
- But cannot find the *alpha* phase !
- (A published work also could not find the alpha phase - visual inspection :-/)
- Found alpha by mimicking nature:  
“Random” musicians are not random:  
Symmetry group probability  
distribution mimics the one of nature  
using DB of molecular crystals. - “Data  
mining”



Lund et al, Chem Phys Lett 262 (2015)



# GLY polymorphism



# Zeta GLY: what did we do right? Accurate energy & volume

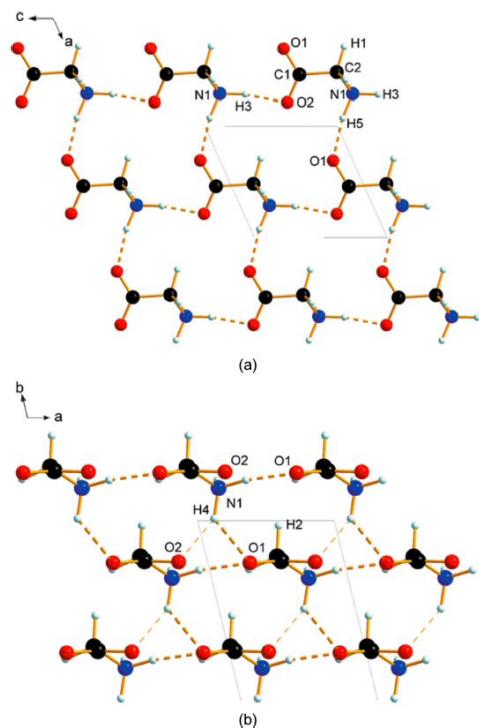
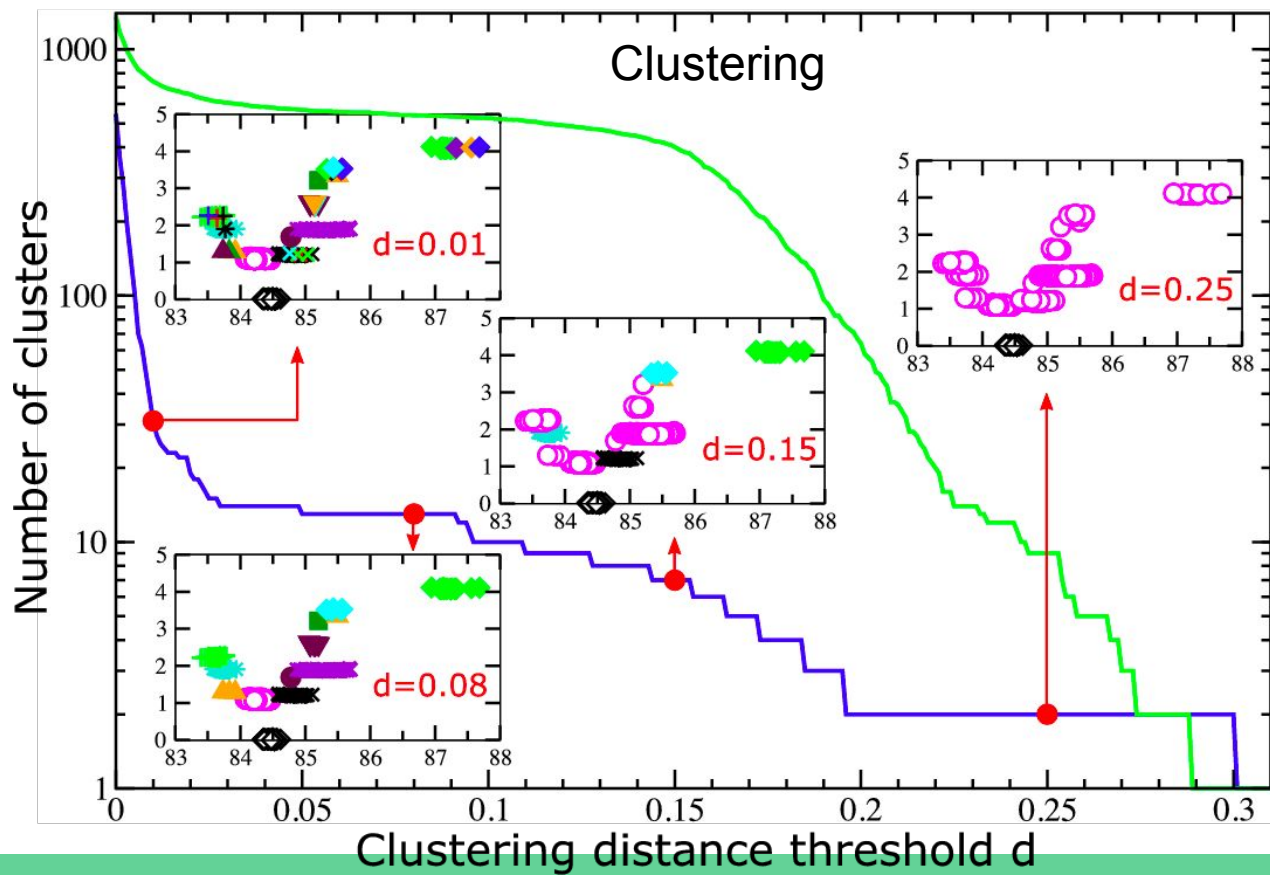


Figure 3  
Intermolecular interactions in  $\zeta$ -glycine. (a) Layers formed in the  $ac$  plane, viewed along  $b$ . (b) Stacking of the layers, viewed along  $c$ .







# Neural Networks for CSP

## 4. Where ML can help:

- Accelerate exploration
- Accelerate characterization
- Recognize descriptors, propose new structures with desired properties  
(Just scratched the surface: Reinf. Learning, Gen. Models, Autoencoders etc.)

Learn a *molecular* forcefield for  $Z=1$ , transfer to  $Z>1$

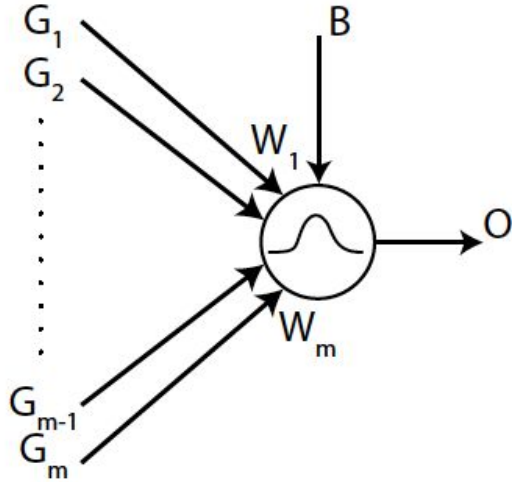
Learn *atomistic* forcefield, transfer to other molecules

Learn *relaxation moves*  
For big systems with many atoms, far from the equilibrium positions

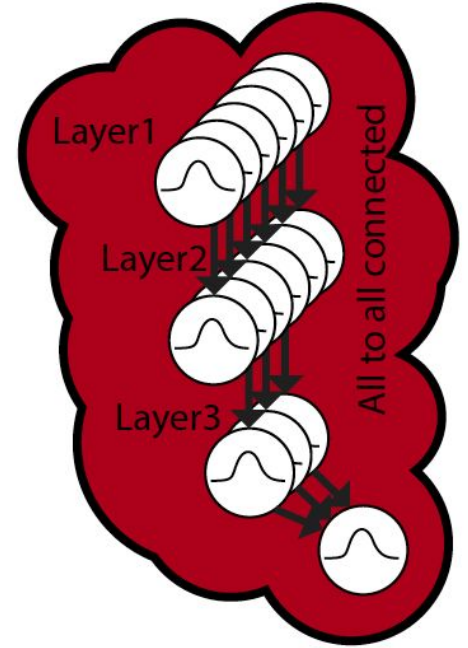
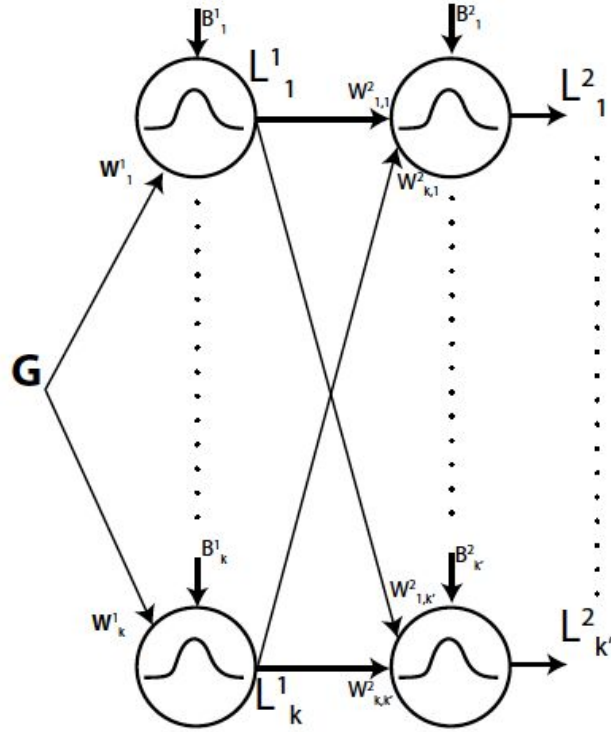
To keep in mind:

- Representation + Architecture + Target are all related
- Model training vs evaluation costs can be very different
- Transferability is good, but data keep changing

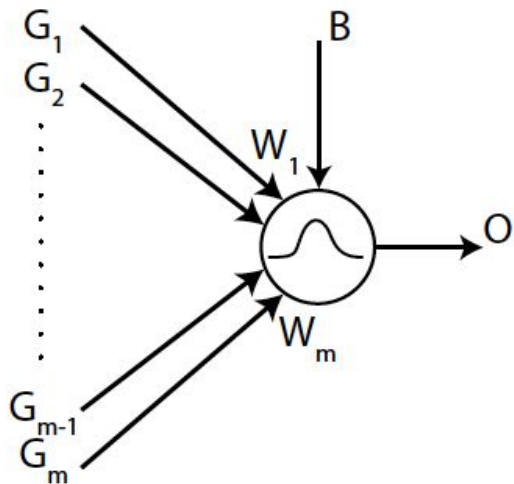
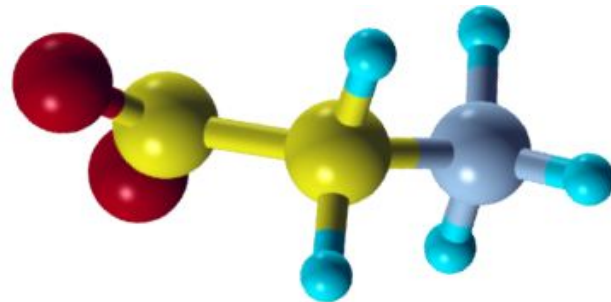
# A2A, FF network with GLY



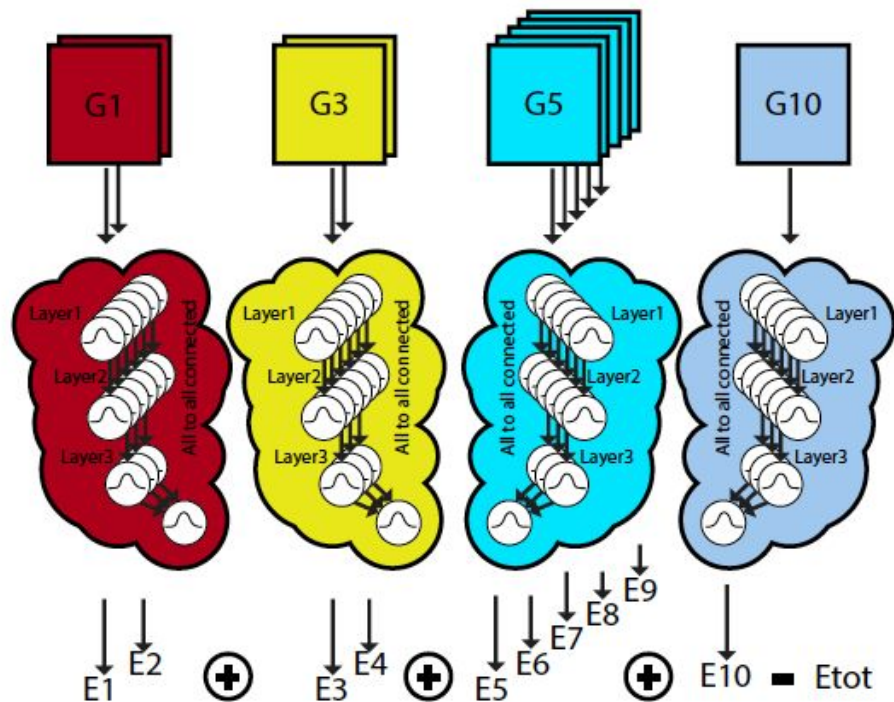
$$O = \exp \left[ - \left( \sum_m G_m W_m + B \right)^2 \right]$$



# A2A, FF network with GLY



$$O = \exp \left[ - \left( \sum_m G_m W_m + B \right)^2 \right]$$



# Representation

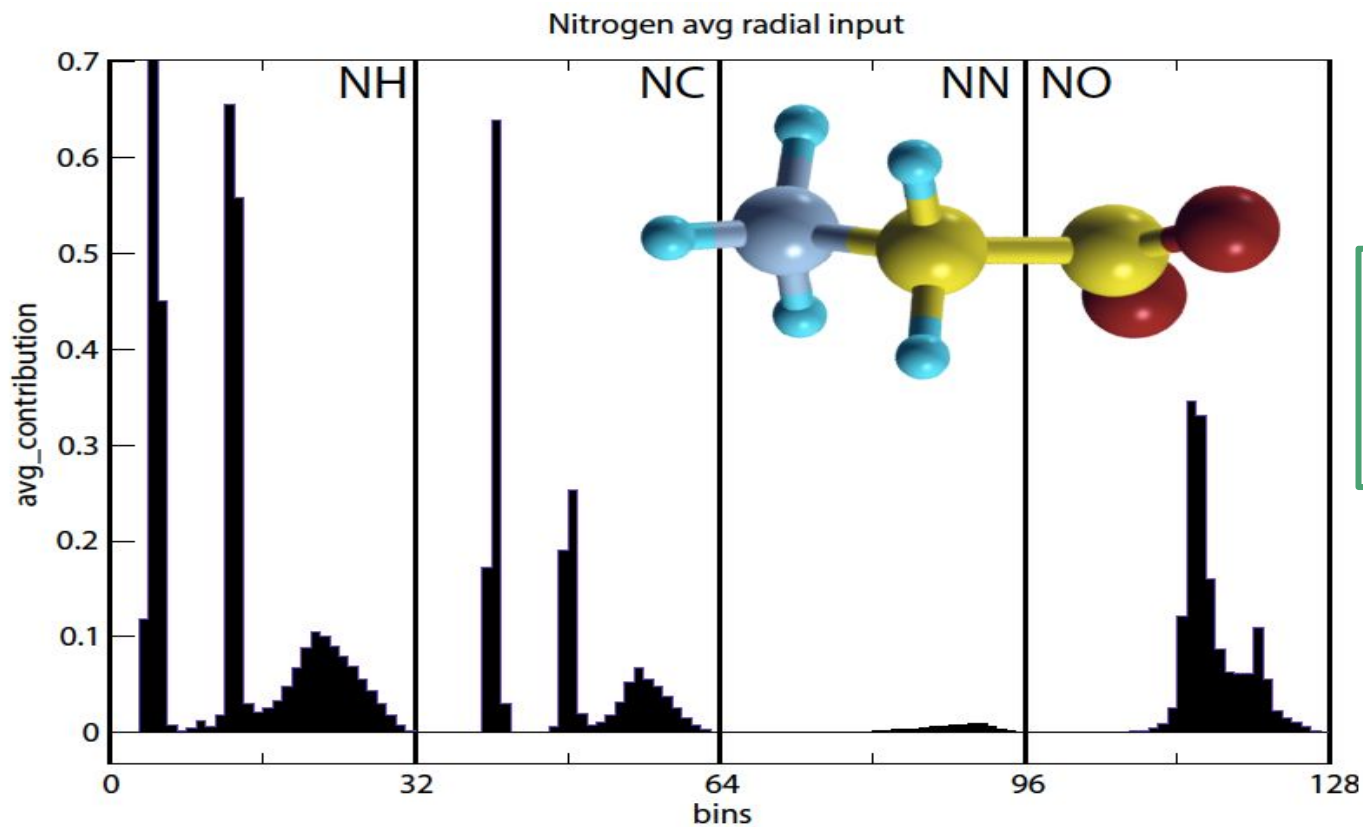
$$G_{m,s;i}^R = \sum_{\substack{\text{All atoms kind s} \\ i \neq j}} e^{-\eta(r_{ij}-R_m)^2} f_c(r_{ij})$$

$$f_c(r_{ij}) = \begin{cases} 0.5 \left[ \cos\left(\frac{\pi r_{ij}}{R_c}\right) + 1 \right] & \text{if } r_{ij} \leq R_c \\ 0 & \text{if } r_{ij} \geq R_c \end{cases}$$

$R_0=0.5\text{\AA}$  ,  $R_c= 4.6\text{\AA}$   
32 bins per pair:  
32x4=128 parameters



# Average G-radial for N in GLY



$R_0=0.5\text{\AA}$  ,  $R_c= 4.6\text{\AA}$   
32 bins per pair:  
 $32 \times 4 = 128$  parameters

# Representation

$$G_{n,m,s;i}^A = 2^{1-\xi} \sum_{\substack{\text{All atom of kind s} \\ j,k \neq i}} (1 + \lambda \cos(\Theta_{ijk} - \Theta_n))^\xi e^{-\eta \left( \frac{r_{ij} + r_{ik}}{2} - R_m \right)^2} f_c(r_{ij}) f_c(r_{ik})$$

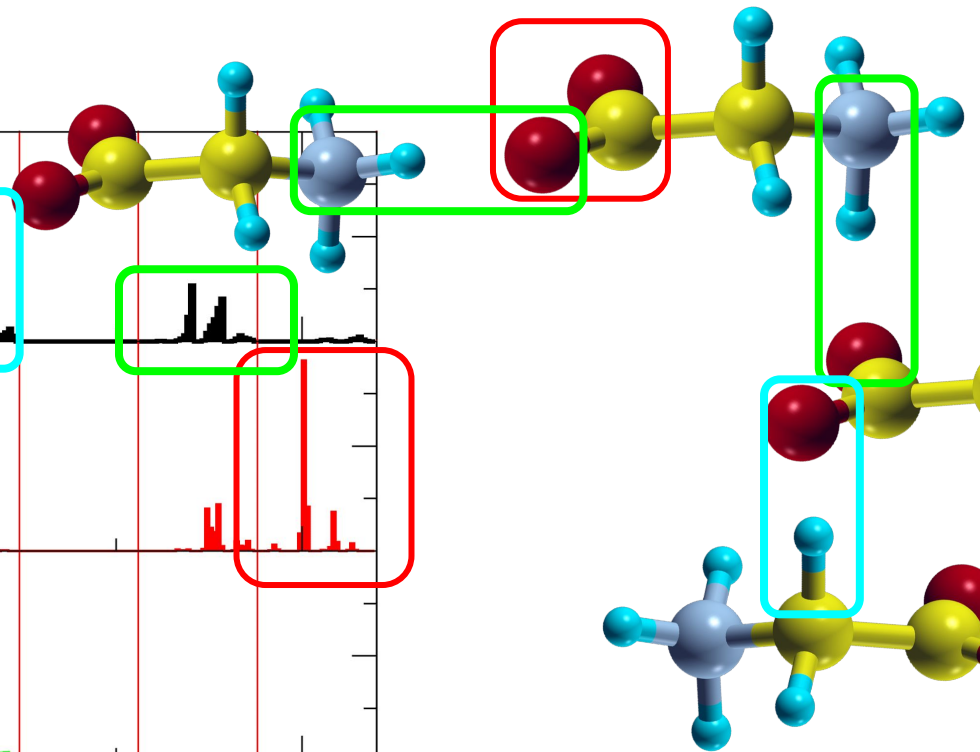
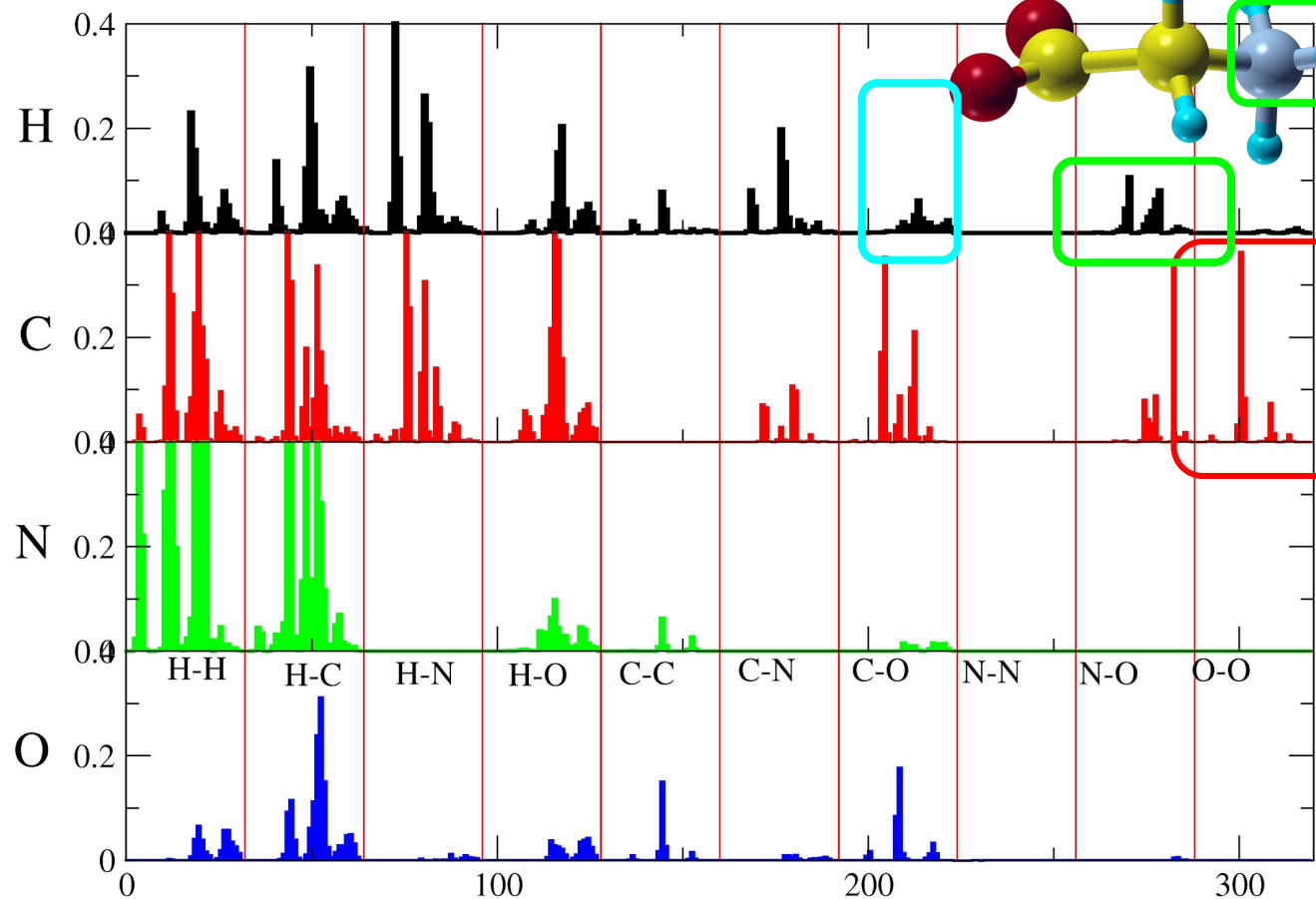
$R_0 = 0.5 \text{ \AA}$  ,  $R_c = 3.1 \text{ \AA}$

8 angular bin for each 8 radial bin

64 bins per trimer:

$64 \times 10 = 640$  parameters

# Average G-angular in GLY

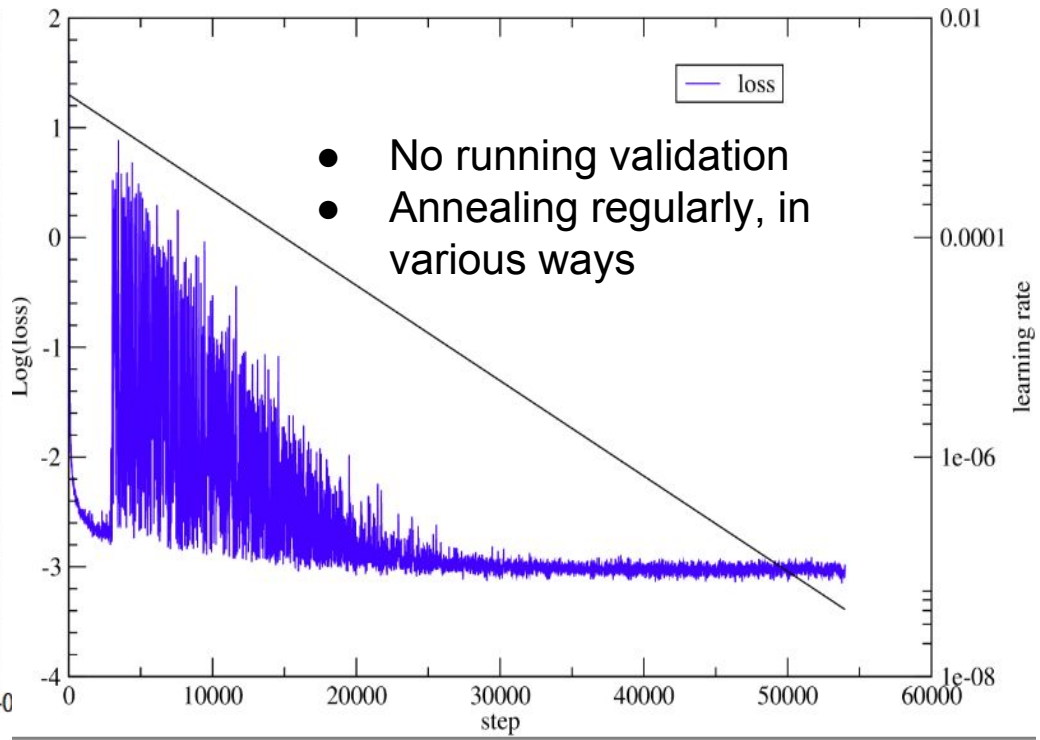
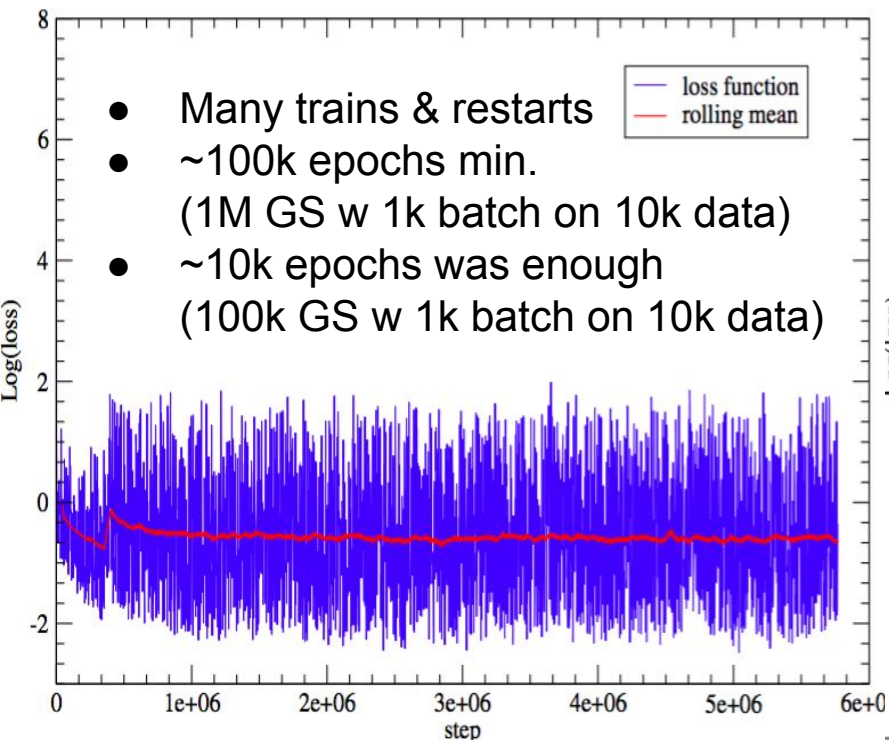


Halved version:  
8 angular for 4 radial  
32 per trimer  
 $32 \times 10 = 320$  parameters

# Remarks about this representation & CSP

- Representation may have *redundant* parts
- Representation may have *irrelevant* parts
- No need for atom centered representations
  - Bond centered, *molecule* centered representations
- No need for equally spaced representations
  - Allow custom gaussian centers
  - Allow gaussian centers to be learned
- $128+640 = 768$ -parameters per G-vector
- G-vector  $\rightarrow$  First layer width  $\approx$  First layer of compression
  - $768 : 128 : 128 : 32 : 1$
  - $\sim 500k$  parameters for the whole network

# Training the A2A-FF, modified BP - GLY data



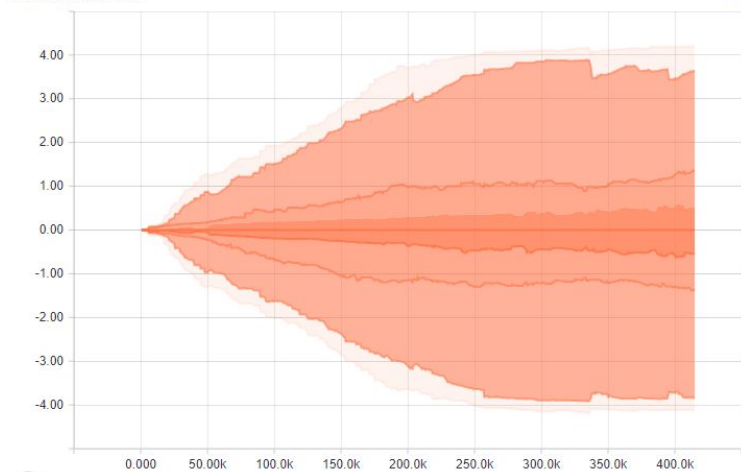
Stochastic gradient descent with Adam Opt.

$T \sim \text{learning rate} / \text{batch size}$

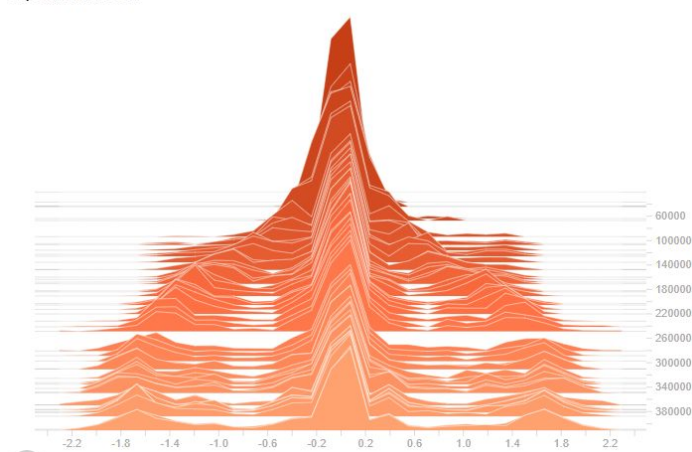
# What network looks like as it trains

## Layer 3 biases in a constant-LR run

Layer\_1/L2\_biases\_1



Layer\_2/L3\_biases\_1



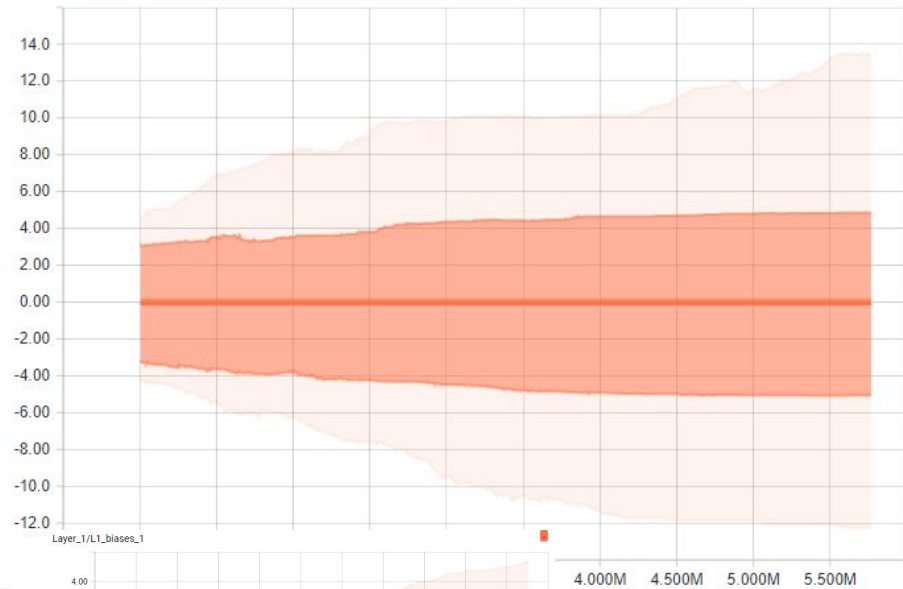
Tensorflow, Abadi et al. (2015)

# Training

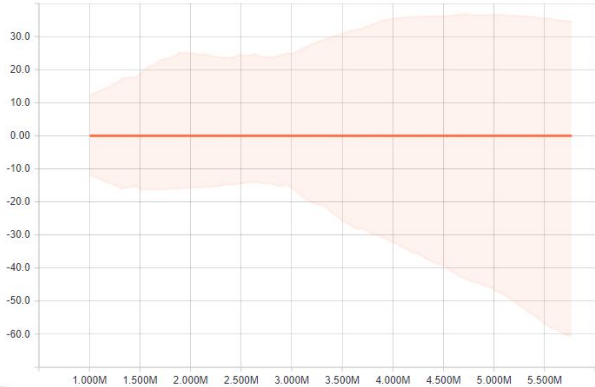
Layer\_1/L2\_W\_1



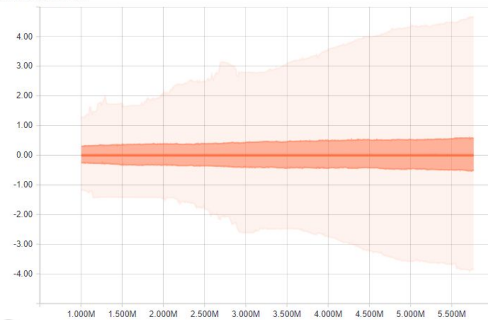
Layer\_1/L2\_biases\_1



Layer\_1/L3\_W\_1

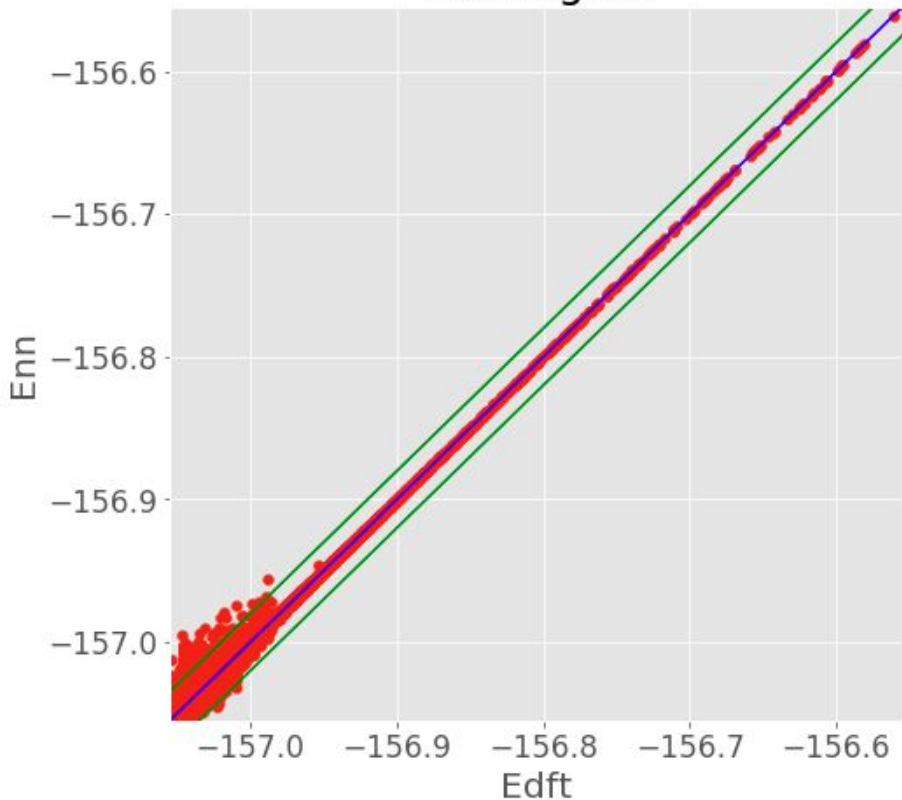


Layer\_1/L1\_biases\_1



# Training

training 20



Line:  $\pm 2$  mRy per atom.

$\sim 10\text{k}$  data,  $\sim 500\text{k}$  parameters

Over-fitting to the training data in an obvious way is not that easy.

RMSE on training set:

**0.33 mRy/atom** @step:130k

0.41 mRy/atom @step:5M

Target **training** accuracy

per molecule

1kcal/mol

(  $\sim 43\text{meV}$   $\sim 3.2\text{mRy}$  )

Roughly

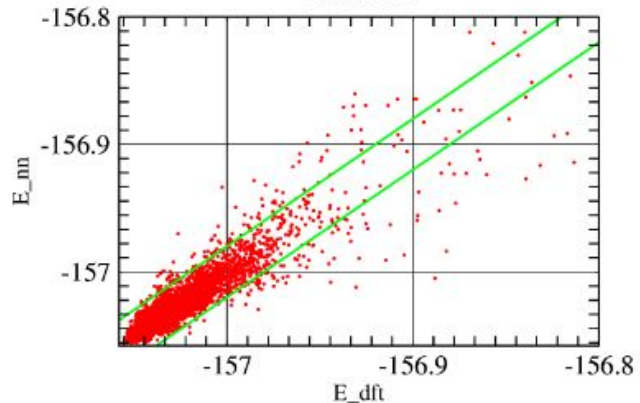
$\sim 4\text{meV}$  per atom

$\sim$  **0.3mRy** per atom

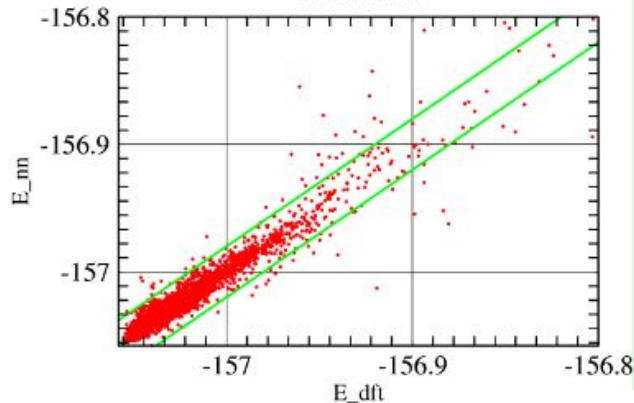


# Training on Z=2, Validating on all Z=1,2,3,4

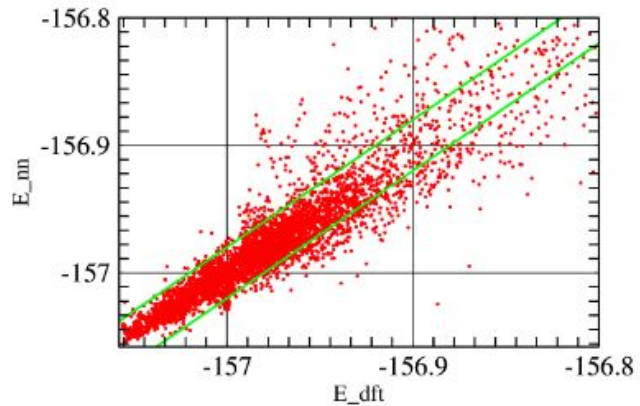
Validation 1



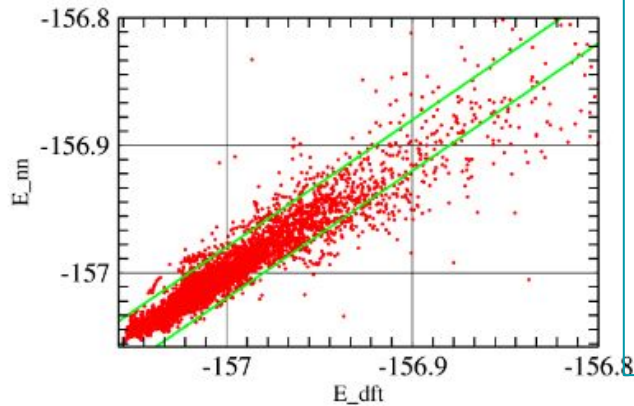
Validation 2



Validation 3



Validation 4



**RMSE @2M steps (@6M>L1)**

(in mRy/a)

**1.21** (1.51 > 1.61) **Z=1**

**1.04** (2.13 > 2.19) **Z=2**

**4.86** (6.14 > 5.84) **Z=3**

**4.06** (4.98 > 4.93) **Z=4**

Target ***extensibility***

accuracy per molecule

1.8kcal/mol

( $\sim 80$ meV  $\sim 5.8$ mRy)

$\sim 8$ meV per atom

$\sim$  **0.6mRy** per atom

# Forces

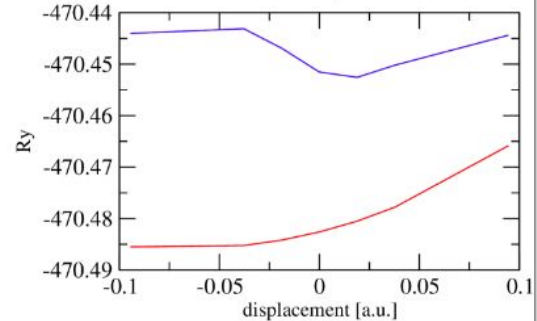
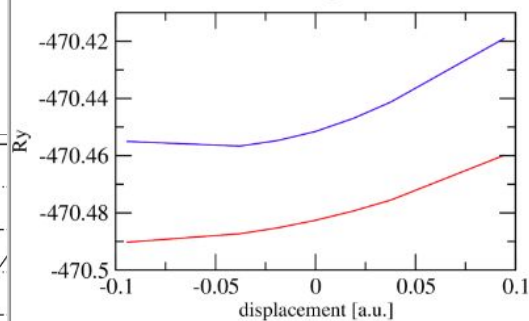
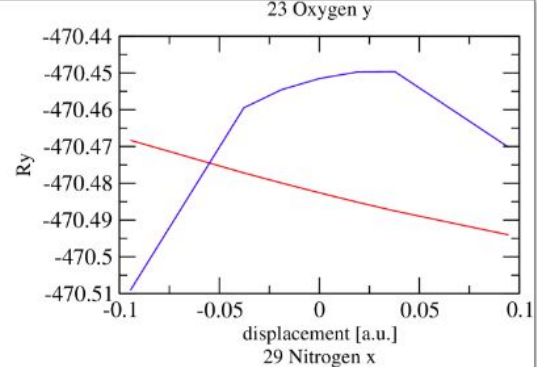
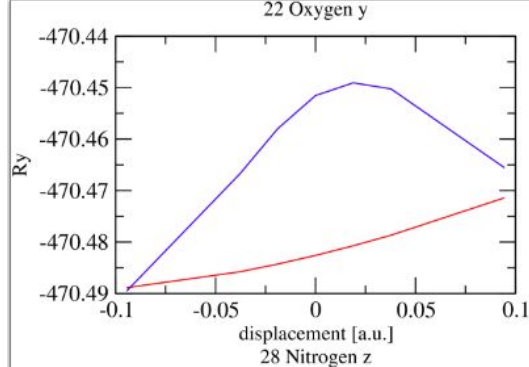
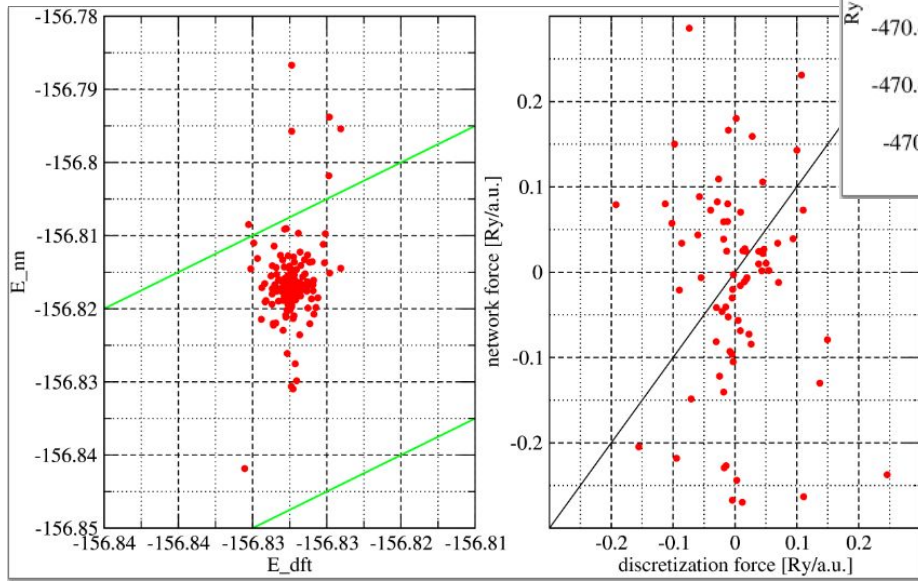
Fitting energies

Forces from finite differences

Terrible :-)

Unsampled atomic displacements

Hope - at least continuous?



Bad forces also for the training set.  
C or H slightly better but not always

Force RMSE  $\sim 0.100$  Ry/au  
(100 x acceptable threshold)

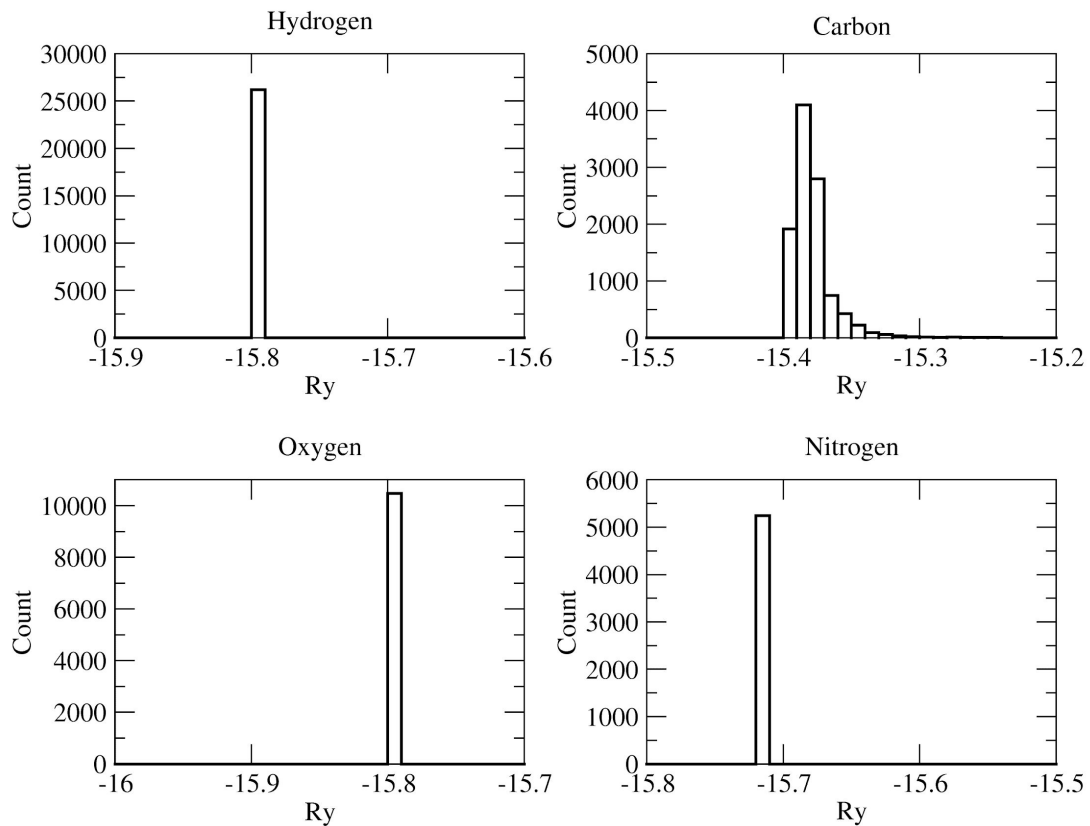
# What does the network learn?

All the variation between different crystal structures is expressed via Carbon atom alone.

*Why?* Limited dataset (one stoichiometry) + redundancy in G-vectors

*Does it matter?* Bad for information transferability between systems, but not significant for single system CSP.

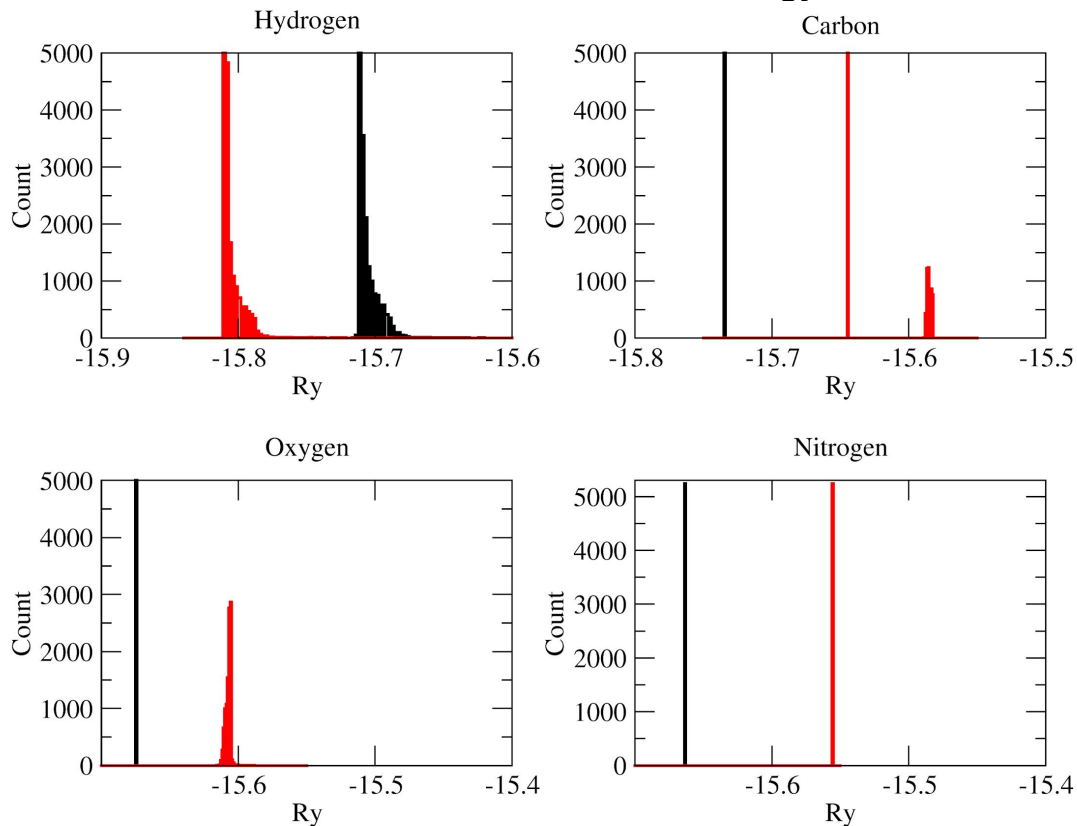
Atomic contribution to energy:



# What does the network learn?

- Restart changes the “informant” atom
- Copying weights may increase variability  
(Before: black, After: red)
- Two things affect this behavior:
  - Reducing  $R_c$
  - Shrinking network
    - Energies get better, (0.5mRy/atom)  
Forces are still 10x threshold

Atomic contribution to energy:



# What we learned from A2A networks + GLY:

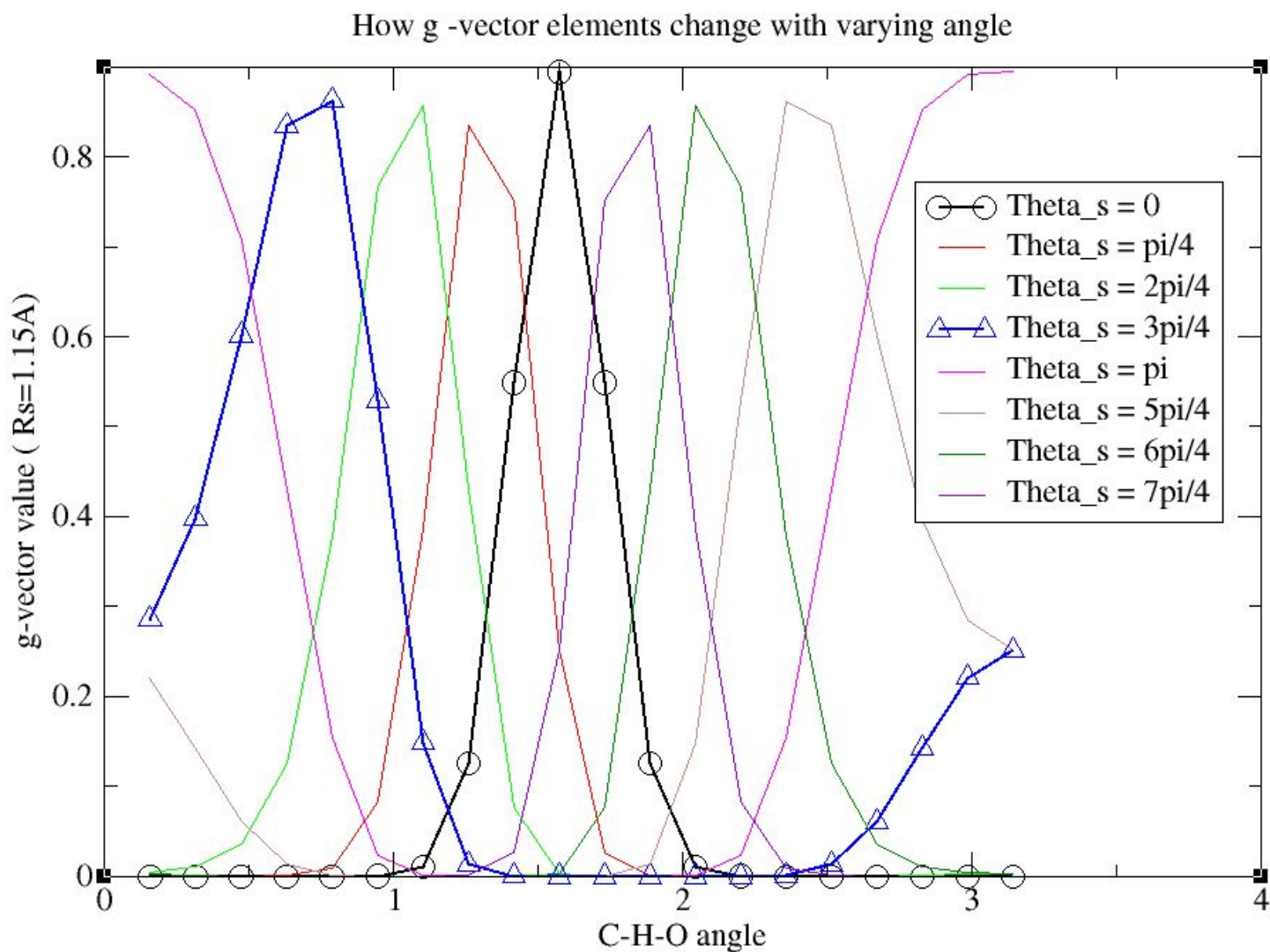
- 10k data, 500k parameters: underfitting & overfitting
- BP representation too redundant/expressive for CSP data
  - + Hyperparameter optimization may be necessary for each CSP
  - + Learning dynamics can be tricky
- Upon shrinking network (down to 10k parameters)
  - Good enough for mild acceleration (0.5mRy/atom),
  - But better forces needed for significant improvement

PANNA: TF based, cpu/gpu, allows different network per atom, freezing the dyn.

# G-vectors

Code-share  
reveals a bug:

The reason why  
so many G-bins  
so much data  
so long training  
was needed?



# Reducing error with increasing data?

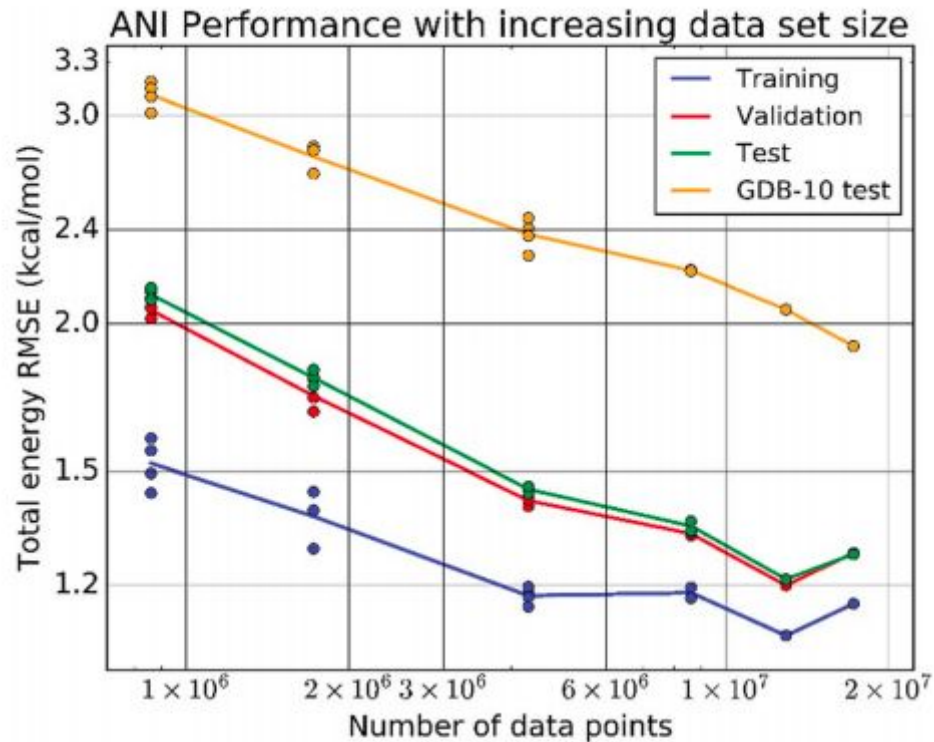


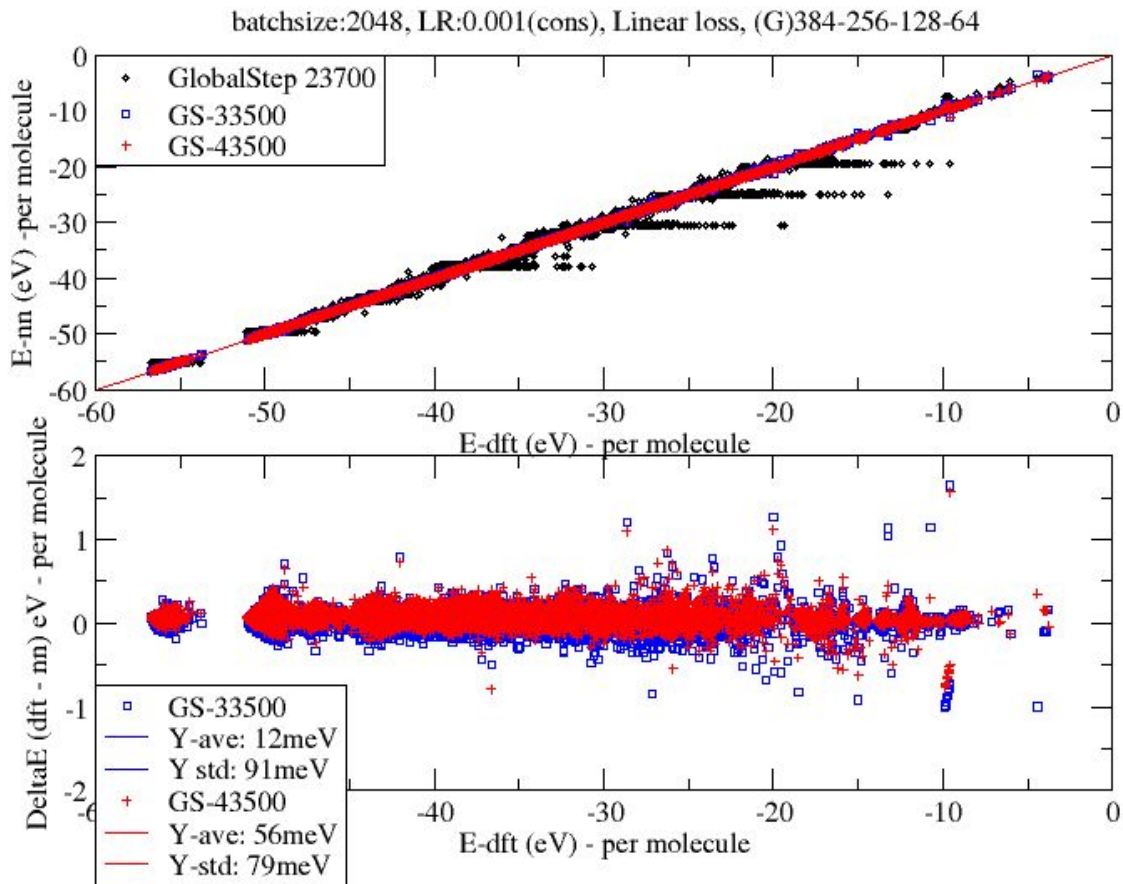
Fig. 3 Log-log plots of the training, validation, testing, and a random

# Training with molecular data

Stoichiometrically varying 100k  
(out of 800k ANI set)  
~500k parameters  
384:256:128:64:1

Target validation accuracy  
per molecule  
2kcal/mol  
( ~90meV )

OK energetics  
much less data.





# Training with molecular data

Stoichiometrically varying 10k data

(out of 800k ANI set)

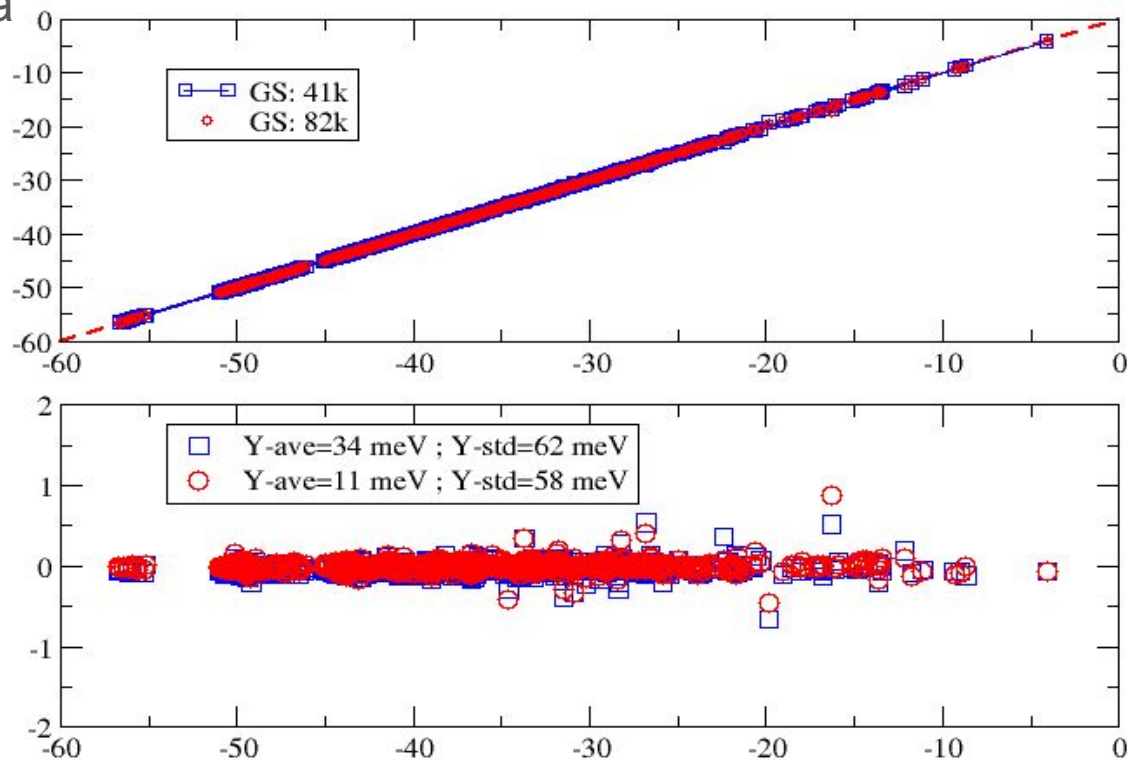
~500k parameters

384:256:128:64:1

Target validation accuracy  
per molecule  
2kcal/mol  
( ~90meV )

OK energetics  
much much less data.

Same network as before. Train on 8k, validate on 1k



# Another comparison

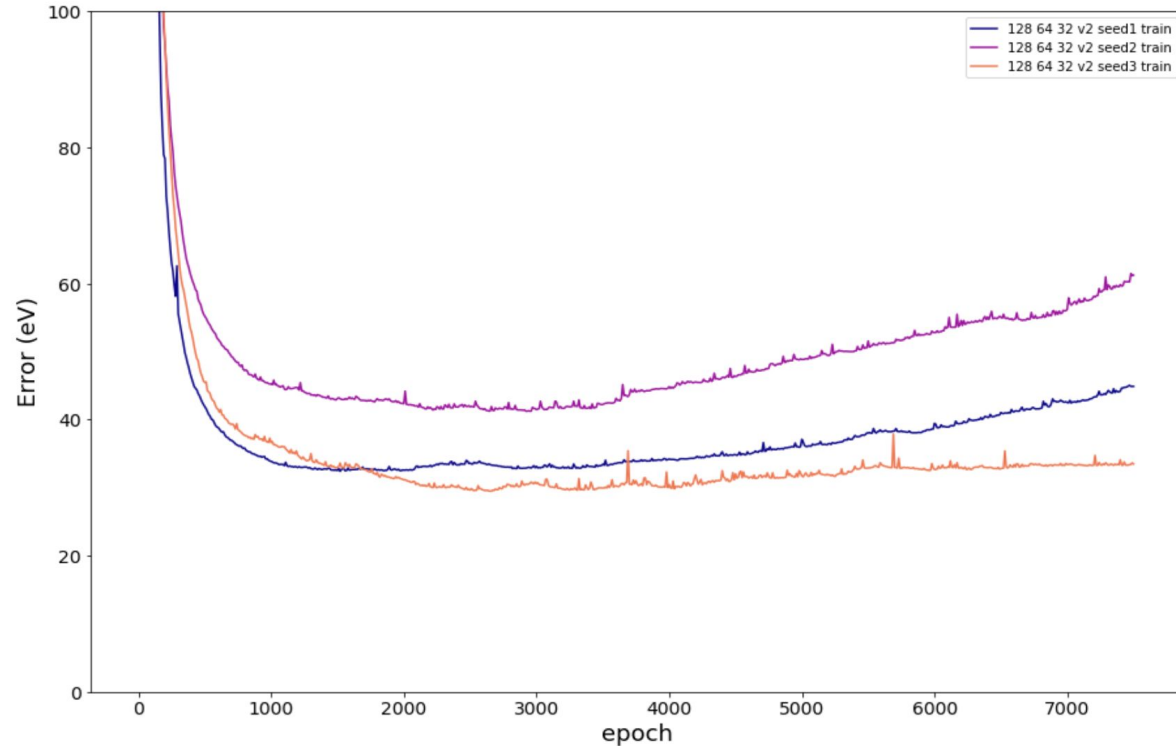
A C++ code, written from scratch

Tested on Silicon, Carbon

Original BP symmetry functions

No mini-batching

>whole data at each step (10k dataset)



# Another comparison

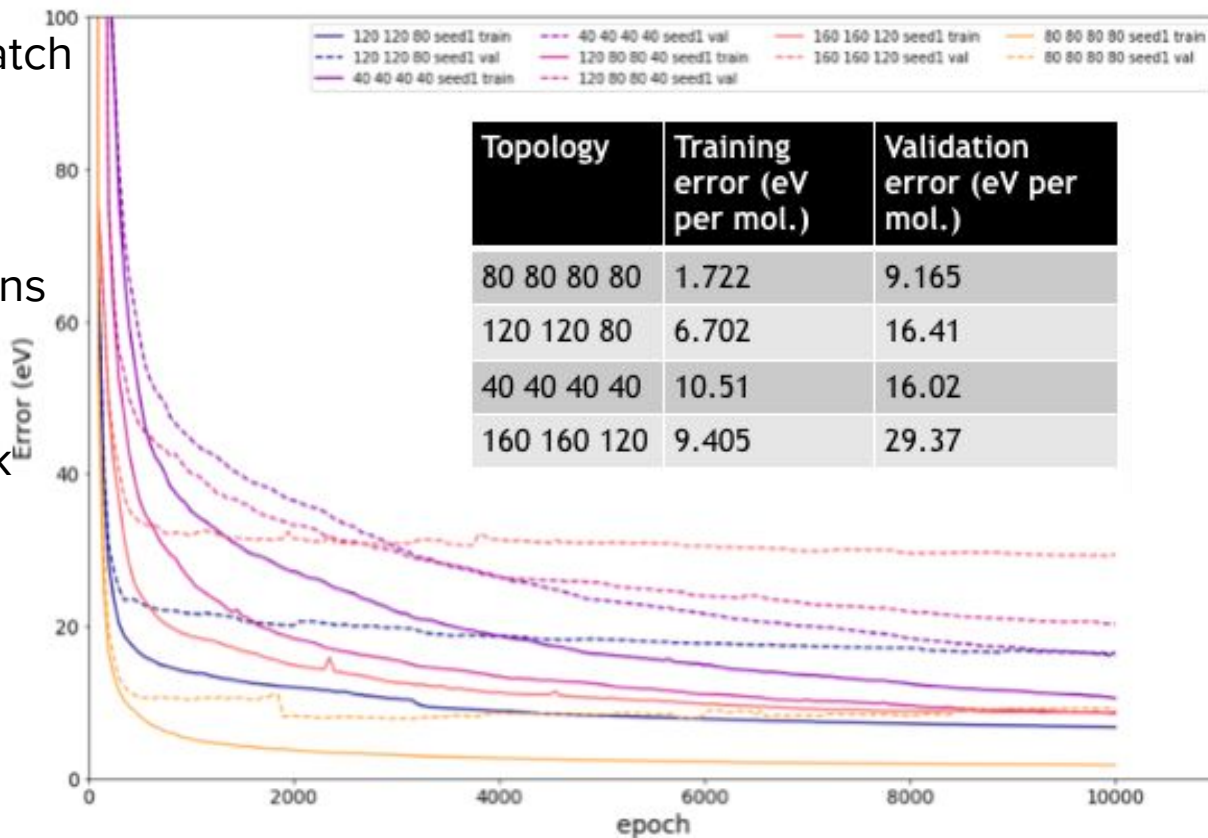
A C++ code, written from scratch

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# Another comparison

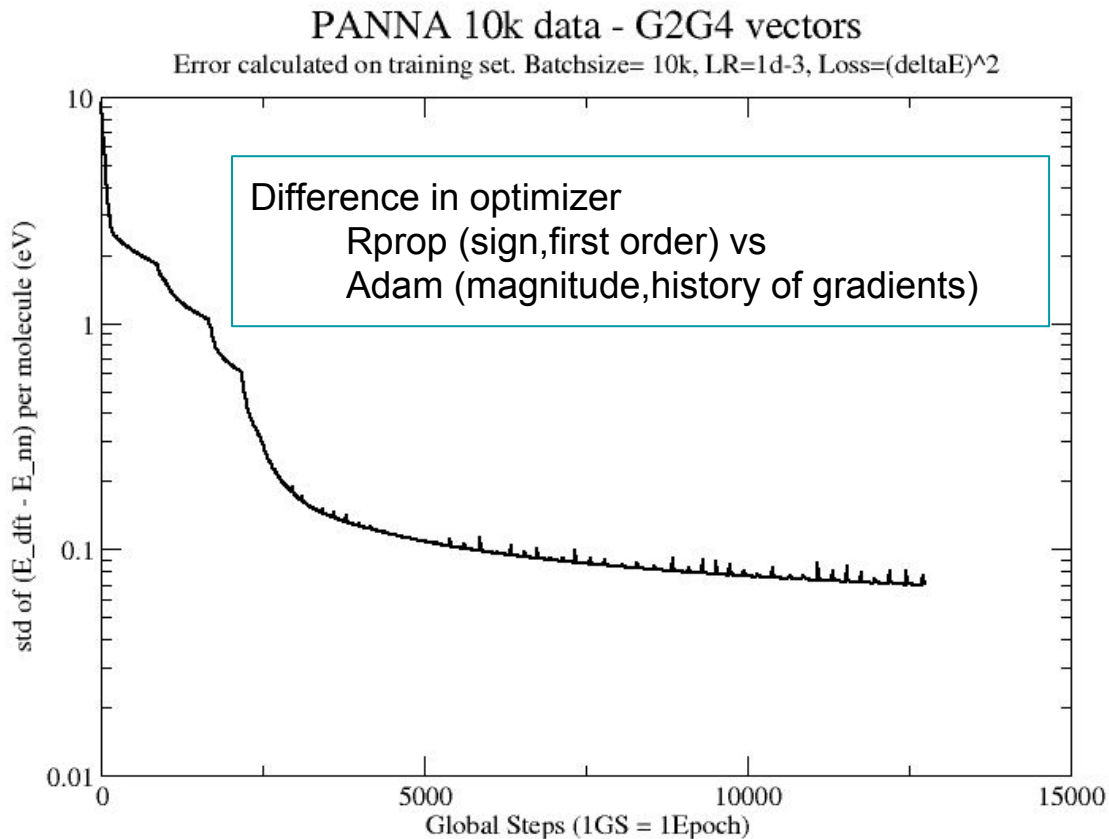
What is the culprit?

Modified PANNA

Original BP symmetry functions

Mini-batch size=whole data

Hits the target validation accuracy ( $\sim 90\text{meV}$ ) at 10k steps



# What else we learnt from A2A networks + ANI data

NN acceleration of CSP is data-wise feasible

Training dynamics is not straightforward - tools to monitor & manipulate dynamics

Reproducibility

# Fingerprinting

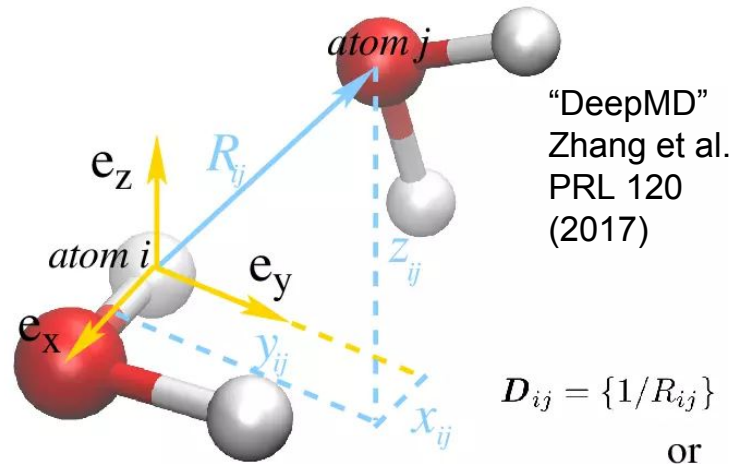
Positions? Distances, angles?

Radial distribution function?

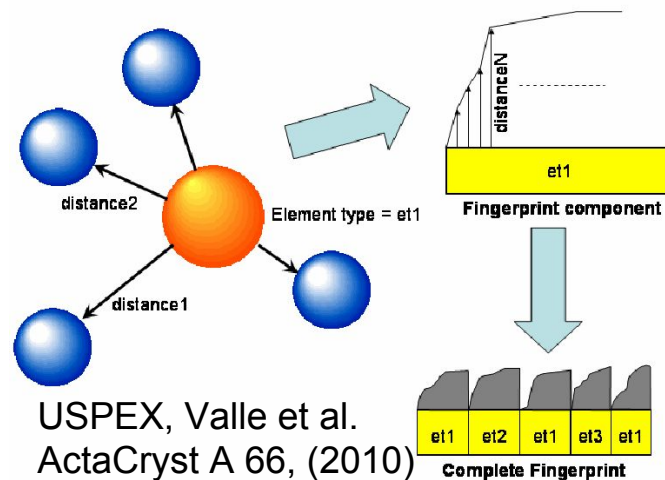
- Ab initio, sometimes-reversible representation of the crystal
- Redundant & not every bin is equally important.
- Doesn't have built-in hierarchical comparison flexibility
- Differentiable

$$\tilde{k}(\mathcal{X}, \mathcal{X}') = \int d\hat{R} \left| \int \sum_{\alpha} \rho_{\mathcal{X}}^{\alpha}(\mathbf{r}) \rho_{\mathcal{X}'}^{\alpha}(\hat{R}\mathbf{r}) d\mathbf{r} \right|^2$$

“SOAP” Bartok et al, PRB 87, 184115 (2013)



$$D_{ij} = \{1/R_{ij}, x_{ij}/R_{ij}^2, y_{ij}/R_{ij}^2, z_{ij}/R_{ij}^2\}$$





# Unsupervised methods & representation

- A good representation -> good distance measure -> successful clustering
- “much of all intelligence is unsupervised clustering”
- If we could do successful clustering all the time, we could solve all classification problems  
(is it a metal? Is it a magnetic material? Is it an 2d-exfoliate-able material?)  
but also make good approximations on other properties as by expanding the property on clusters



# Cluster Expansion for Li-Gr

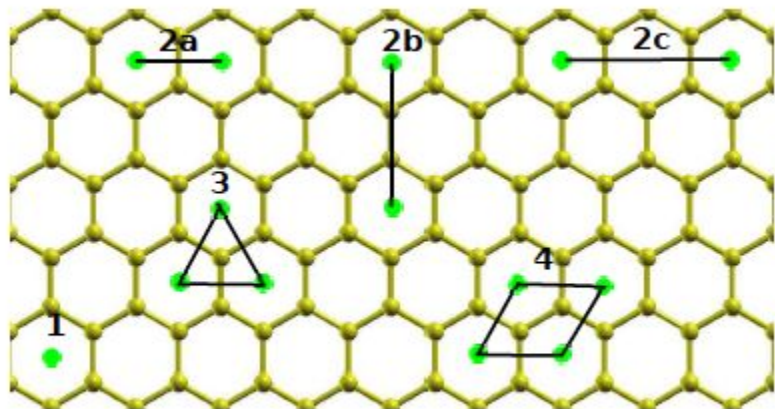


Figure 4: Figures in cluster expansion. (1) is the on-site figure (2) the 2-body figure of (2a) first neighbor (2b) second neighbor (2c) third neighbor (3) 3-body figure (4) 4-body figure

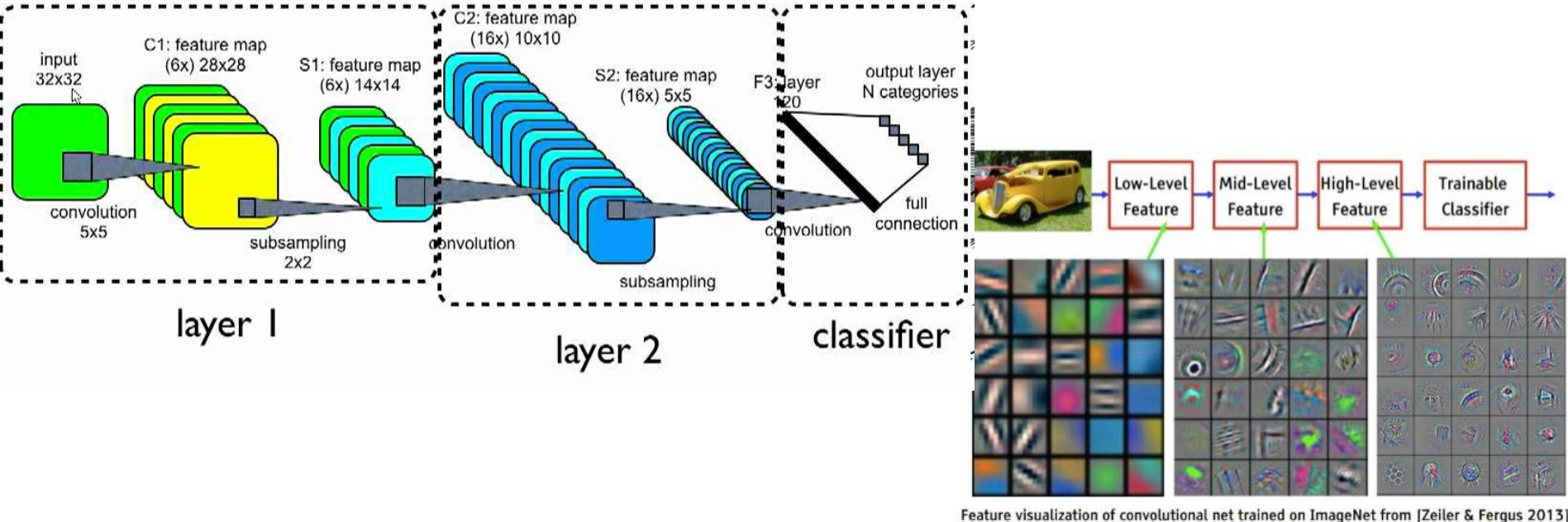
$$\begin{aligned}
 E_{CE}(C) = & J_1 \sum_{i \in f} c_i + \frac{1}{2} \sum_{p=a,b,c} \sum_{\{ij\} \in f} J_{2p} c_i c_j \\
 & + \frac{J_3}{6} \sum_{\{ijk\} \in f} c_i c_j c_k + \frac{J_4}{24} \sum_{\{ijkl\} \in f} c_i c_j c_k c_l \\
 & + \frac{J_d}{2} \sum_{ij, |\mathbf{r}_i - \mathbf{r}_j| > 2a_0} \frac{q_i q_j}{|\mathbf{r}_i - \mathbf{r}_j|^3} c_i c_j
 \end{aligned}$$

Shaidu et al, JPCC 122, 20800 (2018)

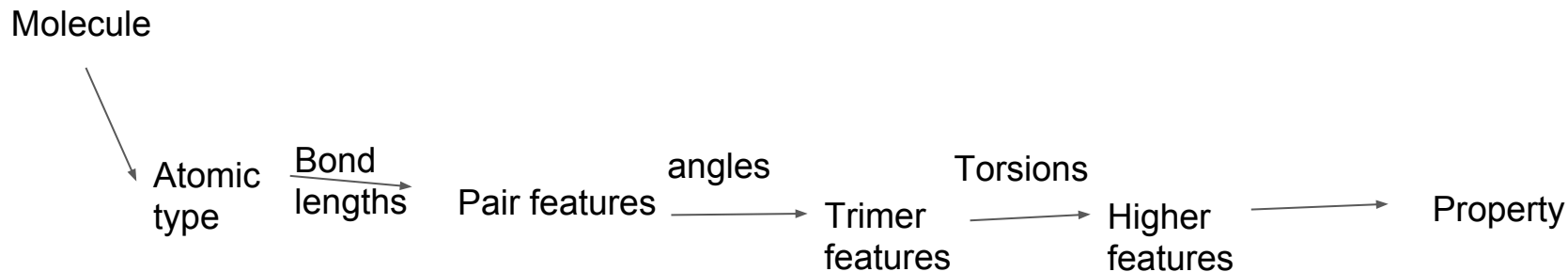
We expand the energy as a sum of “figures” (as in cluster expansion method), which is similar to dimer, trimer, and higher order filters of the convolution.

# Convolutional neural networks

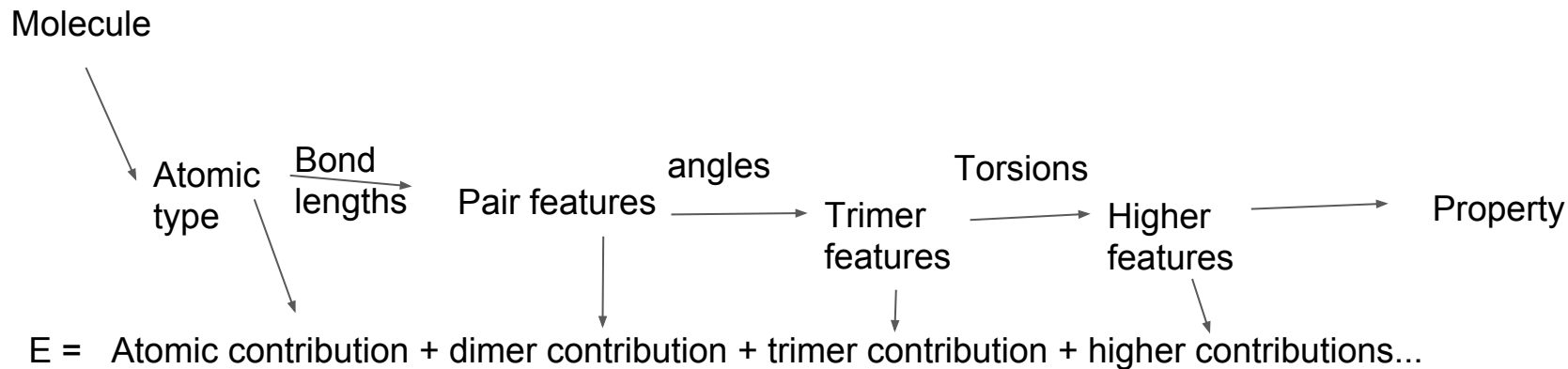
Between the black box learner & handcrafting (e.g. force field fitting)



# Convolutional neural networks for atomistic graphs



# Convolutional neural networks for atomistic graphs



The number of features detected at each layer increase exponentially:

- 1-  $N_{\text{spec}}$
- 2-  $N_{\text{spec}} * N_{\text{spec}} * N_{\text{bond}}$
- 3-  $N_{\text{spec}} * (N_{\text{spec}} * N_{\text{bond}}) * (N_{\text{spec}} * N_{\text{bond}}) * N_{\text{angle}}$

# CNN -- in practice

Input: Atomic positions

Activations:

$$t_{ia}^1 = \delta_{a, \text{type}_i}$$

i. e.  $t_i^1 = [1, 0, 0, 0] = \text{H}$   
+Positions

$$r_i^1 = [x_i, y_i, z_i]$$

Layer 1: Bonds  
(mask index: c)

$\tilde{r}_c$

Activations:

$$t_{ijc}^2 = R(w_c t_i^1 t_j^1 G(|r_{ij}| - \tilde{r}_c))$$

+Positions

$$r_i^2 = [x_i, y_i, z_i]$$

Layer 2: Valence-angle trimer  $\tilde{\theta}_d$   
(mask index: d)

Activations:

$$t_{ijkd}^3 = R(w_d t_{ijc}^2 t_{ikc'}^2 G(\theta_{jik} - \tilde{\theta}_d))$$

+Positions

$$r_i^3 = [x_i, y_i, z_i]$$

R() is for ReLU, G() is gaussian.

All distances, angles, features can be equally weighted activators ( $w_c$  &  $w_d=1$ )  
or separately weighted for a more expressive network (& more nonlinearity)

Batch size: 100; lr=1d-2; mask-eq-weighted network; validation

# MoICNN

Same 10k data

6 nearest neighbors

Nbond =4; Nangle =4

1-10k:only bond

10-20k:only angle

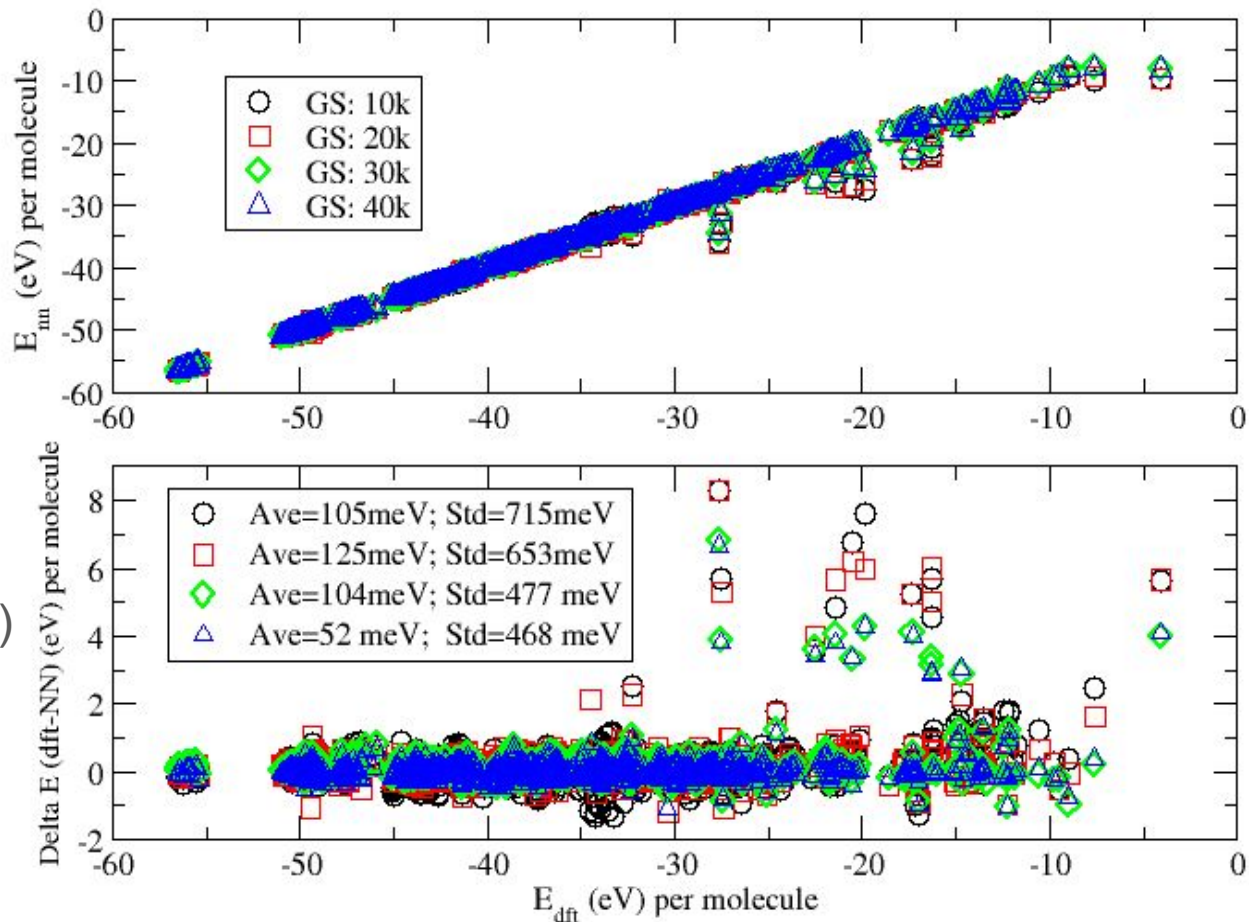
20-30k:both

30-40k:both but slower (1d-3)

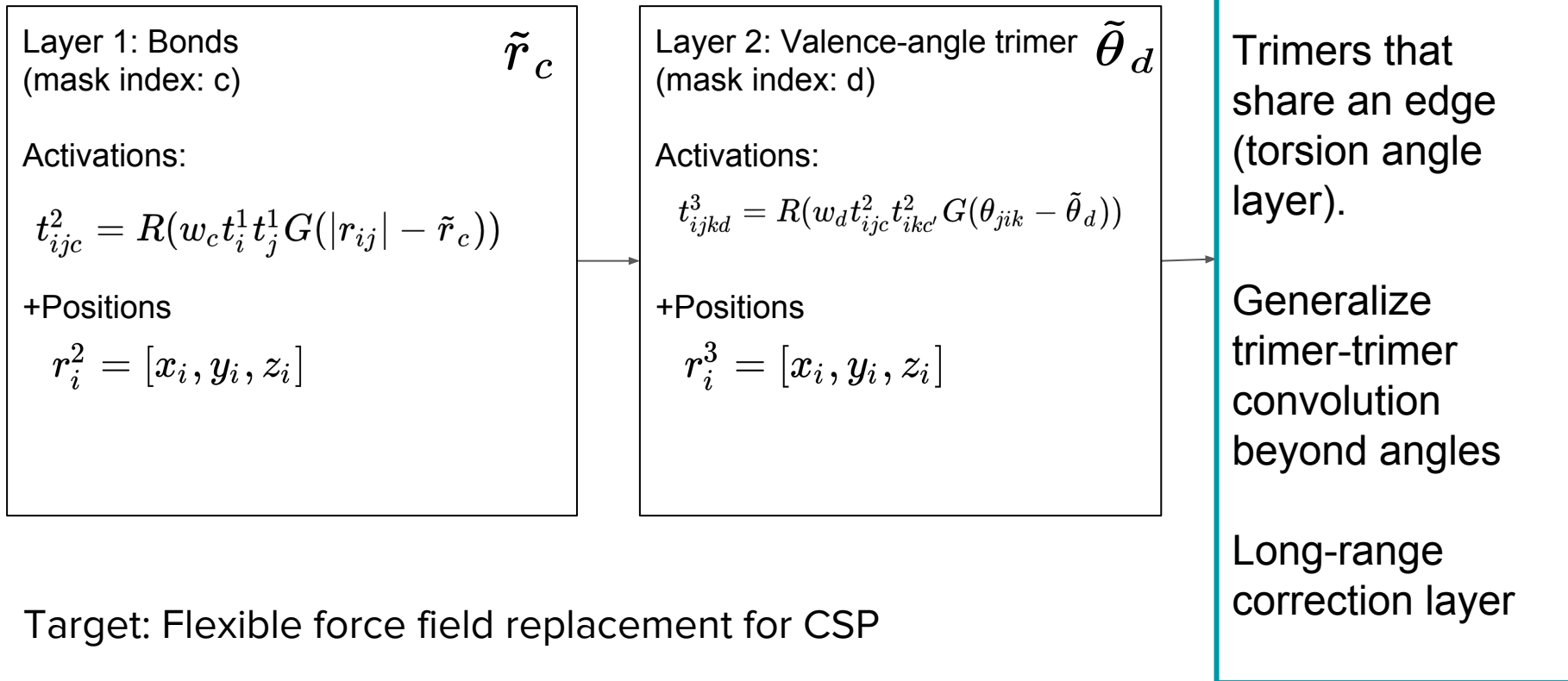
Params

Pbond  $\sim 4*4*4 = 64$

Pangle  $\sim 4*4*4*4 = 256$



# MolCNN -- Where to go?



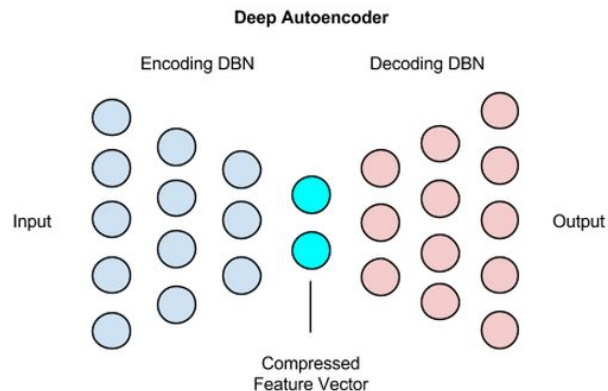
# Autoencoders:

Unsupervised data compression:

Coupled encoder+decoder able to reconstruct input passing from a much smaller representation.

May help finding compact representation, to be then used for learning, clustering or even de-noising.

Already a successful use case published - but with a Graph based representation (SMILES)  
(Gomez-Bombarelli et al ASC Cent. Sci. 2018)





# Reinforcement learning:

Machine learning to decide on “actions” to maximize “reward”.

Main idea: sample environment to estimate “value” of a configuration, based on future expected reward, then pick the best (Q-learning).

Value function can be learnt with a NN.

Proved effective in Atari, Go, Chess...

Useful to learn “intrinsic value” of a configuration  
to guide a complex optimization procedure

Google DeepMind’s AlphaFold used to win the protein folding challenge.

# Take home:

- Crystal structure prediction is a tough problem, NN can help
- Lowest hanging fruit for NN in CSP is exploration and geometry optimization
- GLYdata +A2A network :
  - Variability in data vs the representation
  - Overfitting vs Underfitting & the need to monitoring/control the dynamics
- ANIdata +A2A network:
  - Data variability > amount
  - Open source/Reproducibility
- ANIdata +MolCNN
  - WIP - Few parameters, easy to train, potentially interpretable
- Where next?

## Credits:

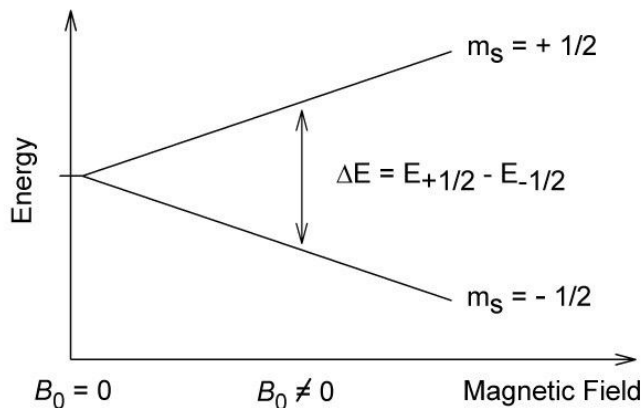
(SISSA) R Lot, F Pellegrini, Y Shaidu, S de Gironcoli

(MIT) S Wyant, A Kolpak

The End.

Notes

# ab initio NMR



$$\Delta E = \hbar \gamma \underline{B_{\text{tot}}} \Delta m_s = \hbar \omega$$

Induced field due to electron shielding

$$\underline{B_{\text{in}}^{(1)}}(\mathbf{r}) = -\underline{\vec{\sigma}}(\mathbf{r}) \mathbf{B}_{\text{ext}}$$

Density Functional Perturbation Theory in Linear Response:

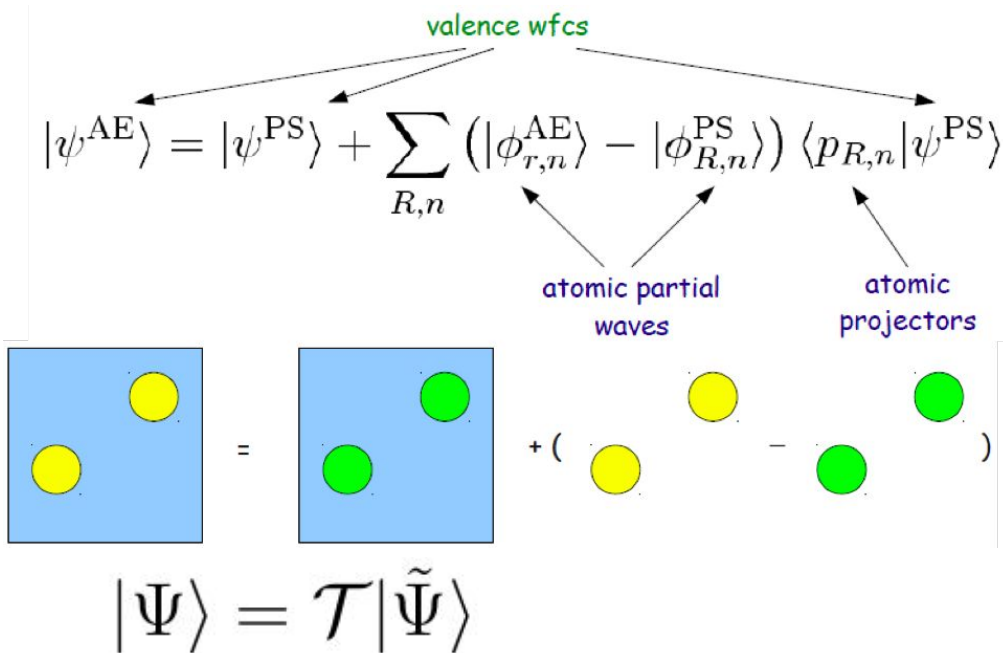
- Perturbation to Hamiltonian
- Perturbed Eigenstates
- Induced stationary current
- Induced field from induced current

Chemical shift

$$\delta = -(\sigma - \sigma_{\text{ref}})$$

Time  $\propto$  (Nat x (Nk x (Ni x (Nbx**NlogN**))))

# *ab initio* NMR with Projector Augmented Wave



$$\mathbf{A}(\mathbf{r}) = 1/2\mathbf{B} \times \mathbf{r}$$

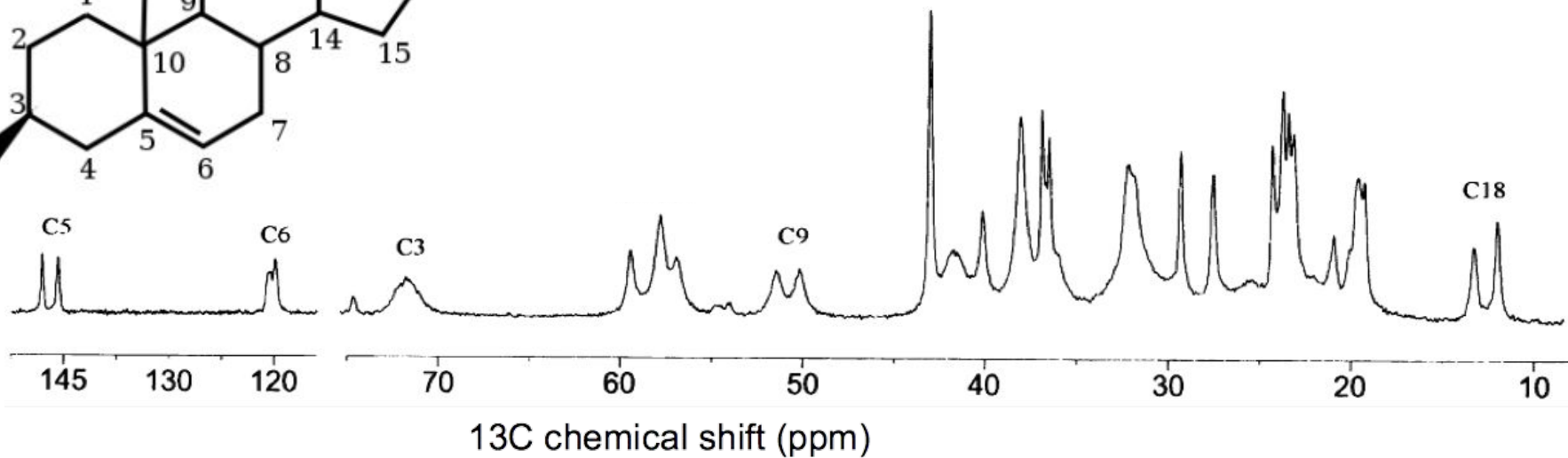
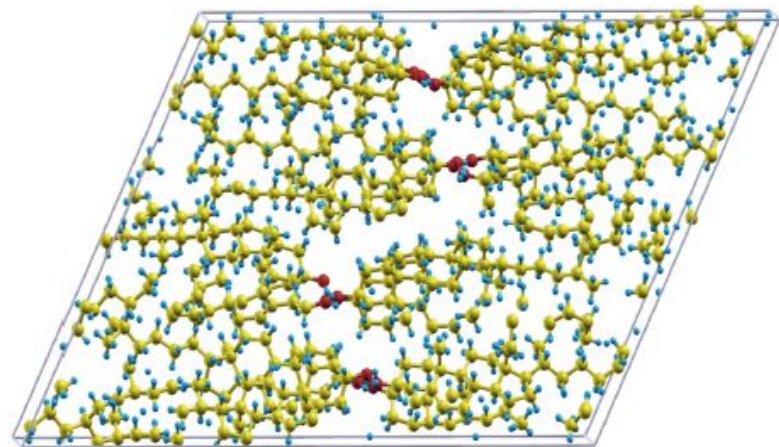
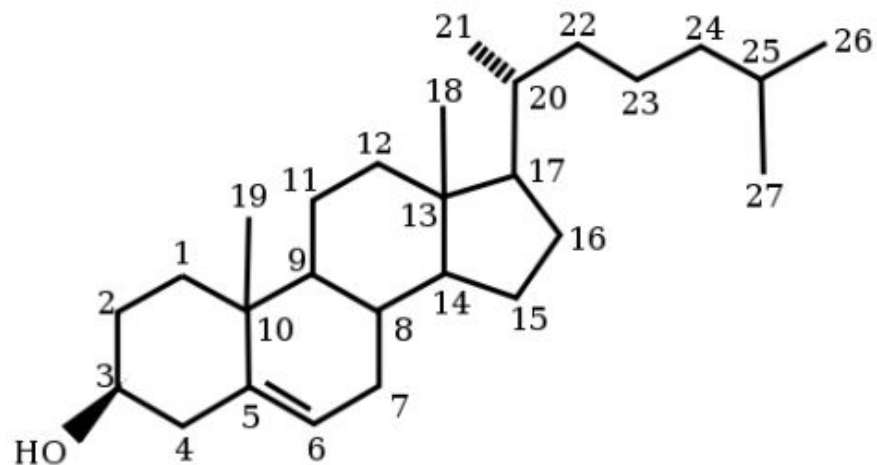
$$\Psi'(\mathbf{r}) = e^{(i/2c)\mathbf{r}\cdot\mathbf{t}\times\mathbf{B}}\Psi(\mathbf{r})$$

$$T_{\mathbf{B}} = 1 + \sum_{\mathbf{R},n} e^{(i/2c)\mathbf{r}\cdot\mathbf{R}\times\mathbf{B}}$$

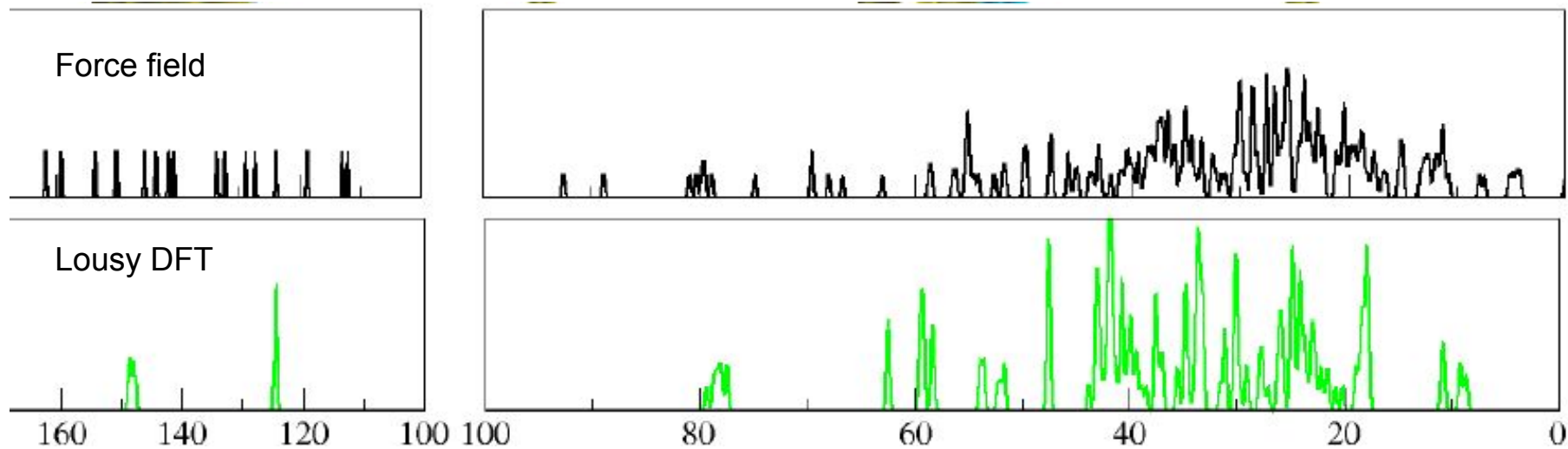
$$(|\phi_{\mathbf{R},n}^{\text{AE}}\rangle - |\phi_{\mathbf{R},n}^{\text{PS}}\rangle)$$

$$\langle p_{\mathbf{R},n}^{\text{PS}} | e^{-(i/2c)\mathbf{r}\cdot\mathbf{R}\times\mathbf{B}}$$

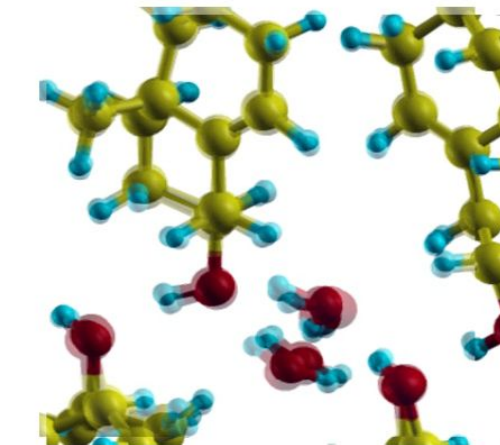
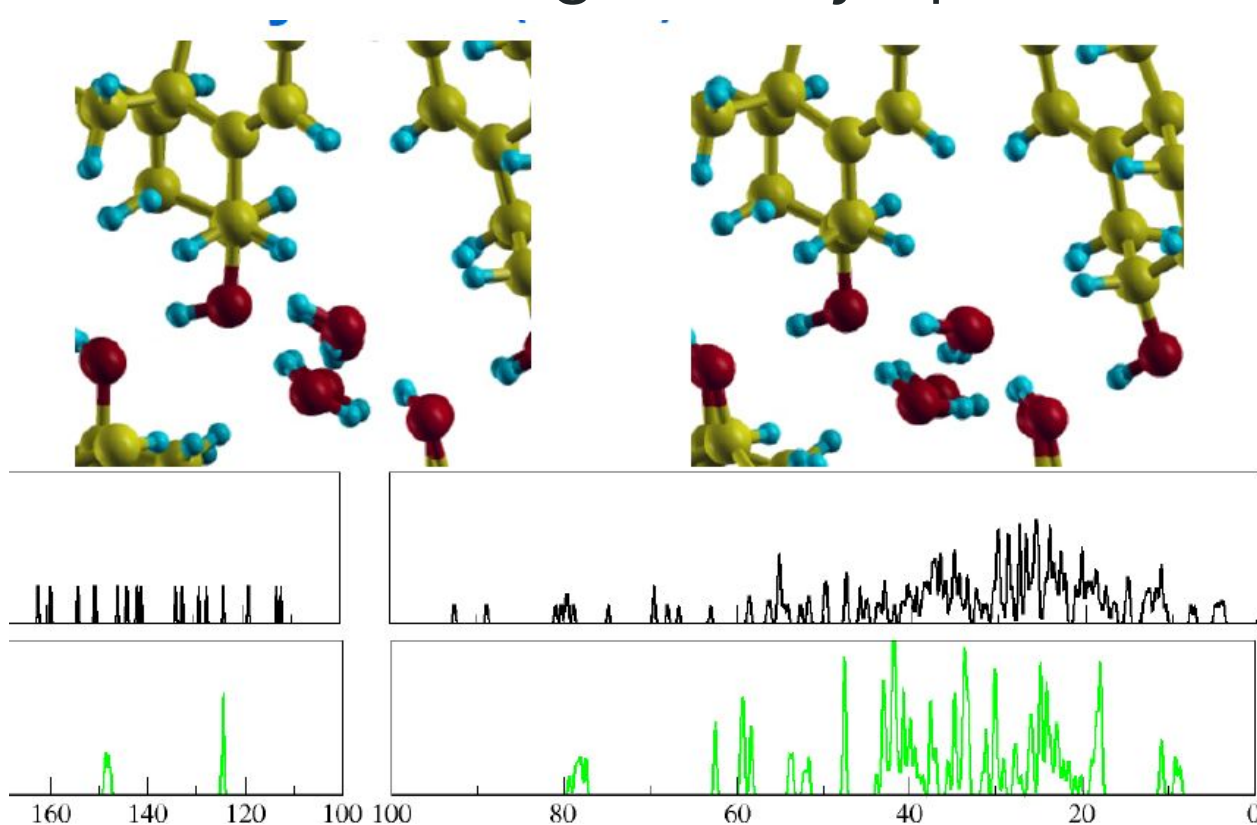
# $^{13}\text{C}$ NMR for CLR



# Geometry optimization



# Force fields for geometry optimization



Force field was not good enough to distinguish the spectra of different polymorphs, or even to verify the same polymorph



# CNN -- in practice

Layer 1: Bonds  
(mask index: c)  $\tilde{r}_c$

Activations:

$$t_{ijc}^2 = R(w_c t_i^1 t_j^1 G(|r_{ij}| - \tilde{r}_c))$$

+Positions

$$r_i^2 = [x_i, y_i, z_i]$$

+Bond directions

$$v_{ij}^2 = [x_j - x_i, y_j - y_i, z_j - z_i]$$

Trimers are convolutions of two 2-atom nodes that share a corner.

An alternative here, would be to add a non-corner-sharing dimer-dimer mask. Ongoing.

Layer 2: Valence-angle trimer  $\tilde{\theta}_d$   
(mask index: d)

Activations:

$$t_{ijkd}^3 = R(w_d t_{ijc}^2 t_{ikc'}^2 G(\theta_{jik} - \tilde{\theta}_d))$$

+Positions

$$r_i^3 = [x_i, y_i, z_i]$$

+Trimer Orientations

$$q_{ijk}^3 = [q_{ijk}^y, q_{ijk}^x, q_{ijk}^z, q_{ijk}^w]$$

Number of possible feature masks  
(and weights to get their contribution to energy) increase dramatically:

- 1- Nspec
- 2- Nspec<sup>2</sup> \* Nbond
- 3- Nspec<sup>3</sup> \* Nangle

We can keep only some trimers that get activated the most (max-pooling ~ coarse graining)

$$t_{id}^{3'} = \max_{jk} t_{ijkd}^3$$

# Fingerprint references

- Ref1: K. Hansen, F. Biegler, O. A. von Lilienfeld, K.-R. Müller, A. Tkatchenko, Nat. Commun. <https://pubs.acs.org/doi/pdfplus/10.1021/acs.jpcclett.5b00831> Ref2: <https://pubs.acs.org/doi/pdfplus/10.1021/ci020345w> Ref3: <https://arxiv.org/pdf/1509.09292.pdf> ref4: <https://journals.aps.org/prl/pdf/10.1103/PhysRevLett.107.085504> ref5: <https://aip.scitation.org/doi/10.1063/1.4818005>

# Brainstorming for NNs in CSP

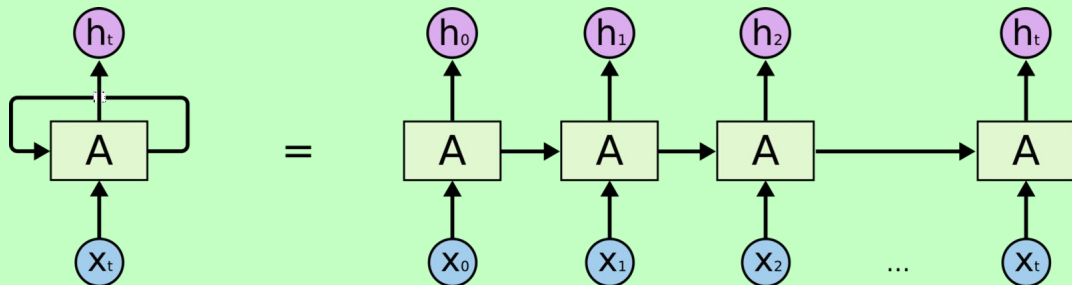
- Neural networks and where CSP can help:
  - Accelerating the exploration of polymorphic phase space -
    - the supposedly low hanging fruit is the geometry relaxation. This means having a very flexible and well performing force field/potential for the molecule at hand. Next step would of course be a force field transferable across different molecules.
    - Relaxation as a winning move: As number of atoms, species, network parameters increase, calculating forces as derivatives might become the bottleneck. Reinforcement learning methods, where the value of each atomistic move is learnt, can be viable alternatives to making quick optimization steps.
    - Evolutionary algorithm accelerated: A jump here would be accelerating the phase space search by exploring more intelligently than evolutionary algorithm using reinforcement learning where the reward this time can be finding a low energy or increasing population diversity or matching the experimental characterization.
  - Accelerating characterization:
    - Characterization could simply be total energy, but also possibly enthalpy, NMR, EPR, elastic constants etc. Challenges: Efficiency & accuracy (representation, ground truth

# Recurrent networks:

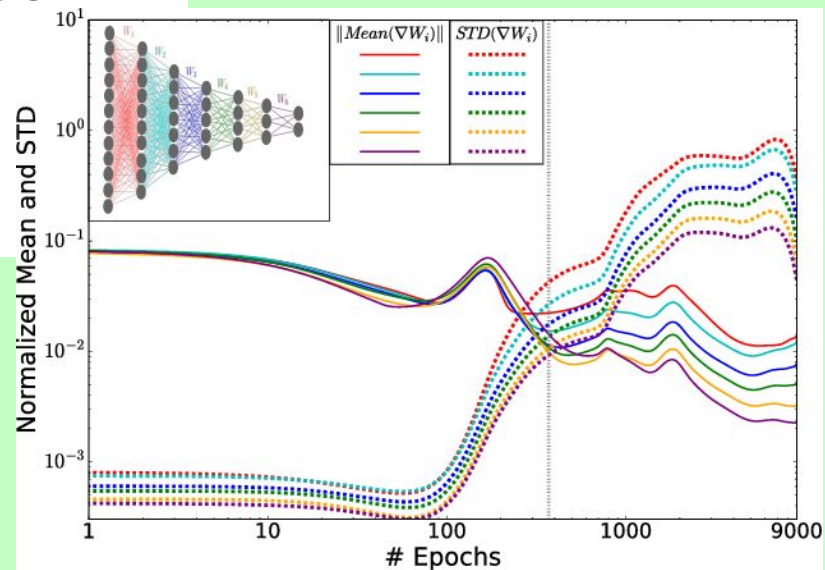
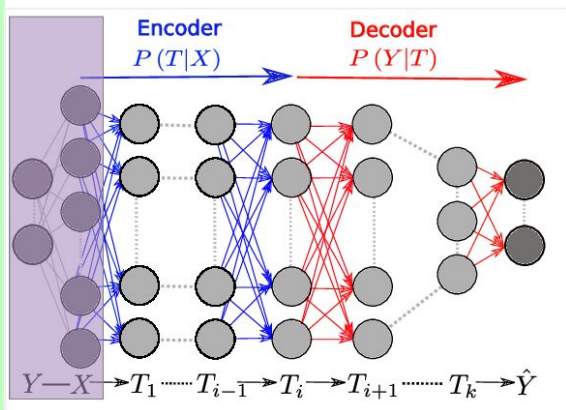
Neural networks useful to analyze “time series”.

Main idea: add to the input at time  $t$  some information from the output at time  $t-1$  → acts like a “memory” for the network.

Typically one takes  $N$  timesteps and optimizes all output with SGD or similar. This can lead to “gradient dilution” for long time correlation → introduce “gated units” to preserve memories for longer times (see GRU, LSTM).

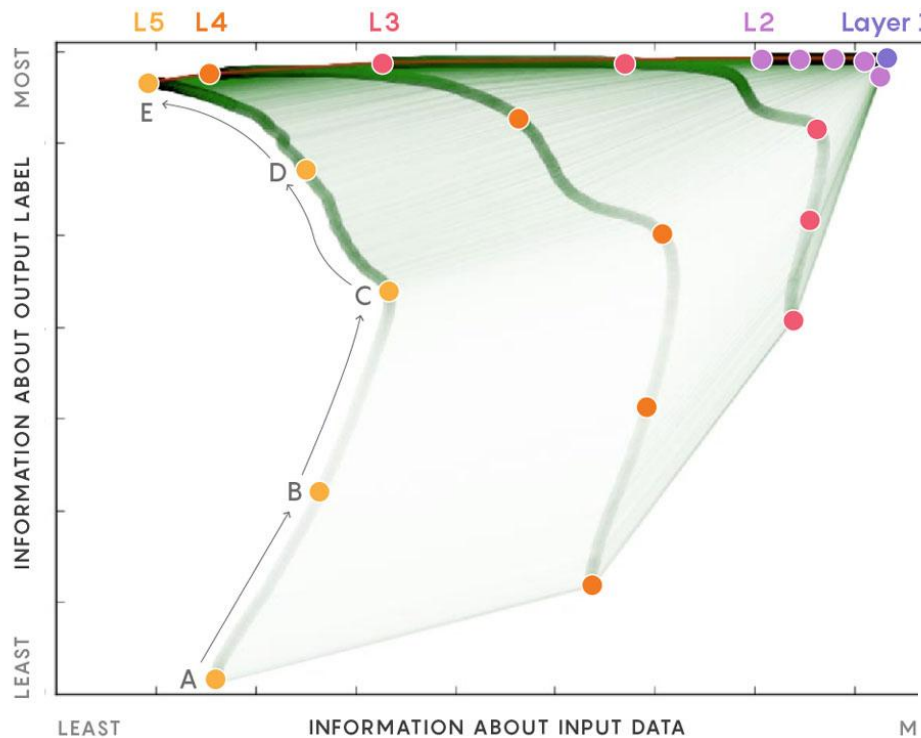


- At each layer information is processed to remove the inessential (data processing inequality)
- In the information plane we can follow the training as deeper layer gain mutual info with output
- There seem to be 2 phases: fitting and compression
- SGD  $\rightarrow$  “compression by diffusion”
- More layers seem to be useful to break diffusion in simpler steps



# Inside Deep Learning

New experiments reveal how deep neural networks evolve as they learn.



**A INITIAL STATE:** Neurons in Layer 1 encode everything about the input data, including all information about its label. Neurons in the highest layers are in a nearly random state bearing little to no relationship to the data or its label.

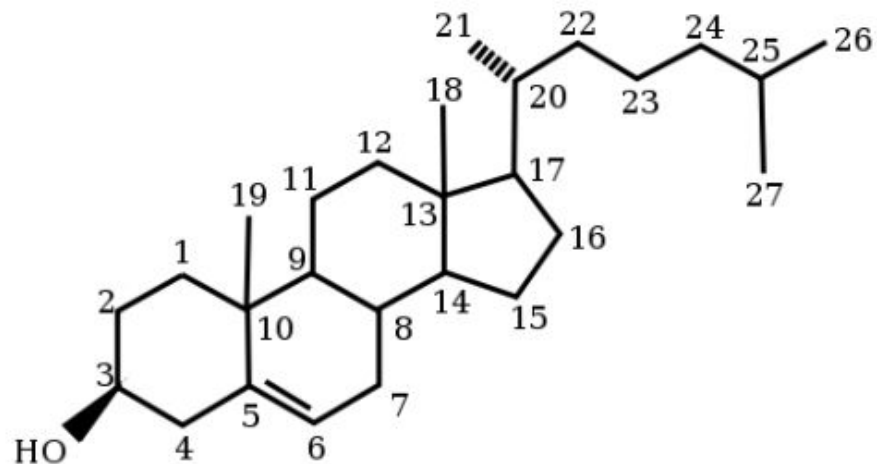
**B FITTING PHASE:** As deep learning begins, neurons in higher layers gain information about the input and get better at fitting labels to it.

**C PHASE CHANGE:** The layers suddenly shift gears and start to "forget" information about the input.

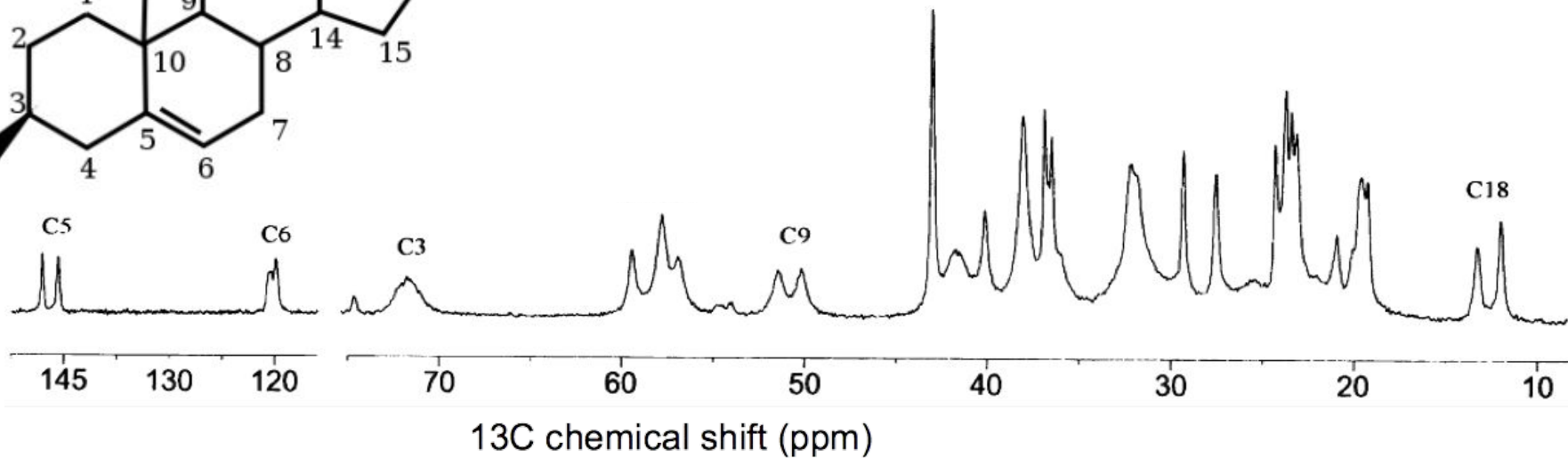
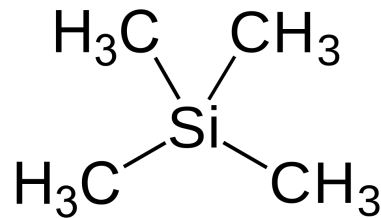
**D COMPRESSION PHASE:** Higher layers compress their representation of the input data, keeping what is most relevant to the output label. They get better at predicting the label.

**E FINAL STATE:** The last layer achieves an optimal balance of accuracy and compression, retaining only what is needed to predict the label.

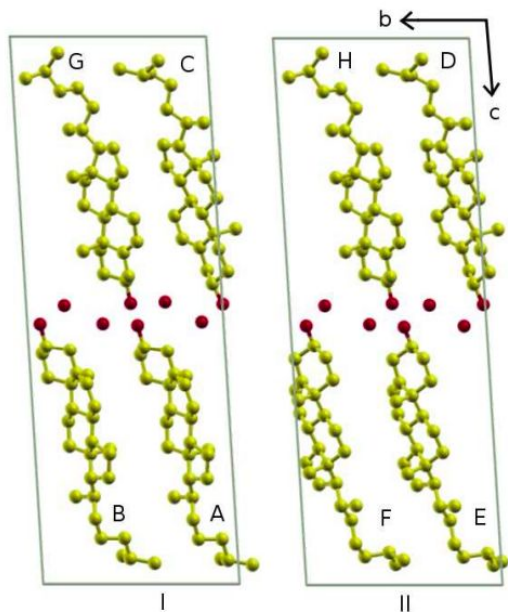
# $^{13}\text{C}$ NMR for CLR



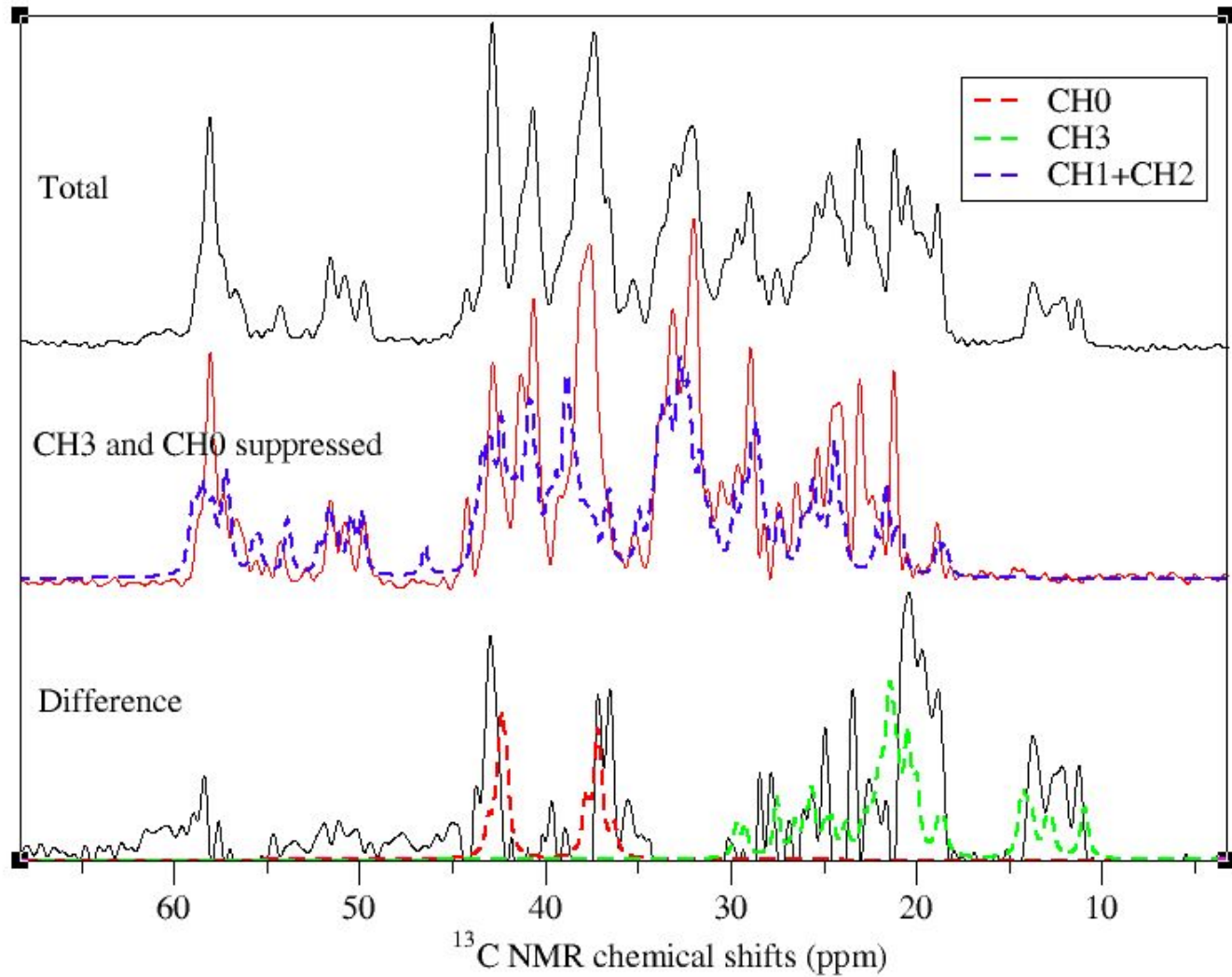
Shift reference:



# Spectral editing



ChM



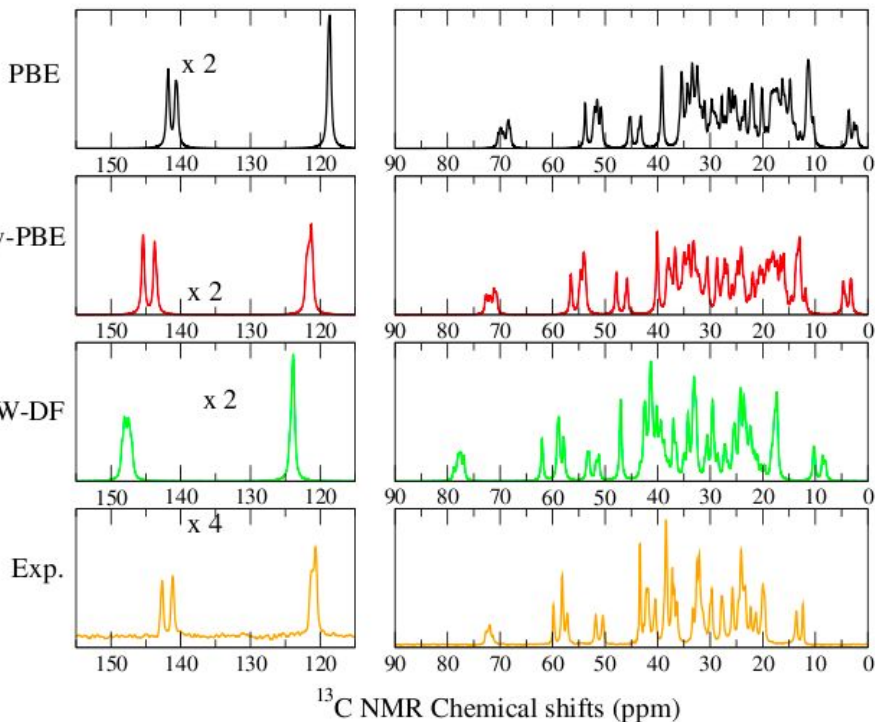


# Different functionals before secondary referencing

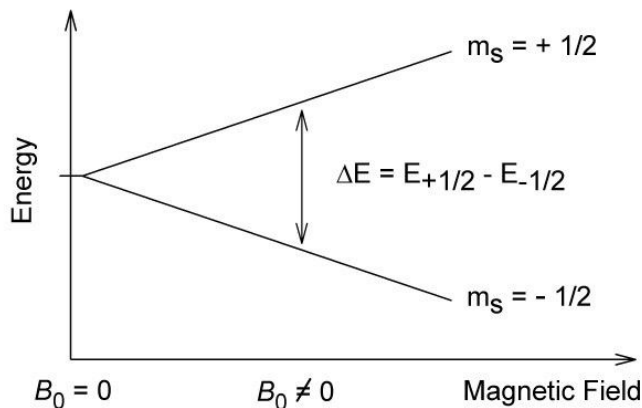
For ab initio non-local functionals (i.e. van der Waals aware functionals) one should be careful in not comparing apples to oranges:

In vdW functionals, we have  $F[n(r),n(r')]$  terms. Hence this means they also include local correlations as well as non-local. Due to this source of 'double counting', when people fit the parameters of a vdW functional, they don't do it for any local XC functional but they choose one in particular. For example vdW-DF is parametrized assuming revPBE xc, while vdW-DF2 is built on rpw86.

People often forget as they compare vdW treatments, that the other parts of the functional change also, and it might be the significant part. So, compare vdw-DF to revpbe first, if you want to see the effect of exchange.



# ab initio NMR



$$\Delta E = \hbar \gamma B_{\text{tot}} \Delta m_s = \hbar \omega$$

Chemical shielding  $H_{\text{cs}} = -\hbar \gamma \mathbf{I} \cdot \boldsymbol{\sigma} \cdot \mathbf{B}_0$

New addition to  
hamiltonian (p):

$$\mathbf{A}(\mathbf{r}) = 1/2 \mathbf{B} \times \mathbf{r}$$

$$\mathbf{j}^{(1)}(\mathbf{r}') = 2 \sum_o [\langle \bar{\psi}_n^{(1)} | \bar{\mathbf{J}}^{(0)}(\mathbf{r}') | \bar{\psi}_n^{(0)} \rangle + \langle \bar{\psi}_n^{(0)} | \bar{\mathbf{J}}^{(0)}(\mathbf{r}') | \bar{\psi}_n^{(1)} \rangle + \langle \bar{\psi}_n^{(0)} | \bar{\mathbf{J}}^{(1)}(\mathbf{r}') | \bar{\psi}_n^{(0)} \rangle]$$

$$\mathbf{B}_{\text{in}}^{(1)}(\mathbf{r}) = -\vec{\sigma}(\mathbf{r}) \mathbf{B}_{\text{ext}} = \frac{1}{c} \int d^3 \mathbf{r}' \mathbf{j}^{(1)}(\mathbf{r}') \times \frac{\mathbf{r} - \mathbf{r}'}{|\mathbf{r} - \mathbf{r}'|^3}$$

$$\delta = -(\sigma - \sigma_{\text{ref}}) \text{ Chemical shift}$$

## Some terms in GIPAW:

$$\Delta \mathbf{J}_{\mathbf{R}}^p(\mathbf{r}') = \sum_{n,m} |\tilde{p}_{\mathbf{R},n}\rangle [\langle \phi_{\mathbf{R},n} | \mathbf{J}^p(\mathbf{r}') | \phi_{\mathbf{R},m} \rangle - \langle \tilde{\phi}_{\mathbf{R},n} | \mathbf{J}^p(\mathbf{r}') | \tilde{\phi}_{\mathbf{R},m} \rangle] \langle \tilde{p}_{\mathbf{R},m} |$$

$$\begin{aligned} \mathbf{j}_{\Delta p}^{(1)}(\mathbf{r}') &= \sum_{\mathbf{R}',o} \text{Re}[\langle \bar{\psi}_o^{(0)} | \Delta \mathbf{J}_{\mathbf{R}'}^p(\mathbf{r}') G(\varepsilon_o^{(0)}) (\bar{H}^{(1)} - \varepsilon_o^{(0)} \bar{S}^{(1)}) | \bar{\psi}_o^{(0)} \rangle \\ &\quad - \langle \bar{\psi}_o^{(0)} | \Delta \mathbf{J}_{\mathbf{R}'}^p(\mathbf{r}') G(\varepsilon_o^{(0)}) \sum_{\mathbf{R}} \frac{\mathbf{B} \times \mathbf{R}'}{2c} \cdot \mathbf{v}(\varepsilon_o^{(0)}) | \bar{\psi}_o^{(0)} \rangle] \\ &\quad - \sum_{\mathbf{R},\mathbf{R}',o,o'} \langle \bar{\psi}_o^{(0)} | \Delta \mathbf{J}_{\mathbf{R}'}^p(\mathbf{r}') | \bar{\psi}_{o'}^{(0)} \rangle \langle \bar{\psi}_{o'}^{(0)} | (\mathbf{R} - \mathbf{R}') \times \frac{1}{2ic} [\mathbf{r}, Q_{\mathbf{R}}] \cdot \mathbf{B} | \bar{\psi}_o^{(0)} \rangle \end{aligned}$$

# How does QE scale?

Time spent in scf calculation:  $T_{scf}$

$N_{iter} * T_{iter} + T_{init}$  ;  $T_{iter} = N_k * T_{diag} + T_{rho} + T_{hxcv}$  ;  $T_{diag} = N_{hpsi} T_{hpsi} + T_{orth} + T_{sub}$

$T_{hpsi} = a N_{band} N_{pw} + b N_{band} N_{123} \log(N_{123}) + c N_{band} N_{pw} \text{Projall}$       $[N_{123} \sim 8 N_{pw}]$

$T_{orth} = d N_{pw} (N_{bandtrial})^2 \sim d N_{pw} (2 N_{bnd})^2$  ;  $T_{sub} = e (2 N_{bndtrial})^3$

$T_{rho} = f N_{bnd} (8 N_{pw}^*) \log(8 N_{pw}^*) + g N_{bnd} (8 N_{pw}^*) + T_{paw}$

$T_{hxcv} = j (8 N_{pw}^*) + k 8 N_{pw}^* \log(8 N_{pw}^*)$

For NMR via linear response  $\sim N_{atom}$  (as an approximation to  $N_{proj}$ ) x (7 directions x  $N_k$ ) x ( $N_{band}$  x  $N_{123} \log N_{123}$ )