arXiv: 1007.1903 ~~ADS~~ ADS, M. MAGGIORE, A. RIOTTO.

- Excursion Set Theory
- P.F.
- Our results for generic barrier

■ PROBLEM to ADDRESS is an important problem of modern cosmology: the formation of structures (DM halos, galaxies, clusters) and the impact of primordial NG.

- The halo mass function (# of halos with a given mass) is a sensitive probe of NG.

- The formation and evolution of DM halos is a very complex phenomenon ~~and~~ → quantitative results from large-scale N-body sim
  ↳ Need to have analytical control and understanding.

EXCURSION SET THEORY

- Study the evolution of the density contrast as a function of the smoothing scale R:

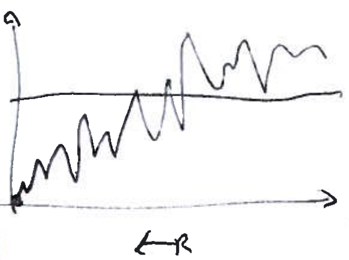
$$\delta(R) = \frac{\delta\rho}{\rho} = \int \frac{d^3k}{(2\pi)^3} \underbrace{\tilde{\delta}_{\bar{k}}}_{\text{STOCHASTIC VARIABLE}} \underbrace{\tilde{W}(\bar{k},R)}_{\text{WINDOW FUNCTION ("FILTER")}} e^{-i\bar{k}\cdot\bar{x}}$$ (at fixed position $\bar{x}=0$)

- Imagine to start at $R=\infty \rightarrow \delta=0$ and then lower R:

$$\frac{\partial\delta(R)}{\partial R} = \zeta(R) \quad \text{(stochastic noise)} \quad \text{(Langevin eq.)}$$

■ $\delta(R)$ evolves stochastically with R $\delta(S)$

$S \equiv \sigma^2 = \langle \delta^2_R \rangle$



■ $\boxed{\text{Halo formation probability is mapped into a FIRST-PASSAGE TIME PROBLEM}}$

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HALO MASS FUNCTION (# of halos with mass $M, M+dM$)

$$\frac{dn}{dM} = 2S\, \boxed{f(S)}\, \frac{\bar{\rho}}{M^2} \left| \frac{d\ln S^{1/2}}{d\ln M} \right|$$

↳ FIRST CROSSING RATE
(prob. of first crossing between $S, S+dS$)

FIRST-CROSSING RATE

# of trajectories that did not cross the barrier before S

$$F(S) = -\frac{\partial}{\partial S} \int_{-\infty}^{\delta_c} d\delta\, \boxed{\Pi(\delta, S)}$$

↳ PROB. THAT THE DENSITY CONTRAST ARRIVES AT δ IN A TIME S.

→ DETERMINATION OF $\Pi(\delta, S)$ SOLVES THE PROBLEM.

ASSUMPTIONS:

[WHITE NOISE $\frac{\partial \delta}{\partial S} = \eta(S)$
$\langle \eta(S_1)\eta(S_2)\rangle = \delta(S_1 - S_2)$

1. ~~FOR~~ $\tilde{W}_{(k)} = \Theta(k_F - k)$ (→ uncorrelated steps → Brownian random walk
→ MARKOVIAN DYNAMICS)

2. SPHERICAL COLLAPSE (δ_c)

3. GAUSSIAN FLUCTUATIONS.

$$\Rightarrow \quad FP \quad \boxed{\frac{\partial \Pi}{\partial S} = \frac{1}{2} \frac{\partial^2 \Pi}{\partial \delta^2}}, \quad \Pi(\delta_c, S) = 0$$

$$\hookrightarrow \quad F_{PS}(S) = \frac{\delta_c}{\sqrt{2\pi S^3}}\, e^{-\delta_c^2/2S}$$

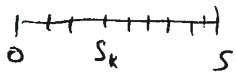
IF ANY OF THE ASSUMPTIONS ABOVE IS NOT SATISFIED
⇒ FP NON-LOCAL!! → GO TO P.I. FORMULATION!

PATH INTEGRAL APPROACH (at a more fundamental level)

- It allows to construct Π by summing over all paths that never exceed the threshold.

- Consider an "ensemble" of trajectories all starting at $\delta(0)=0$. Follow them for a time S

- Discretize S : $S_k = k\epsilon$



$\delta(S_k) = \delta_k$

- A given trajectory is defined by the set $\{\delta_1, \dots, \delta_k\}$.

Let:

$$\Pi_\epsilon(\delta_n, S_n) = \int_{-\infty}^{\delta_c} d\delta_1 \dots d\delta_{n-1} \underbrace{W(\delta_1, \dots, \delta_n, S_n)}_{\text{PROB. DENSITY IN TRAJECTORY SPACE}}$$

$$\langle \delta_D(\delta(S_1)-\delta_1) \cdots \delta_D(\delta(S_n)-\delta_n) \rangle$$

$$\| $$

$$\int \prod_{i}^{n} \frac{d\lambda_i}{2\pi} e^{i\lambda_i \delta_i} \underbrace{\langle e^{-\lambda_i \delta(S_i)} \rangle}_{\text{GENERATING FUNCTIONAL OF CONNECTED CORRELATORS.}}$$

- THE PROBLEM IS REDUCED TO THE EVALUATION OF A PATH INTEGRAL (with BOUNDARIES)

- IN THIS WAY IT WAS POSSIBLE TO COMPUTE (MR)
  - NON-MARKOVIAN CORRECTIONS
  - NG (for spherical collapse)

$\longrightarrow$ OUR PAPER: $\boxed{\text{NG WITH GENERIC BARRIER}}$

RESULTS

NOW $\boxed{\text{ELLIPS. COLLAPSE} \rightarrow B(S)}$ HAVING B

We have derived $F_G(S)$, $F_{NG}(S)$ for generic $B(S)$ (important because of ellipsoidal collapse)

for GAUSSIAN AND NON GAUSSIAN initial conditions.

$$\left\{ \Pi = \int_{-\infty}^{B(S_1)} dS_1 \cdots \int_{-\infty}^{B(S_{n-1})} dS_{n-1} \, W \right\}$$ then change variables → expand in derivatives of B

WHAT DO WE LEARN?

1. WE UNDERSTAND WHY THE EMPIRICAL ST FORMULA WORKS

2. WE PROVE THAT THE "FORM FACTOR" METHOD FOR NG FLUCTUATIONS IS INCORRECT.

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1. For Gaussian fluctuations:

$$F_{ST}(S) = \frac{e^{-B^2/2S}}{\sqrt{2\pi S^3}} \underbrace{\sum_{p=0}^{5} \frac{(-S)^p}{p!} \frac{\partial^p B}{\partial S^p}}_{p \to \infty}$$

ARTIFICIAL

NO JUSTIFICATION BUT IT FITS DATA WELL.

$p \to \infty$ ↓ $B(0)$ NO GOOD FIT.

It also reproduces the exact results for constant and linear barriers.

- Our result is rigorous and numerically agrees with ST to better than 10% → good fit as well.

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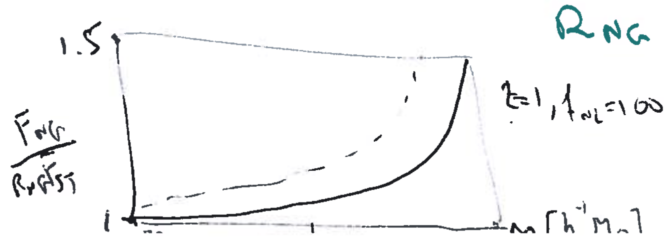
2. $$F_{NG} = F_G \underbrace{\left(\frac{F_{NG}}{F_G}\right)_{PS}}_{R_{NG}}$$ PS ← SPHERICAL COLLAPSE → NO JUSTIFICATION. (WE PROVED IT IS INCORRECT)



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