International School for Advanced Studies

1999 Entrance Examination: Condensed Matter

Multiple choice quizzes

1. Rare gas atoms, which attract each other very little, solidify in close-packed crystal structures. All other elements, where atoms attract each other much more, and therefore could be expected to close-pack at least as much, only rarely do. Why?
    - a) too much attraction deforms the crystal structure
    - b) it is due to temperature effects. Indeed, at zero temperature all materials are close-packed.
    - c) chemical bonding cannot be reduced to pairwise attractions
    - d) it's a surface effect: the surface applies an inwards pressure, which is minimised by the actual crystal structure
2. Magnetism is due to quantum mechanics, and would disappear if Planck's constant were hypothetically to vanish.
    - a) This is false, magnetism would in that case get even stronger
    - b) In that case, the effective temperature would be infinite, hence above the Curie point
    - c) The free energy can be shown to be independent of the vector potential in classical mechanics
    - d) Electrons would pair off, due to vanishing of the Fermi pressure
3. Para-hydrogen H_2 is a strange molecule, since it can only have even rotational angular momenta.
    - a) This is false
    - b) The reason is that vibrations couple with a Berry phase
    - c) An odd angular momentum would require molecular dissociation
    - d) Nuclear spins make the molecule antisymmetric to proton exchange

4. Oxygen O_2 is a magnetic molecule. Why does this happen?

   - a) because of Hund's rule, as in many atoms
   - b) because it has a nonzero orbital momentum state
   - c) it is nuclear magnetism, the nuclear spins line up in a triplet state
   - d) it is not really magnetic, only paramagnetic; but the susceptibility is very high

5. A gas of identical quantum particles can condense, under certain conditions. When can free (i.e., non interacting) identical particles condense ?

   - a) When the particles are fermions.
   - b) When the particles are bosons.
   - c) For both fermions and bosons.
   - d) For neither fermions nor bosons.

6. In thermal equilibrium any system

   - a) is usually found in its ground state.
   - b) is more likely to be in the ground state than in any other single state.
   - c) is more likely to have an energy near the ground state energy than near any other value.
   - d) is equally likely to be in any single state.

7. The Hilbert space of a system of two $s = 1/2$ spins (treated as distinguishable particles) consists of 4 states. How large is the Hilbert space of a system of 10 spin-1/2 ?

   - a) 20
   - b) 100
   - c) 1024
   - d) 10^{10}

8. Consider two angular momentum variables $\mathbf{L}_1$ and $\mathbf{L}_2$ and their difference $\mathbf{L} = \mathbf{L}_1 - \mathbf{L}_2$. What are the allowed values for the magnitude L of $\mathbf{L}$?

- a) $-min(L_1, L_2) \leq L \leq max(L_1, L_2)$
- b) $0 \leq L \leq min(L_1, L_2)$
- c) $0 \leq L \leq |L_1 - L_2|$
- d) $|L_1 - L_2| \leq L \leq L_1 + L_2$

9. A molecule at temperature $T = 300K$ can be either in the ground state, or in any of <u>three</u> excited states each with energy $k_B T \ln 2$ above the ground state energy. What is the probability that the molecule is in the ground state?
    - a) 1/4
    - b) 2/5
    - c) 3/2
    - d) 2/3

10. Ferromagnets are usually found to form several domains of uniform magnetization (i.e., macroscopic regions within which the spins are ferromagnetically oriented) which, however, are oriented in such a way that the resulting total magnetization is zero. This is due to:
    - a) exchange forces between spins
    - b) long-ranged dipolar spin-spin interactions
    - c) electron-electron Coulomb interactions
    - d) the presence of the surface which breaks spin rotational invariance

11. N distinguishable quantum spins of magnitude S are coupled ferromagnetically. The zero temperature entropy is:
    - a) zero;
    - b) $N \ln(2S + 1)$;
    - c) $N \ln[S(S + 1)]$;
    - d) $\ln(2NS + 1)$.

12. By a dipole transition an electron can go from an orbital of p-like symmetry to
    - a) p or s or d-orbitals;

- b) p or d orbitals;
- c) s or d orbitals;
- d) just d orbitals.

13. A perfectly spin-isotropic ferromagnet in one dimension is

    - a) ordered at zero temperature but disordered otherwise;
    - b) always disordered;
    - c) ordered for temperatures below a finite Curie temperature;
    - d) ordered only at finite temperature.

14. The low-temperature specific heat of a certain material is found to behave like $aT^{\frac{3}{2}} + bT^3$. How do you interpret this measurement:

    - a) The effect is due to electrons and phonons.
    - b) The effect is due to optical and acoustic phonons.
    - c) The effect is due to magnons and phonons.
    - d) The effect is due to magnons and electrons.

15. How can you measure the dipole moment of a molecule?

    - a) Measuring the intensity of the rotational spectrum.
    - b) Measuring the intensity of the vibrational spectrum.
    - c) Measuring the intensity of the electronic excitations.
    - d) None of the above.

16. Let $H = p^2/(2m) + V(x)$ be the Hamiltonian of a spinless particle in one dimension with $V(x) = V(-x)$, and $\psi(x)$ be a trial wavefunction which is odd ($\psi(x) = -\psi(-x)$). The expectation value $\langle\psi|H|\psi\rangle/\langle\psi|\psi\rangle$ is:

    - a) lower than the ground state energy.
    - b) lower than the first excited state energy.
    - c) exactly zero.
    - d) higher than or equal to the first excited state energy.

17. For an electron in a crystal, which of the following statements is true?

- a) The Bloch functions $\psi_{\mathbf{k}}$ computed at two different $\mathbf{k}$ points in the Brillouin zone are orthogonal.
- b) The periodic part of the Bloch functions $u_{\mathbf{k}}$ computed at two different $\mathbf{k}$ points in the Brillouin zone are orthogonal.
- c) The Bloch functions $\psi_{\mathbf{k}}$ and $\psi_{\mathbf{k}'}$ computed at two different $\mathbf{k}$ points in the Brillouin zone differ just in the phase factor $\exp i(\mathbf{k}\cdot\mathbf{r})$
- d) All Bloch functions can be taken to be periodic for an appropriate choice of the phase.

18. Physical quantities at a continuous phase transition exhibit in general power-law singularities, but the true exponents are not classical, as in simple mean field theory. This is due to the fact that:

    - a) forces are generally long-ranged power laws
    - b) critical fluctuations destroy classical behaviour
    - c) boundary effects become dominant near long-range ordering
    - d) in reality all transitions are slightly first-order

The following questions should be answered only by candidates interested in the bio-simulation curriculum

19. Water undergoes a reversible ionization to yield a hydrogen ion and a hydroxide anion. In pure water, at 300 K

    - a) about one of every 10 molecules is ionized.
    - b) about one of every 10^{-5} molecules is ionized.
    - c) about one of every 10^{-7} molecules is ionized.
    - d) about one of every 10^{-14} molecules is ionized.

20. Consider the peptide Glu-Gly-Ser-Ser-Ala at physiological pH. What is its net electric charge?

    - a) +1
    - b) 0
    - c) -1

- d) -2

21. Disulfide bonds in proteins are formed:
    - a) between two Gly residues.
    - b) between two Ser residues.
    - c) between two Met residues.
    - d) between two Cys residues.
22. Hydrogen bonds are crucial for structure and function of biological systems. Usually, the bond holding a hydrogen bond complex:
    - a) is weaker than both covalent bonds and van der Waals forces.
    - b) is weaker than covalent bonds but stronger than van der Waals forces.
    - c) is weaker than van der Waals forces but stronger than covalent bonds.
    - d) is stronger than both covalent bonds and van der Waals forces.