Spring 2008 - Entrance Examination: Condensed Matter
Multiple choice quizzes

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1. In the ammonia molecule (NH_3) the three H atoms define a plane whose position oscillates about the N atom, by a tunneling effect. Which is the experimental consequence of this oscillation?

   A. The molecule absorbs radiation in the microwave energy region.

   B. Each vibrational infrared absorption peak is actually split in three.

   C. Tunneling destroys the permanent dipole of the molecule, and the dielectric constant of liquid ammonia is zero.

   D. There is no measurable consequence, it is only a nice theoretical prediction.

2. How does the rotational entropy of a molecular gas depend on the individual molecular moments of inertia I:

   A. It is independent of I.

   B. It is an increasing function of I.

   C. It is an decreasing function of I.

   D. It diverges because of quantum fluctuations.

3. Consider the liquid-gas phase transition of a simple substance such as Ar. What statement is correct?

   A. Because liquid and gas have the same symmetry, the transition can only occur as a smooth crossover.

   B. Because liquid and gas have the same symmetry, the transition can only occur as either first order, or a smooth crossover.

   C. The transition is always critical, with the density as order parameter.

   D. Because liquid and gas have the same symmetry, the transition can only be either first order, or a smooth crossover, except for a single critical point between the two.

4. Consider a $N \times N \times N$ cubic arrangement of N^3 sites. Each site carries a classical magnetic moment ("spin") whose value can be $S_i = +1$ or -1, and neighboring sites (i, j) interact with the Ising coupling $-JS_iS_j$, with $J > 0$. At small but finite temperature T and N large but finite, the total magnetization of the cube $M = \langle \sum_i S_i \rangle$ is

A. Always zero

B. Nonzero only at $T \to 0$, and zero at all finite temperatures

C. Nonzero up to the Ising critical temperature T_c

D. Always nonzero

5. At a first-order phase transition:

   A. The free energy has a jump.

   B. The free energy is continuous, but its first derivatives have a jump

   C. The free energy and its first derivatives are continuous, but higher derivatives can have a jump

   D. The first derivatives of the free energy have power-law singularities with certain critical exponents

6. The zero-point energy of CH_4 is 27 kcal/mol, while the zero-point energy of CH_3 is 17 kcal/mol. If, neglecting zero-point energy, you calculate that the difference between the atomization energy of CH_3 and of CH_4 is 113 kcal/mol, which is the atomization energy difference that you expect to measure?

   A. 103 kcal/mol.

   B. 113 kcal/mol.

   C. 123 kcal/mol.

   D. 69 kcal/mol.

7. The entropy density $S(T)$ at low temperature T of a free electron gas confined in $d \leq 3$ dimensions:

   A. goes to zero with a power law $S(T) \sim T^{d-2}$ since the energy dispersion goes like $\sim k^2$;

   B. is finite at $T = 0$ because of the spin degrees of freedom, specifically $S(0) = \ln 2$;

   C. goes to zero exponentially because of Pauli principle.

   D. goes to zero with universal power law $S(T) \sim T$;

8. The first and second Hund's rules state that the lowest energy configuration of an isolated atom with a partially filled shell:

   A. has the highest spin and, compatibly with that, the highest orbital angular momentum;

   B. has the highest orbital angular momentum and, compatibly with that, the highest spin;

C. has the highest spin but the lowest orbital angular momentum;

D. has the highest orbital angular momentum but the lowest spin.

9. The Ising model in one dimension, in the thermodinamic limit, is

   A. ordered below a finite critical temperature T_c;

   B. disordered at any finite temperature and only ordered at zero temperature;

   C. disordered at any finite temperature but ordered at zero temperature if it is ferromagnetic, while disordered if it is antiferromagnetic;

   D. ordered up to a critical temperature T_c if it is ferromagnetic, while, if it is antiferromagnetic, ordered at zero temperature but disordered at any finite temperature.

10. The magnetization of a normal metal, i.e. not superconducting, in the presence of a small uniform magnetic field B is

    A. finite and antiparallel to B;

    B. finite and parallel to B;

    C. zero;

    D. finite and perpendicular to B.

11. Approximately, how many H_2O molecules are contained in a liter of water?

    A. 3×10^{15}

    B. 3×10^{20}

    C. 3×10^{25}

    D. 3×10^{30}

12. The linear dimensions of an ink drop, dropped in otherwise clean water, are observed to increase in time proportionally to $t^{\frac{1}{2}}$. This fact indicates that:

    A. The ink molecules strongly repel each other.

    B. The ink molecules slightly attract each other.

    C. The polar water molecules mediate a weak repulsive interaction among the ink molecules which would not otherwise interact in vacuum.

    D. The interaction among the ink molecules is negligible, and the motion is purely diffusive.

13. Consider N classical particles held together in free 3-dimensional space by some mutual interaction, and vibrating around their equilibrium state. Their 3N equations of motion possess a certain number n of zero-frequency modes. How many are they?

A. $n = 0$.

B. $n = 3$.

C. $n = 6$.

D. $n = N$.

14. Upon application of a uniaxial compression to a crystal, two of its opposite surfaces are observed to acquire an opposite charge. Which one of the following statements is correct?

    A. Migration of charge from one surface to the opposite requires a high electric conductivity: the crystal is therefore a metal.

    B. The existence of an electric field due surface charges and generated by a disturbance that does not break inversion symmetry is only compatible with a non centro-symmetric insulator.

    C. The crystal may be either a metal or an insulator, according to the different mechanism by which the surface charges are generated.

    D. None of the above. It is impossible that a macroscopic piece of matter displays an electric response to a mechanical disturbance, because this would imply a violation of the conservation of parity by electromagnetic interactions.

15. The ionization potential I of oxygen ($Z = 8$, $I = 13.6$ eV) is smaller than the ionization potential of nitrogen ($Z = 7$, $I = 14.5$ eV) and of fluorine ($Z = 9$, $I = 17.4$ eV). Can you guess why?

    A. The above data cannot be right. In all the three atoms the valence electrons fill the same $2p$ shell. Therefore the ionization potential must increase by increasing the nuclear charge Z.

    B. The value of the ionization potential is due to subtle electron correlations effects. There are no rules, it changes randomly from one atom to the next.

    C. This is due to Hund's rules. The $2p$ shell of N is half filled. The new electron added to the $2p$ shell of O must have opposite spin with respect to the other three. It has therefore higher energy and this compensates the increased nuclear attraction.

    D. This is due to spin-orbit splitting which is larger in O than in F, due to the larger number of vacancies in the $2p$ shell, whereas N has none.

16. The Hamiltonian of a system of interacting electrons contains a term $V \propto \mathbf{L}\cdot\mathbf{S}$, where $\mathbf{L} = \sum_i \mathbf{l}_i$ and $\mathbf{S} = \sum_i \mathbf{s}_i$ are the total orbital and spin angular momenta, respectively. Let us indicate with $\mathbf{J} = \mathbf{L} + \mathbf{S}$ the total angular momentum of the system. Which of the following operators commute with V?

A. J^2, J_z, L^2, and S^2.

B. L^2, L_z, S^2, and S_z.

C. J^2, J_z, L_z, and S_z.

D. None of the above.

17. The specific heats of two different sistems, A and B, are observed to behave differently at very low temperature: $C_A \propto T$ and $C_B \propto T^3$. One can conclude that:

    A. The energy gap of system A is larger than system B's.

    B. The energy gap of system A is smaller than system B's.

    C. Both systems are gapless: system A has a larger density of states at low energy than system B.

    D. Both systems are gapless: system A has a smaller density of states at low energy than system B.

18. A small metal spherical and neutral particle of radius ρ is placed at distance R from a point charge. For large R the force acting on the metal particle behaves as:

    A. $|\mathbf{f}| \propto \mathrm{e}^{-\frac{R}{\rho}}$, attractive.

    B. $|\mathbf{f}| \propto R^{-12}$, repulsive.

    C. $|\mathbf{f}| \propto R^{-5}$, attractive.

    D. $|\mathbf{f}| \propto R^{-5}$, repulsive.

19. ^{3}He and the electron are both spin-$\frac{1}{2}$ fermions. Let us consider an electron gas and an ^{3}He fluid at a same homogeneous density. What can be said about the Fermi energies of the two systems:

    A. The electron gas has a well defined Fermi energy, whereas ^{3}He has not, because the latter is made of neutral particles.

    B. In the absence of interparticle interactions the two Fermi energies are the same.

    C. The Fermi energy of electrons is a few thousands times larger than ^{3}He's.

    D. The Fermi energy of electrons is a few thousands times smaller than ^{3}He's.

20. The grass is green because:

    A. Chlorophyll absorbs visible light with almost equal intensity, but for two dips in the yellow and blue regions where light is transmitted.

    B. Chlorophyll has a strong absorption band in the green spectral region, and it absorbs very little visible light in other spectral regions.

C. Green is known to be a mix of yellow and blue: chlorophyll has two strong absorption lines in the yellow and in the blue.

D. Chlorophyll has strong absorption bands in the red and blue spectral regions, and very little absorption in the green.