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March 2012 - Entrance Examination: Condensed Matter
Multiple choice quizzes

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1. Hamiltonians in physics usually conserve energy during time evolution. However, we are all familiar with energy dissipation and friction phenomena. Where does that come from?

   A. Coupling with vacuum fluctuations makes the effective Hamiltonian non-Hermitian.

   B. Integrating away certain "bath" degrees of freedom amounts to making the remaining Hamiltonian degrees of freedom non conservative.

   C. Chaos is responsible for the irreversibility.

   D. Dissipation is an illusion, if you wait a long enough time the energy will come back.

2. The molecule C_6D_6 is a benzene molecule in which hydrogen is substituted by deuterium. The energy needed to remove a deuterium from this molecule is:

   A. Higher than the energy needed to remove a hydrogen from benzene C_6H_6.

   B. Identical to the energy needed to remove a hydrogen from benzene C_6H_6.

   C. Lower than the energy needed to remove a hydrogen from benzene C_6H_6.

   D. This cannot be decided without comparing the electronic structure of C_6D_6 and C_6H_6.

3. An energy band of a periodic chain of atoms at distance a is given by $\varepsilon(k) = -cos(ka)$ (k is the Bloch wave-vector). Which one of the following statements is true?

   A. The average speed of the electron at k and $-k$ is the same.

   B. The average speed of the electron is maximum at $k = 0$.

   C. The average speed of the electron at k and $-k$ is equal in magnitude but opposite in direction.

   D. The average speed of the electron with spin up at k is the same as the average speed of the electron with spin down at $-k$.

4. In an electromagnetic plane wave the electric and magnetic fields are:

   A. Parallel to each other and perpendicular to the propagation direction both in vacuum and in anisotropic solids.

   B. Parallel to each other and perpendicular to the propagation direction in vacuum but not necessarily in anisotropic solids.

   C. Perpendicular to each other and to the propagation direction in vacuum but not necessarily in anisotropic solids.

   D. Perpendicular to each other and to the propagation direction both in vacuum and in anisotropic solids.

5. Free electrons have an energy versus momentum $\mathbf{k}$ dispersion relation $\varepsilon(\mathbf{k}) = \hbar^2 k^2/2m$, density of states $\rho(\varepsilon)$ and chemical potential μ. The electronic specific heat at low temperature is linear in T if

   A. The density of states $\rho(\varepsilon \simeq \mu) \sim \varepsilon - \mu$ is also linear in $\varepsilon - \mu$.

   B. The chemical potential $\mu(T \to 0) \sim T$ is linear in T.

   C. The density of states is finite at the chemical potential: $\rho(\mu) = \text{constant}$.

   D. Always, whatever $\rho(\varepsilon)$ and μ are.

6. Consider the ground state of a gas of free electrons with a finite density and dispersion relation $\varepsilon(\mathbf{k}) = \hbar^2 k^2/2m$ in presence of a magnetic field that is assumed to couple only with the spin degrees of freedom. Which of the statements below is correct?

   A. The ground state is fully polarized in the direction of the magnetic field.

   B. The ground state is not polarized at all.

   C. The ground state is never fully polarized whatever the amplitude of the magnetic field.

   D. The ground state is fully polarized only when the magnetic field exceeds a threshold.

7. The ground state of a system of quantum spins coupled among each other ferromagnetically is a ferromagnet:

   A. Always.

   B. Only in dimensions greater than one.

   C. Only in dimensions greater than two.

   D. Only for particular lattices.

8. The valence electronic configuration of the Vanadium atom is $3d^3 4s^2$. The eigenvalues of the square of the orbital and spin angular momentum are $\hbar^2 L(L+1)$ and $\hbar^2 S(S+1)$ respectively. According to Hund's rules, what are the values of L and S in the ground state?

   A. $S = 3/2$ and $L = 3$.
   B. $S = 1/2$ and $L = 3$.
   C. $S = 3/2$ and $L = 1$.
   D. $S = 1/2$ and $L = 1$.

9. Two ideal gases A and B have the same internal energy. Gas A has twice the number of moles of gas B. This means that:

   A. The temperature of A is the same as the temperature of B.
   B. The temperature of A is twice the temperature of B.
   C. The temperature of A is one half of the temperature of B.
   D. One cannot say. It depends on the volume of the two gases.

10. Which of the sentences below is correct for the spin-orbit split band structures of a nonmagnetic solid in a general point $\mathbf{k}$ of the Brilluoin zone?

    A. Each band is at least spin degenerate: At each energy one can put two electrons with opposite spin. In general the bands at $\mathbf{k}$ are different from the bands at $-\mathbf{k}$.
    B. The bands at $\mathbf{k}$ are not spin degenerate, but the bands at $\mathbf{k}$ are degenerate with the bands at $-\mathbf{k}$.
    C. The bands at $\mathbf{k}$ are spin degenerate and the bands at $\mathbf{k}$ are degenerate with the bands at $-\mathbf{k}$.
    D. The bands at $\mathbf{k}$ are not spin degenerate and the bands at $\mathbf{k}$ are different from the bands at $-\mathbf{k}$.

11. Which one of the following relations is wrong:

    A. $\left(\frac{\partial S}{\partial V}\right)_T = \left(\frac{\partial P}{\partial T}\right)_V$.
    B. $\left(\frac{\partial T}{\partial V}\right)_S = -\left(\frac{\partial P}{\partial S}\right)_V$.
    C. $\left(\frac{\partial T}{\partial P}\right)_S = \left(\frac{\partial V}{\partial S}\right)_P$.
    D. $\left(\frac{\partial S}{\partial T}\right)_P = -\left(\frac{\partial V}{\partial P}\right)_T$.

12. Two metallic spheres are connected by a thin metallic wire but are sufficiently distant that their mutual interaction can be neglected. A charge q is added to one sphere and at equilibrium the charge on the sphere A is $q/3$ while the charge on the sphere B is $2q/3$. The radii (R_A and R_B) of the two spheres are:

    A. $R_A = 2R_B$.

    B. $R_A = \frac{1}{\sqrt{2}} R_B$.

    C. $R_A = \frac{1}{2} R_B$.

    D. $R_A = \frac{1}{3} R_B$.

13. Two identical, neutral, spherically symmetric, and electrically polarizable classical *objects* are placed at a mutual distance, R, which is much larger than their size ($R \gg l$). Let $V(R)$ be the interaction energy of the system, *i.e.* the energy of the two particles at distance R, minus the energy they would have at infinite distance. Ignoring any quantum effects ($\hbar = 0$), which one of the following statements is true:

    A. $V(R) = -\frac{C}{R^6}$, where C is a positive constant that is different from zero at zero temperature and initially decreases with increasing temperature.

    B. $V(R) = -\frac{C}{R^6}$, where C is a positive constant that vanishes at zero temperature and initially increases with increasing temperature.

    C. $V(R)$ is attractive and decreases exponentially with R.

    D. $V(R)$ is repulsive and decreases exponentially with R.

14. A free electron and a photon have the same energy (without accounting for the electron rest energy). Which one has the longest wavelength.

    A. The electron.

    B. The photon.

    C. It depends on the energy. At low energies the photon, at high energies the electron.

    D. It depends on the energy. At low energies the electron, at high energies the photon.

15. Order the following molecules CO, NO, and O_2 in order of increasing atomization energy:

    A. $E_{\mathrm{CO}} < E_{\mathrm{NO}} < E_{\mathrm{O_2}}$.
    B. $E_{\mathrm{NO}} < E_{\mathrm{CO}} < E_{\mathrm{O_2}}$.
    C. $E_{\mathrm{O_2}} < E_{\mathrm{CO}} < E_{\mathrm{NO}}$.
    D. $E_{\mathrm{O_2}} < E_{\mathrm{NO}} < E_{\mathrm{CO}}$.

16. The isolated NO molecule has an odd electron number. That implies that

    A. The molecule must have a semi-integer spin, such as $S = 1/2$, etc.
    B. The molecule must have an integer spin, such as $S = 1, 2, 3$, etc.
    C. Such a molecule cannot be stable.
    D. Like all such molecules, NO will quantum mechanically fluctuate between the two states $S = +1/2$ and $-1/2$, ending up with effective zero spin.

17. The classical prediction of Rayleigh and Jeans for the frequency spectrum of the radiation emitted by a black body is

$$\rho(\nu, T) = \frac{8\pi\nu^2 kT}{c^2} \tag{1}$$

    where k is the Boltzmann constant, c the speed of light, ν the frequency and T the temperature. This formula is correct:

    A. At all frequencies but only at low temperature.
    B. At low frequencies and at all temperatures.
    C. At high frequencies and at low temperature.
    D. At low frequencies and at low temperature.

18. A solenoid has n turns per unit length. The current I on the solenoid is changing and dI/dt is constant (t is the time). The induced electric field is:

    A. Zero everywhere. There is only a magnetic field.
    B. Zero inside the solenoid, and different from zero outside.
    C. Different from zero inside the solenoid and zero outside.
    D. Different from zero both inside and outside.

19. Fermi's famous golden rule of time-dependent perturbation theory gives the rate of decay of a given initial state into a continuum of final states, in presence of a coupling V. The decay rate

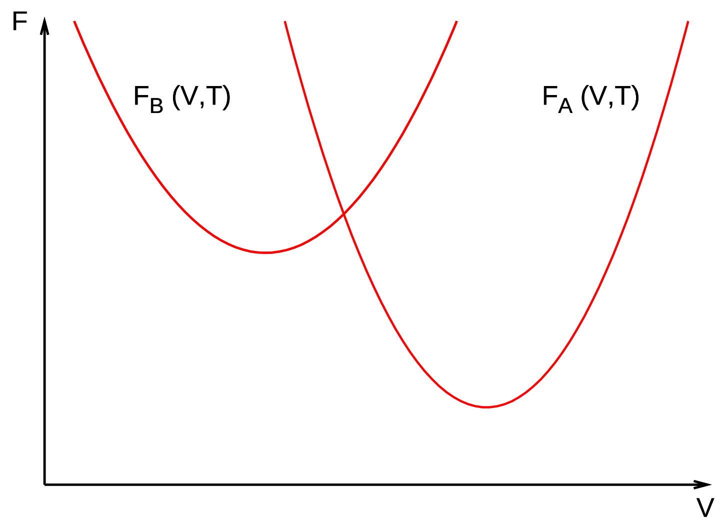
A. is proportional to $|V|^2$ and to the inverse density of final states.

B. is proportional to $|V|^{-2}$ and to the density of final states.

C. is proportional to $|V|^{-2}$ and to the inverse density of final states.

D. is proportional to $|V|^2$ and to the density of final states.

20. A given chemical compound can exist in two crystal structures, A and B. The Helmholtz free energies $F_A(V,T)$ and $F_B(V,T)$ of the two structures as a function of the volume V and at a given temperature T are shown in the figure:



Which one of the following statements is true:

A. A pressure $\bar{P}$ exists such that $\bar{P} \times (V_A - V_B) = F_B(V_B,T) - F_A(V_A,T)$, the stable crystal structure is A for $P < \bar{P}$ and B for $P > \bar{P}$, and a first-order transition would occur at $P = \bar{P}$.

B. The stable crystal structure is A at any pressure because this structure has always the smallest free energy.

C. No phase transformation can occur from one structure to the other as a function of pressure, unless the symmetry group of the high-pressure phase is a subgroup of that of the low-pressure one. When this is the case, the phase transition is second order and the volume is conserved across the transition.

D. A pressure-induced transformation from structure A to structure B can only occur as a consequence of quantum fluctuations, and would take a time of the order of $\tau \sim (\omega_D)^{-1} e^{\frac{P \times (V_A - V_B)}{\hbar \omega_D}}$, where ω_D is the Debye frequency: too long to be observed in a real macroscopic sample.