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Non-Gaussianities in Halo Clustering properties

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Based on:

ADS, M. Maggiore, A. Riotto - 1102.0046

ADS, M. Maggiore, A. Riotto - MNRAS (1007.1903)

OUTLINE

- Excursion Set Theory
- Path Integral formulation
- Computation of NG corrections to:
    - Halo mass function
    - Halo bias
    - Halo formation time distribution

Introduction

- The formation and evolution of structures is a very complex phenomenon.
- Clustering properties of DM haloes (e.g. mass function) can be sensitive probes of NG and will be tested in the near future.
- At present, quantitative knowledge comes mainly from N-body sims.
- A full theoretical understanding is still lacking. A successful theory of structure formation must be able to make predictions.
  Need for an analytical control.
- Focus on:
  analytical description of the formation of DM haloes and impact of NG.

→ **Excursion Set Theory**

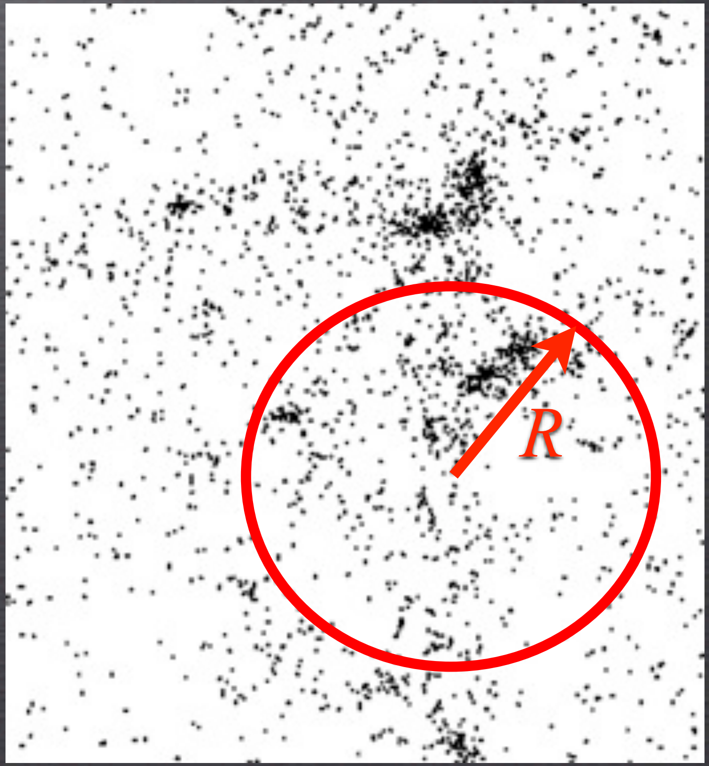
Excursion Set Theory

[Bond, Cole, Efstathiou, Kaiser 1991]
[Peacock, Heavens 1990]

- It allows to map the statistics of initial conditions with the subsequent formation of structures.
- Smooth out the density pert. $\delta = \dfrac{\delta\rho}{\rho}$ on a sphere of radius R

$$\delta(R, \mathbf{x}) = \int d^3x' \, W(|\mathbf{x} - \mathbf{x}'| \, , R) \, \delta(\mathbf{x}')$$

- Study the evolution of δ as a function of R
    - At $R=\infty$, $\delta(R)=0$.



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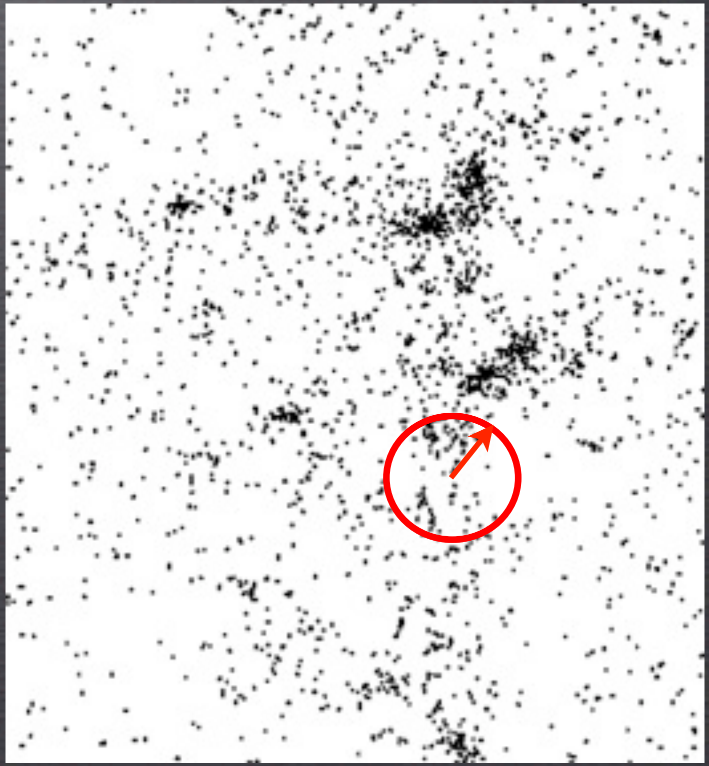

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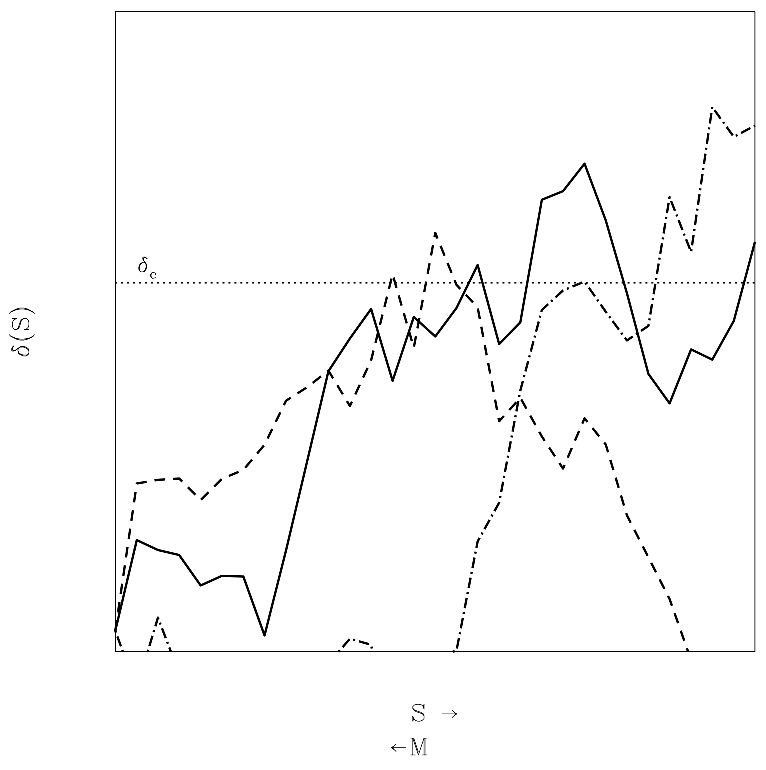
- Study the evolution of δ as a function of R
  - At $R=\infty$, $\delta(R)=0$. Lowering R, $\delta(R)$ evolves stochastically and performs a random walk.
  - Use $S=\sigma^2(R)$ as “time”. S increases as R decreases.



Excursion Set Theory

$$\delta(R, \mathbf{x}=0) = \int \frac{d^3k}{(2\pi)^3} \tilde{\delta}_{\mathbf{k}} \widetilde{W}(\mathbf{k}, R)$$

Window function ("filter")

stochastic variable

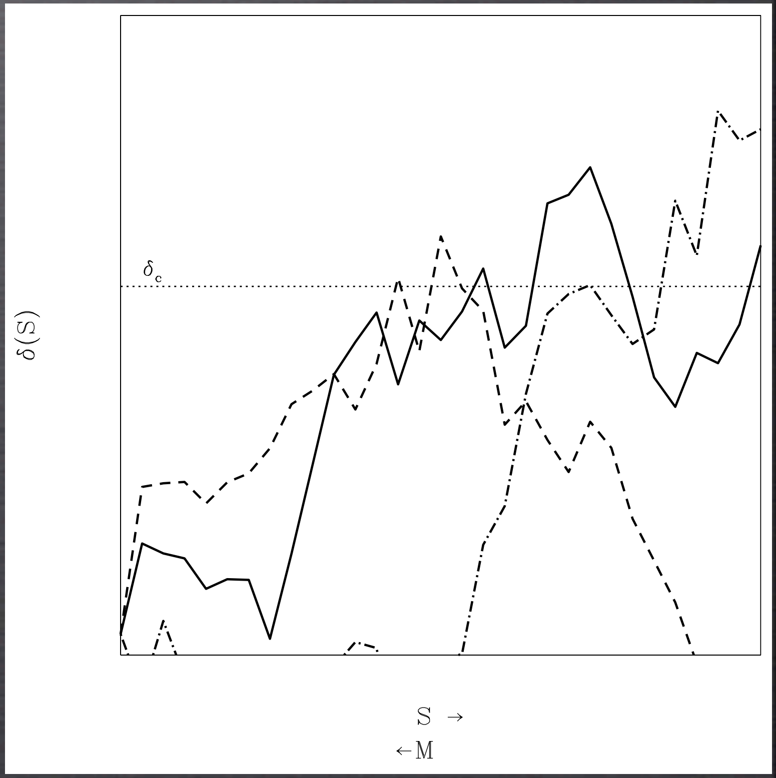
$$\frac{\partial \delta(S)}{\partial S} = \eta(S)$$ (Langevin eq.)

"noise"

Ex: for sharp k-space filter

$$\langle \eta(S_1)\eta(S_2)\rangle = \delta_D(S_1 - S_2)$$

"white noise"



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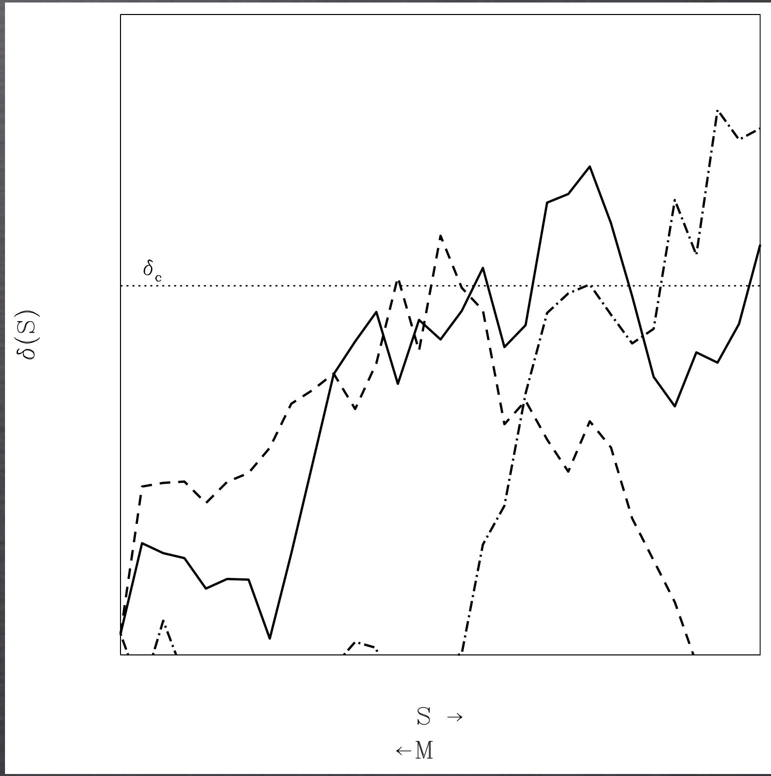
A region collapses if it is dense enough: when δ crosses δ_c the first time

Halo formation probability is mapped into a *first - passage time problem*



Excursion Set Theory

Problem: find the probability that a particle subject to a random walk passes for the first time through a given point

$\Pi(\delta, S)$: prob. that the density contrast arrives for the first time at δ in a time S

Knowledge of Π solves the problem and allows computation of the mass function

First-crossing rate:

$$\mathcal{F}(S) = -\frac{\partial}{\partial S}\int_{-\infty}^{\delta_c} d\delta\, \Pi(\delta, S)$$

number of trajectories that did not cross the barrier before S

Halo mass function: $$\frac{dn}{dM}dM = \frac{\bar{\rho}}{M}\mathcal{F}(S)\left|\frac{dS}{dM}\right|dM$$

Excursion Set Theory

Assumptions:

1. Top-hat filter in k-space: $\widetilde{W}(\mathbf{k}, R) = \theta(R^{-1} - |\mathbf{k}|)$
2. Spherical collapse (δ_c)
3. Gaussian initial conditions

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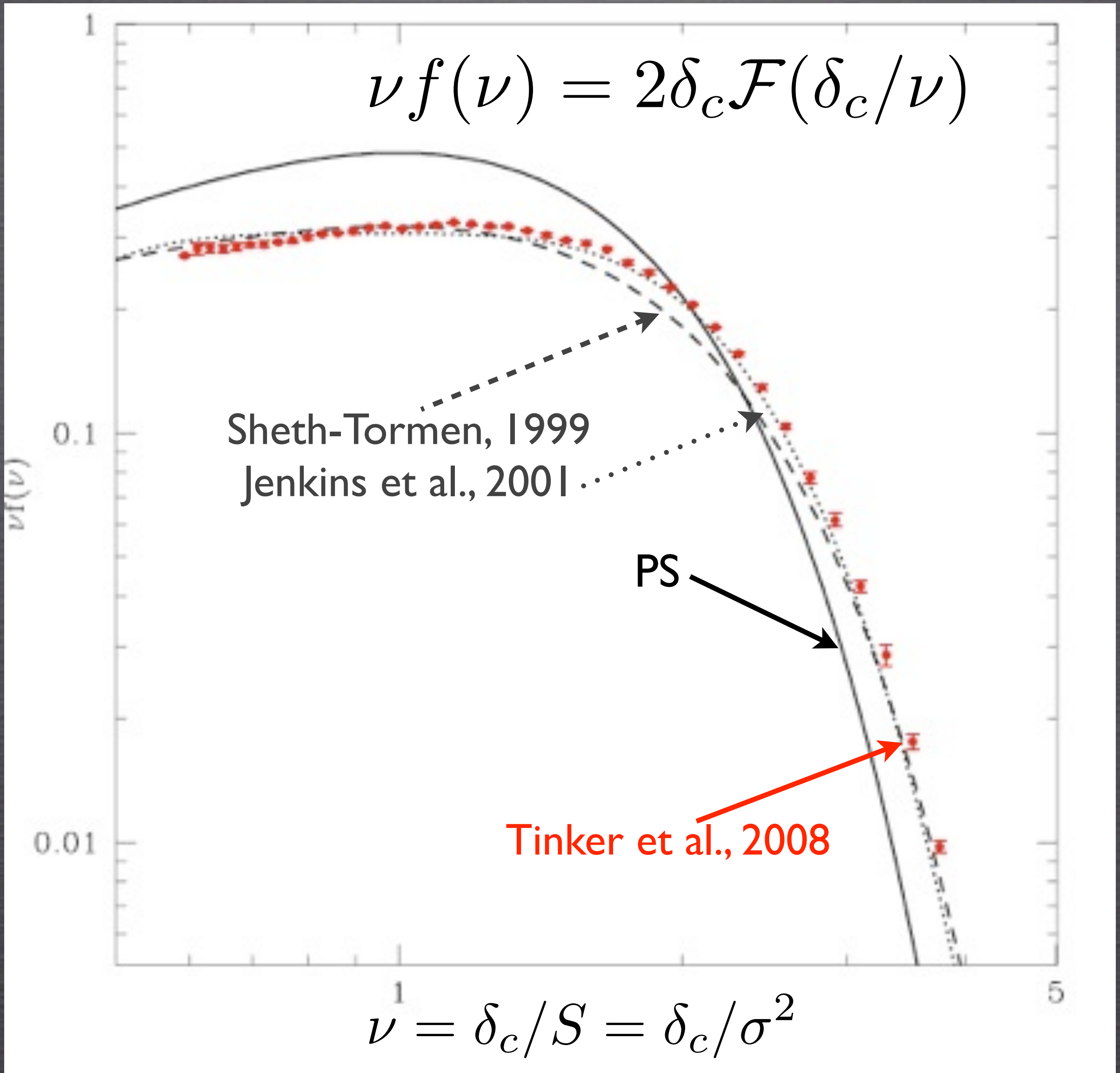
→ $$\frac{\partial \Pi(\delta, S)}{\partial S} = \frac{1}{2} \frac{\partial^2 \Pi(\delta, S)}{\partial \delta^2}$$ (Fokker-Planck eq.)

$$\Pi(\delta_c, S) = 0$$

solution → $$\Pi(\delta, S) = \frac{1}{\sqrt{2\pi S}} \left[e^{-\delta^2/(2S)} - e^{-(2\delta_c - \delta)^2/(2S)} \right]$$

→ $$\mathcal{F}(S) = \frac{\delta_c}{\sqrt{2\pi S^3}} e^{-\delta_c^2/(2S)}$$ (Press-Schechter)



At large masses, the PS theory underestimates the halo masses by a factor ~ 10;
at small halo masses it overestimates them by a factor ~ 2

Excursion Set Theory

1. Top-hat filter in k-space ?

- Unphysical, one may not identify well-defined mass. N-body sim. use top-hat filter in real space.

- Smoothing with the top-hat filter in real space and/or dealing with non-Gaussianities makes the various random steps correlated: the dynamics is non-Markovian and memory effects are introduced.

Excursion Set Theory

2. Spherical collapse ?

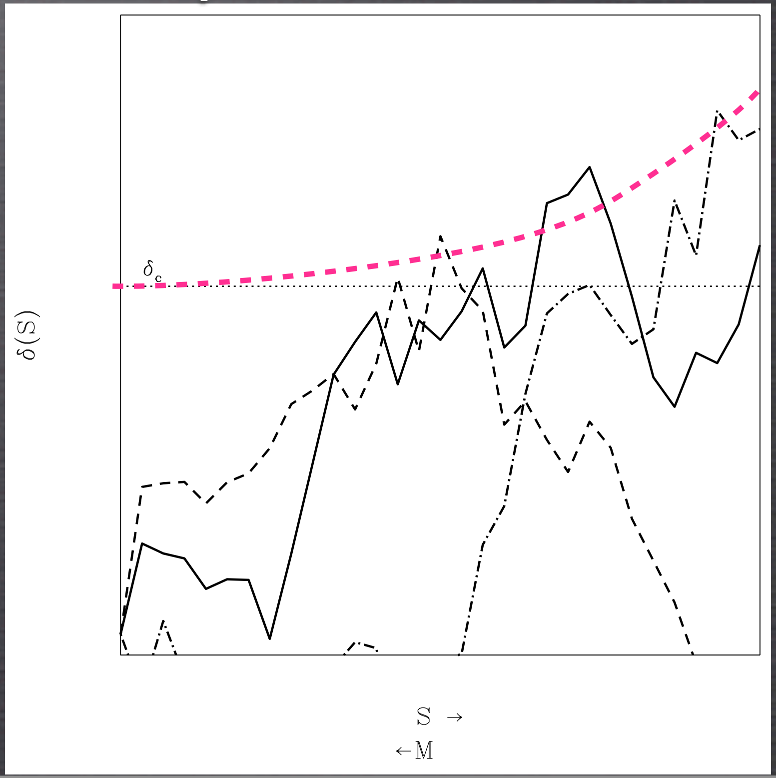
Formation of DM haloes proceeds through an ellipsoidal collapse along each of the principal ellipsoidal axes under the action of external tides (DM haloes carry angular momentum)

One can describe it by changing the collapse barrier into a moving collapse barrier

$$\delta_c \rightarrow B(S)$$

$$B(S) = \delta_c \left[1 + 0.4 \left(\frac{S}{\delta_c^2} \right)^{0.6} \right]$$

[Sheth, Tormen 2001]



It tends to sph. coll. in large mass limit

Excursion Set Theory

3. Gaussian initial conditions ?

- Want to include effects of primordial NG on structure formation.
- NG introduces non-markovian dynamics and memory effects

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If any of the assumptions is not met, FP eq is non-local. Need to go to a more fundamental level.

Excursion Set Theory has stuck for years because of this technical difficulty.

Path Integral Formulation

[Maggiore, Riotto 2009]

- Take "ensemble" of trajectories of the smoothed density contrast $\xi(S)$ and follow them for a time S.

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- Discretize S: $S_k = k\,\varepsilon$. $\xi(S_k) = \delta_k$. A given trajectory is defined by the set $\{\delta_{1,\ldots,}\delta_n\}$. The prob. density in the space of trajectories is

$$W(\delta_0;\delta_1,\cdots,\delta_n;S_n) \equiv \langle \delta_D(\xi(S_1)-\delta_1)\cdots\delta_D(\xi(S_n)-\delta_n)\rangle$$

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- Π is constructed by summing over all paths that never exceed the threshold

$$\Pi_\epsilon(\delta_0;\delta_n,S_n) = \int_{-\infty}^{\delta_c} d\delta_1 \cdots \int_{-\infty}^{\delta_c} d\delta_{n-1}\, W(\delta_0;\delta_1,\cdots,\delta_{n-1},\delta_n;S_n)$$

The problem is reduced to the evaluation of a path integral with boundaries.

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$$\Pi_\epsilon(\delta_0;\delta_n,S_n) = \int_{-\infty}^{\delta_c} d\delta_1\cdots d\delta_{n-1}\int_{-\infty}^{\infty}\frac{d\lambda_1}{2\pi}\cdots\frac{d\lambda_n}{2\pi}$$

$$\times\; e^{i\sum_{i=1}^{n}\lambda_i\delta_i+\sum_{p=2}^{\infty}\frac{(-i)^p}{p!}\sum_{i_1=1}^{n}\cdots\sum_{i_p=1}^{n}\lambda_{i_1}\cdots\lambda_{i_p}\langle\xi_{i_1}\cdots\xi_{i_p}\rangle_c}$$

$$\equiv e^{Z}$$

$$Z = i\sum_i \lambda_i\delta_i - \frac{1}{2}\sum_{i.j}\lambda_i\lambda_j\langle\xi(S_i)\xi(S_j)\rangle_c + \frac{(-i)^3}{3!}\sum_{i,j,k}\lambda_i\lambda_j\lambda_k\langle\xi(S_i)\xi(S_j)\xi(S_k)\rangle_c + \cdots$$

Generating functional of connected correlators!

PATH INTEGRAL FORMULATION

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Generating functional of connected correlators!

- NG: $\langle\xi(S_i)\xi(S_j)\xi(S_k)\rangle_c \neq 0$
- Non-spherical collapse: $\delta_c \to B(S)$ in the integrals

Halo Mass Function

[DS, Maggiore, Riotto 2010]

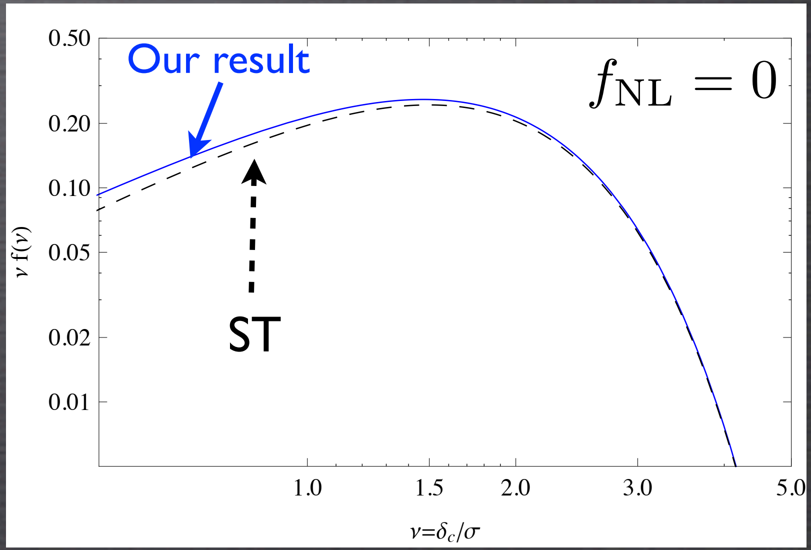
- Using path integrals, we computed the halo mass function with and without NG, for a generic barrier *B(S)*.

- Sheth&Tormen (2002) showed that the empirical formula fits well numerical sims:

$$\mathcal{F}_{\rm ST}(S) = \frac{e^{-B^2(S)/(2S)}}{\sqrt{2\pi}S^{3/2}} \sum_{p=0}^{5} \frac{(-S)^p}{p!} \frac{\partial^p B(S)}{\partial S^p}$$

- For NG=0, recover (numerically) the ST ansatz (better than 10%) and put it on firmer grounds.

$$\mathcal{F}(S) = \frac{B(S)}{\sqrt{2\pi}S^{3/2}} e^{-B^2(S)/(2S)} - \frac{B'(S)}{\sqrt{2\pi S}} e^{-B(S)^2/(2S)} + \frac{B''(S)}{4\pi}\left\{\sqrt{2\pi S} e^{-B(S)^2/(2S)} - \pi B(S) \mathrm{Erfc}\left[\frac{B(S)}{2S}\right]\right\} + \cdots$$



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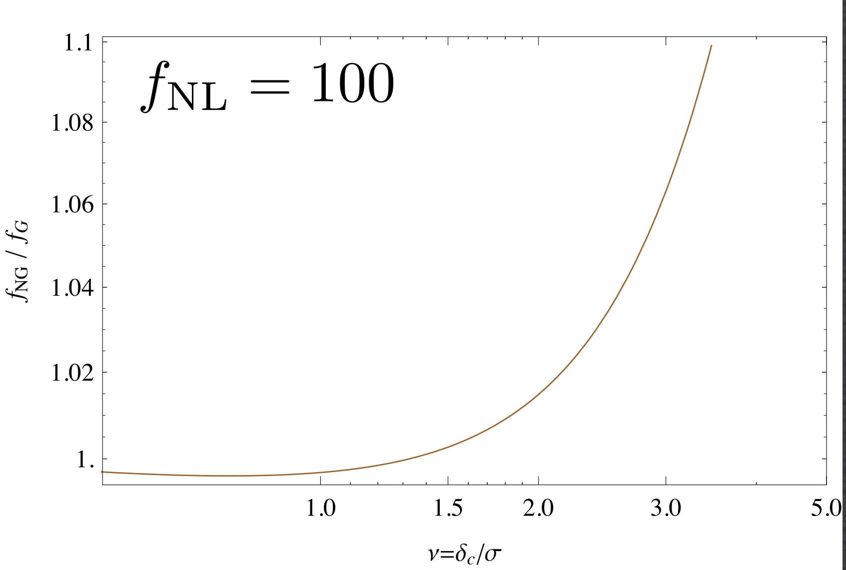


- For NG≠0, ~10% corrections

Halo Mass Function with NG

[DS, Maggiore, Riotto 2010]

- The halo mass function with NG is usually computed hoping that

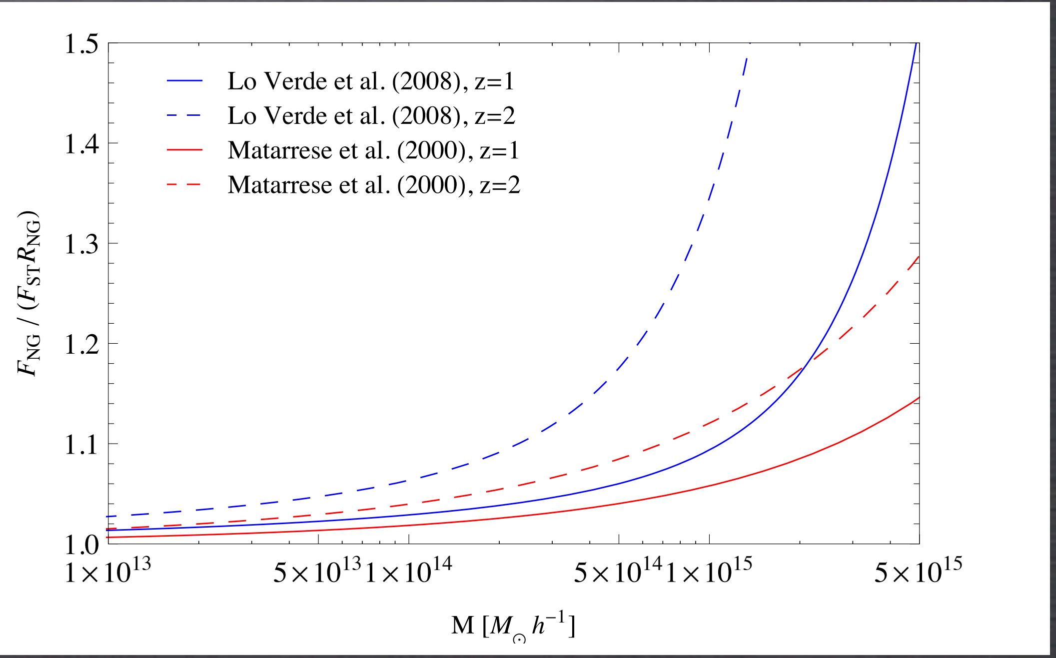
$$\frac{\mathcal{F}_{\rm NG}(f_{\rm NL},S)}{\mathcal{F}_{\rm G}(S)} = \left.\frac{\mathcal{F}_{\rm NG}(f_{\rm NL},S)}{\mathcal{F}_{\rm G}(S)}\right|_{\rm PS} \equiv \mathcal{R}(S)$$

- No rigorous justification.

- We computed directly F_{NG} for ellipsoidal collapse, with "saddle-point" improvement

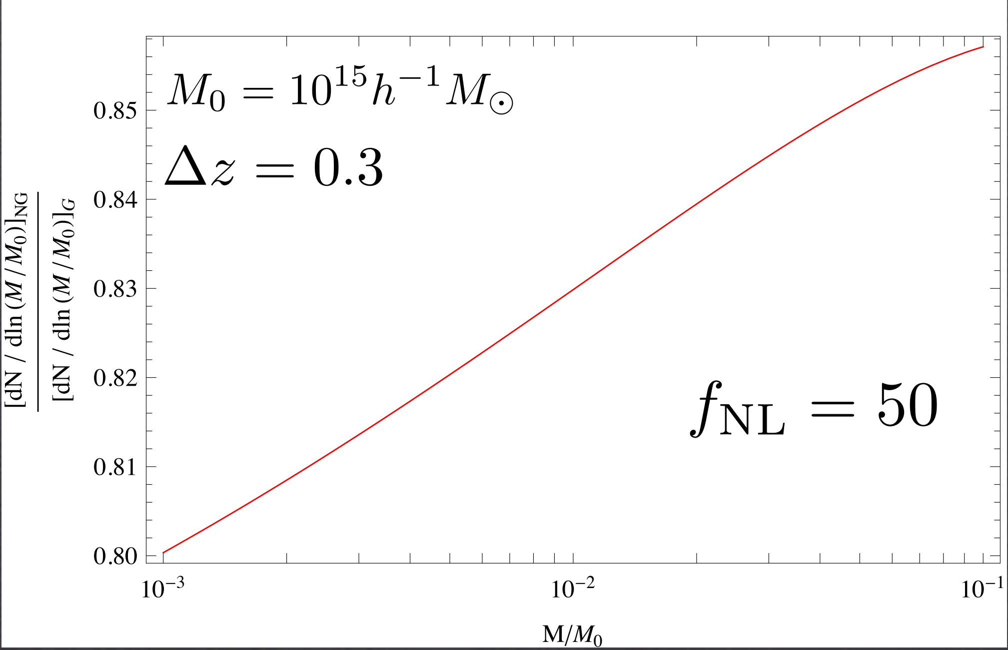
[D' Amico, Musso, Norena, Paranjape 2010]

- Proved that the above *ansatz* is incorrect.



Halo Mass Function with NG

- Conditional probabilities are useful for formation history.
- Given a halo of mass M_0 at redshift z_a, how was its mass partitioned among smaller haloes of mass M at redshift $z_b > z_a$.
- In Excursion Set Theoy: two-barrier problem. [Lacey, Cole 1993]
- Conditional mass function for generic barrier, with/without NG



[DS, Maggiore, Riotto 2010]

Halo Bias

The bias is the proportionality of halo overabundance wrt matter overdensity:

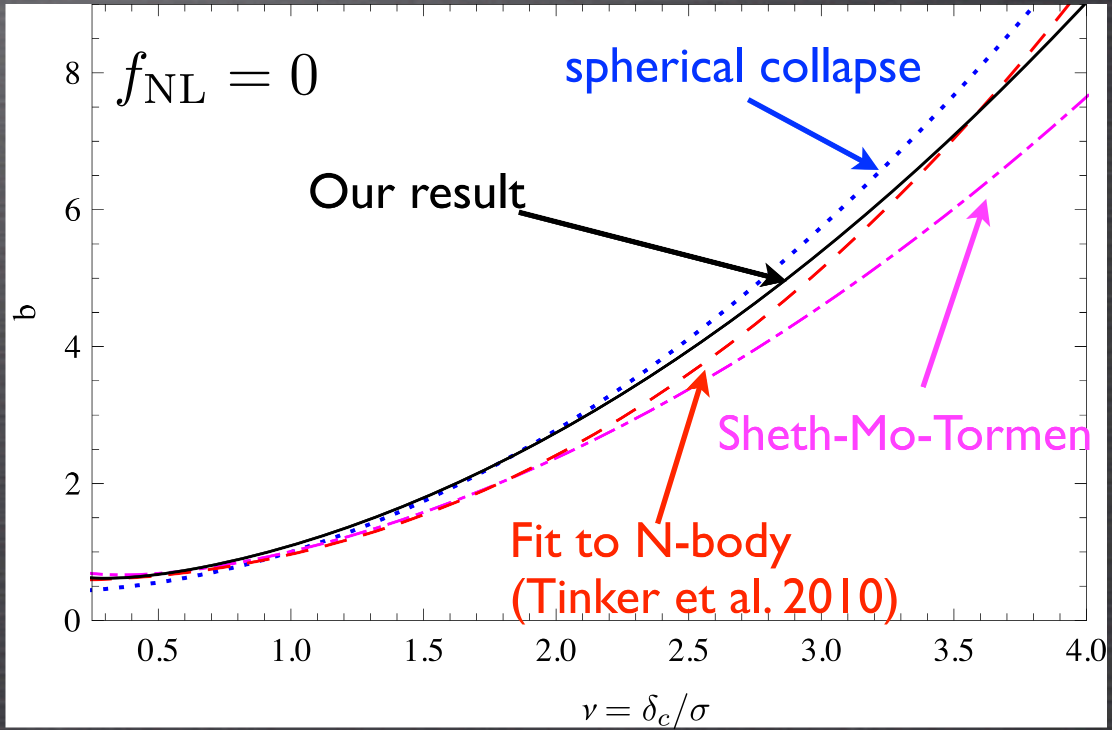
$$\left(\frac{\delta\rho}{\rho}\right)_{\rm galaxies} = b\left(\frac{\delta\rho}{\rho}\right)_{\rm mass}$$

$$b(S) = 1 + \frac{\nu(S)^2 - 1}{\delta_c} = 1 + \frac{\delta_c}{S} - \frac{1}{\delta_c}$$

[Cole, Kaiser 1989]
[Mo, White 1996]

Our result for generic barrier, without NG:

$$b(S) = 1 + \frac{B(S)}{S} - \frac{1}{B(S) + \sum_{p=1}^{\infty} \frac{(-S)^p}{p!} \frac{\partial^p B(S)}{\partial S^p}}$$



Halo Bias with NG

- We computed the (linear and quadratic) bias for generic barrier and with NG. For ellipsoidal collapse:

$$\Delta b^{(1)}_{\rm NG} \simeq -\frac{1}{6}\mathcal{S}_3\left[3a\nu^2 - 1.6(a\nu^2)^{0.4}\right]$$

$$\mathcal{S}_3 = \langle \delta^3(S)\rangle / S^2$$

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- Factor 2/3 discrepancy with [Desjacques, Marian, Smith 2009] which uses the "form factor" prescription.
  Our calculation is from "first principles".

- Non-Markov corrections due to filter: [Ma, Maggiore, Riotto, Zhang 2010]

- N-body sims began to study the bias with NG. [Wagner, Verde 2011]

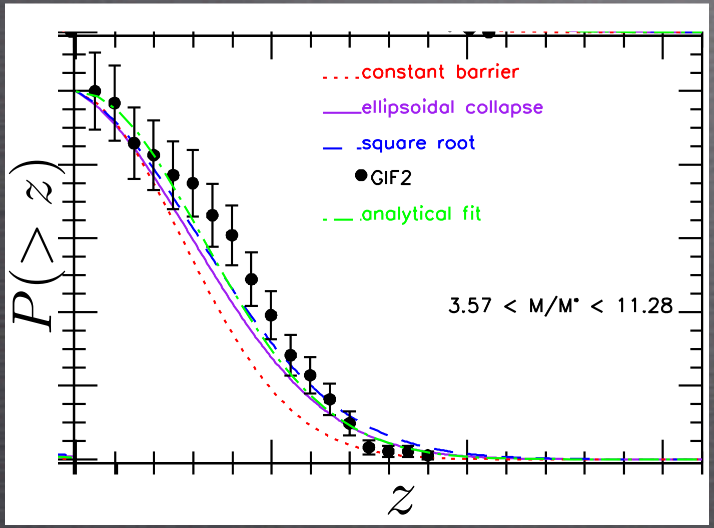
HALO FORMATION TIME WITH NG

- Formation time: the earliest time when at least half of its mass was assembled into a single progenitor.

$$p(z_b) = 2\omega(z_b)\mathrm{Erfc}\left[\frac{\omega(z_b)}{\sqrt{2}}\right]\frac{d\omega(z_b)}{dz_b}$$

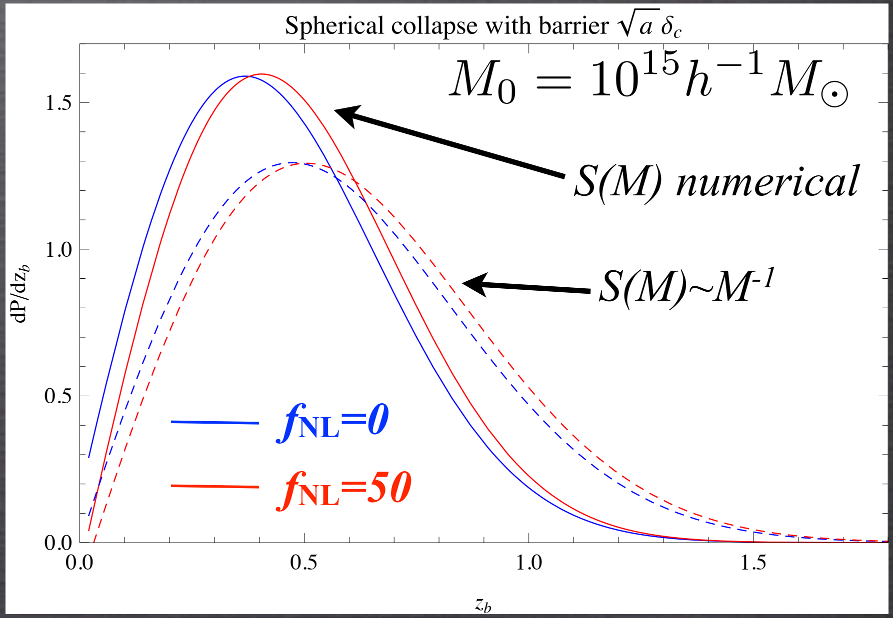
$$\omega(z_b) = \frac{\delta_c(z_b) - \delta_c(z_a)}{\sqrt{S(M_0/2) - S(M_0)}}$$

[Lacey, Cole 1993]



[Giocoli, Moreno, Sheth, Tormen 2006]

- Our computation: prob. of formation redhifts with NG. We also found an analytical generalization of Lacey&Cole for NG.



- *Caveat* on *S(M)*

< 10% shift

Conclusions

- Excursion Set Theory (with path integrals) is a convenient and powerful framework to compute properties of LSS analytically.

- It allowed a consistent derivation of the effects of NG.

- "First-principles" calculation of halo mass function, bias and formation time for generic barrier, with and without NG.

- Look forward to comparing with N-body sims and new data.

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