Universal Dynamics of a Soliton after a Quantum Quench

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SEVENTH FRAMEWORK PROGRAMME

MARIE CURIE ACTIONS

SISSA

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arXiv:1408.3618

Outline

- Introduction & Motivations:

  Out of equilibrium & Quantum Quenches

- Our Quench Protocol

- Hydrodynamis & KdV Reduction

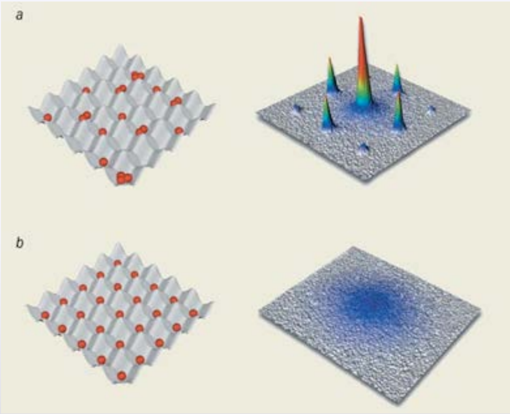
- Results: Universal splitting

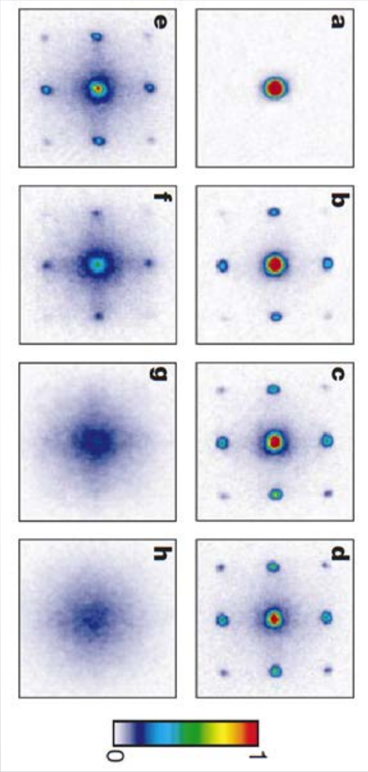
- Conclusions

Out-of-Equilibrium

- Experimental progresses challenge us with new questions:

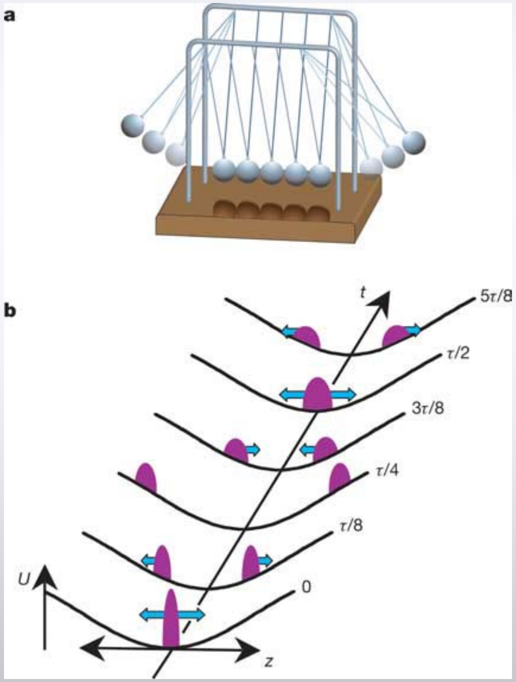
Transition from Superfluid to Mott Insulator

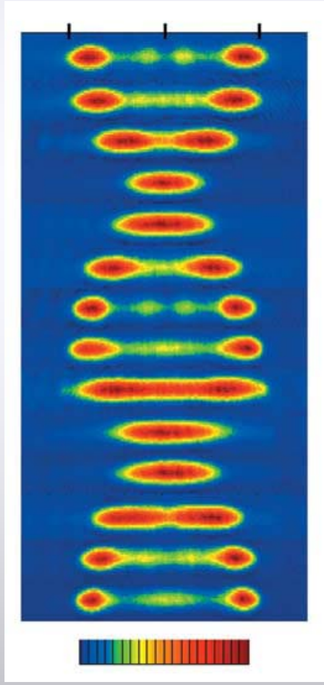




Greiner, Mandel, Esslinger, Haensch & Bloch, **Nature 415** (2002)

Quantum Newton's Cradle





Kinoshita, Wenger, & Weiss, **Nature 440** (2006)

Preaching to the Choir…

- Quest for recurring structures in out-of-equilibrium systems:

Equilibration?

- **Yes**
  - ❑ **Thermalization?**
  - ❑ **Pre-Thermalization?**
  - ❑ **GGE?**
- **No**
  - ❑ **Non-equilibrium State?**
  - ❑ **GGE?**

❑ Aging

❑ (Dynamical) Quantum Phase Transitions

❑ Work Statistics

Out-of-Equilibrium Stat. Mech.?

- Reductionist Approach (universalities?)
- Different Set-ups to be considered
- Typical protocol: Quantum Quench
  - Initial condition: ground state of local Hamiltonian
  - Evolution: different Hamiltonian
- Extended excited states also considered (free fermions)

Bucciantini, Kormos, Calabrese, JPA **47** (2014)

Quenching a Soliton

Our question:

What happens if you change the

interaction strength

in a system prepared

in a (moving) localized excitation?

Our Answer:

Short time dynamics is Universal !

Quantum Quenches

- Take a system in its Ground State $|\Psi_0\rangle$
- Let it evolve according to different Hamiltonian $H \neq H_0$
- Unitary evolution: $|\Psi(t)\rangle = \sum_j \langle j|\Psi_0\rangle \, e^{iE_j t} \, |j\rangle$

$$H|j\rangle = E_j \, |j\rangle$$

Does the system reach a stationary state, in some sense?

Gibbs Ensemble

$$|\Psi(t)\rangle = \sum_j c_j \, e^{iE_j t} \, |j\rangle$$

- Restricted to local observables, most quantum quenches result in an effective stationary mixed state

$$\lim_{t\to\infty} \langle\Psi(t)|\mathcal{O}|\Psi(t)\rangle = \mathrm{Tr}\left[\rho_{\text{eff.}}\, \mathcal{O}\right]$$

- Moreover, generally: $\rho_{\text{eff}} = \frac{e^{-\beta_{\text{eff}} H}}{\mathcal{Z}}$

(Gibbs distribution consequence of Eigenstate Thermalization Hypothesis)

Deutsch, PRA **43** (1991); Srednicki, PRE **50** (1994); Rigol, Dunjko, & Olshanii, Nature **452** (2009)

Generalized Gibbs Ensemble

- If system has local conservation laws (f.i. integrabilty), these should be included → **G.G.E.**

$$\lim_{t\to\infty} \langle \Psi(t) | \mathcal{O} | \Psi(t) \rangle = \mathrm{Tr}\left[\rho_{\mathrm{eff.}}\, \mathcal{O}\right]$$

$$\rho_{\mathrm{eff}} = \frac{e^{-\sum_l \beta_l I_l}}{\mathcal{Z}}$$

- Open problem: find all local charges

Countless efforts from

- SISSA (Mussardo, Silva, Gambassi & collaborators);
- Pisa (Calabrese & collaborators);
- Oxford (Cardy, Essler & collaborators);
- Amsterdam (Caux & collaborators);
- Many more (Polkovnikov, Mitra, Kehrein, Andrei, Prosen)…

Unitary Dynamics

- Quantum dynamics → Unitary Evolution
- A pure sate evolve into a pure state
- However, locally, the asymptotic state can be approximated by a mixed one:

$$\lim_{t\to\infty} \langle \Psi(t)|\mathcal{O}|\Psi(t)\rangle = \mathrm{Tr}\left[\rho_{\text{eff.}}\,\mathcal{O}\right]$$

- Out-of-equilibrium quantum systems act as their own bath
- Locality allows transition from quantum to classical

Our Protocol

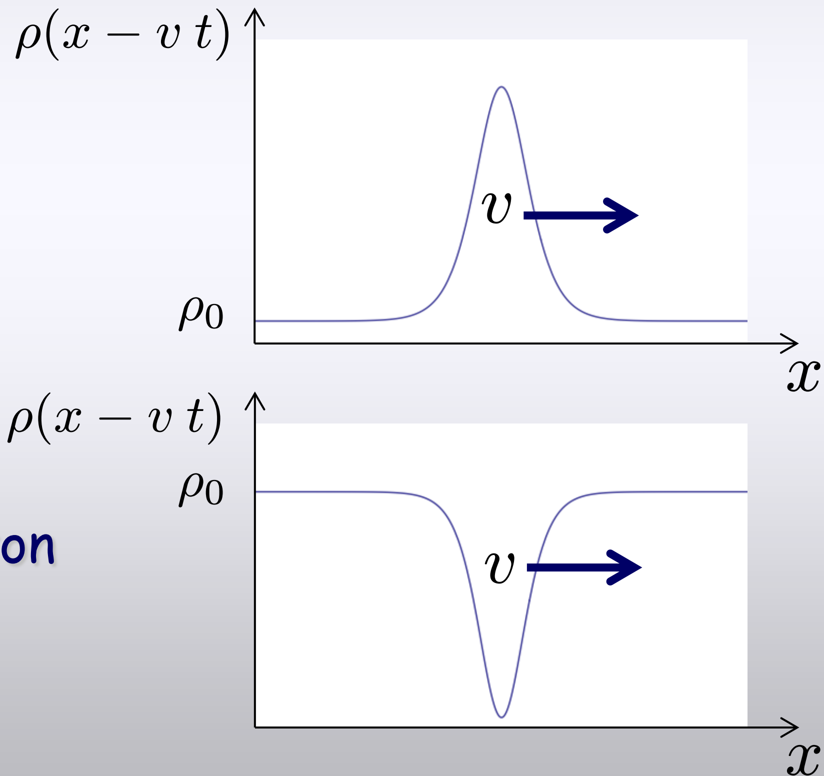
- Instead of a ground state, let's start with a localized excited state in interacting system
- Let it evolve with a different Hamiltonian
- Previously: local quenches or extended excited states (in free systems)
- Universality emerges for short times!

Our Set-Up

- Consider a cold-atom system
- Prepare a single solitonic state

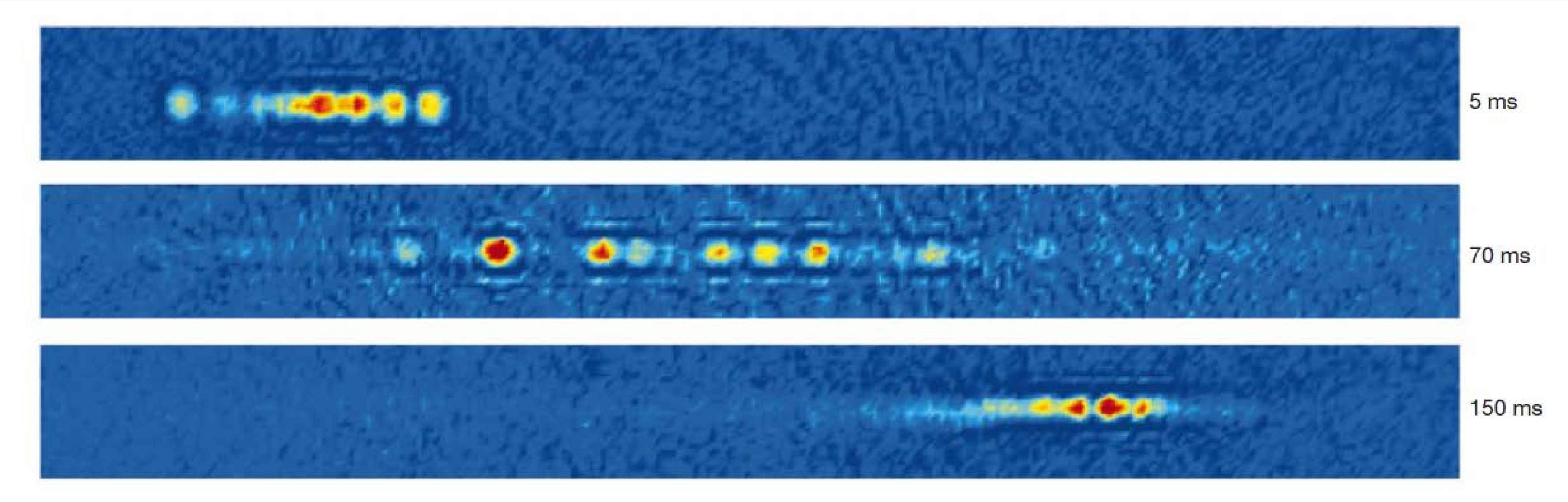
Bright Soliton
(v > c)

Dark (gray) Soliton
(v < c)



Solitons?

- A localized excitation cannot be eigenstate of translational invariant Hamiltonian
- Nonetheless, long-lived localized excitations are observed in cold atom systems:

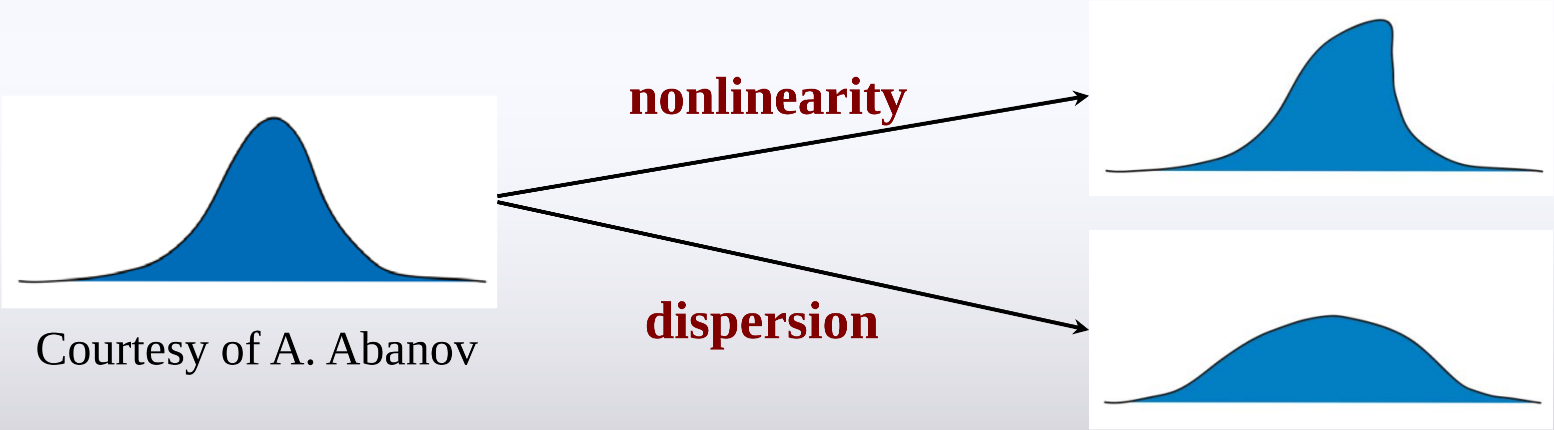


Strecker, Partridge, Truscott, & Hulet, Nature **417** (2002)

Solitons

- Soliton: "Localized excitation that propagates at constant velocity while maintaining its shape"
- Stable solutions of certain PDE

  → balancing of dispersive and non-linear terms



- Multi-soliton solutions exist only for integrable systems
- Solitonic states are ubiquitous

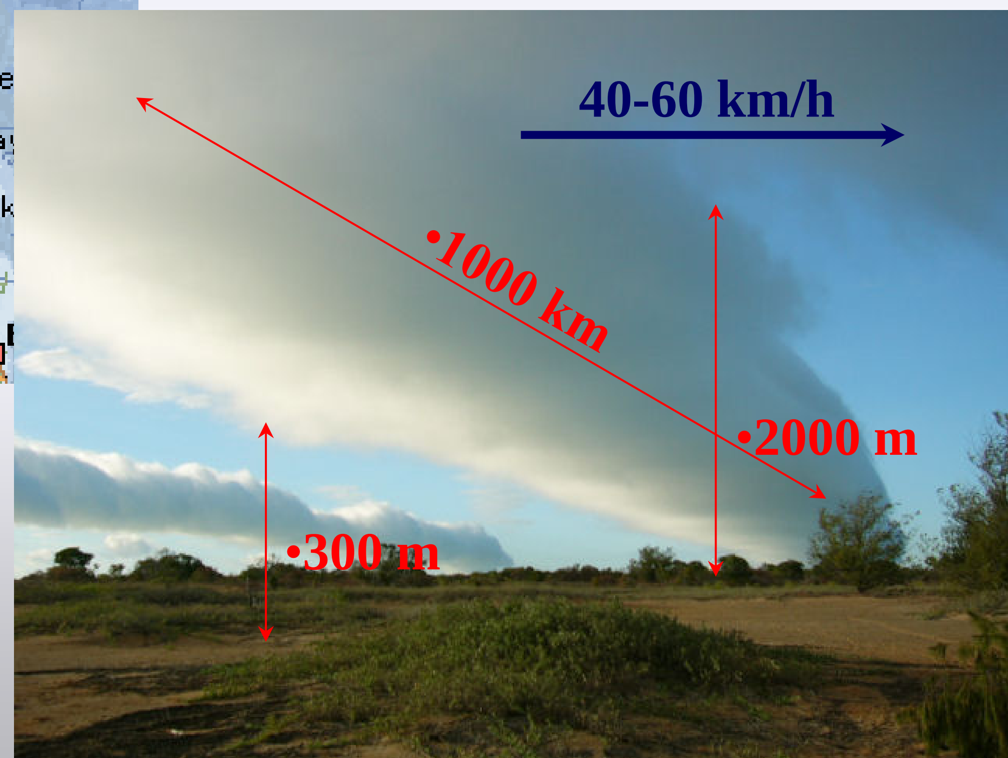
Soliton on Scott Russell Aqueduct

Dugald Duncan/Heriot-Watt University, Edinburgh

https://www.youtube.com/watch?v=SknvLa8qEu0

Morning Glory

Rolling Clouds in
the Gulf of Carpentaria,
Northern Australia

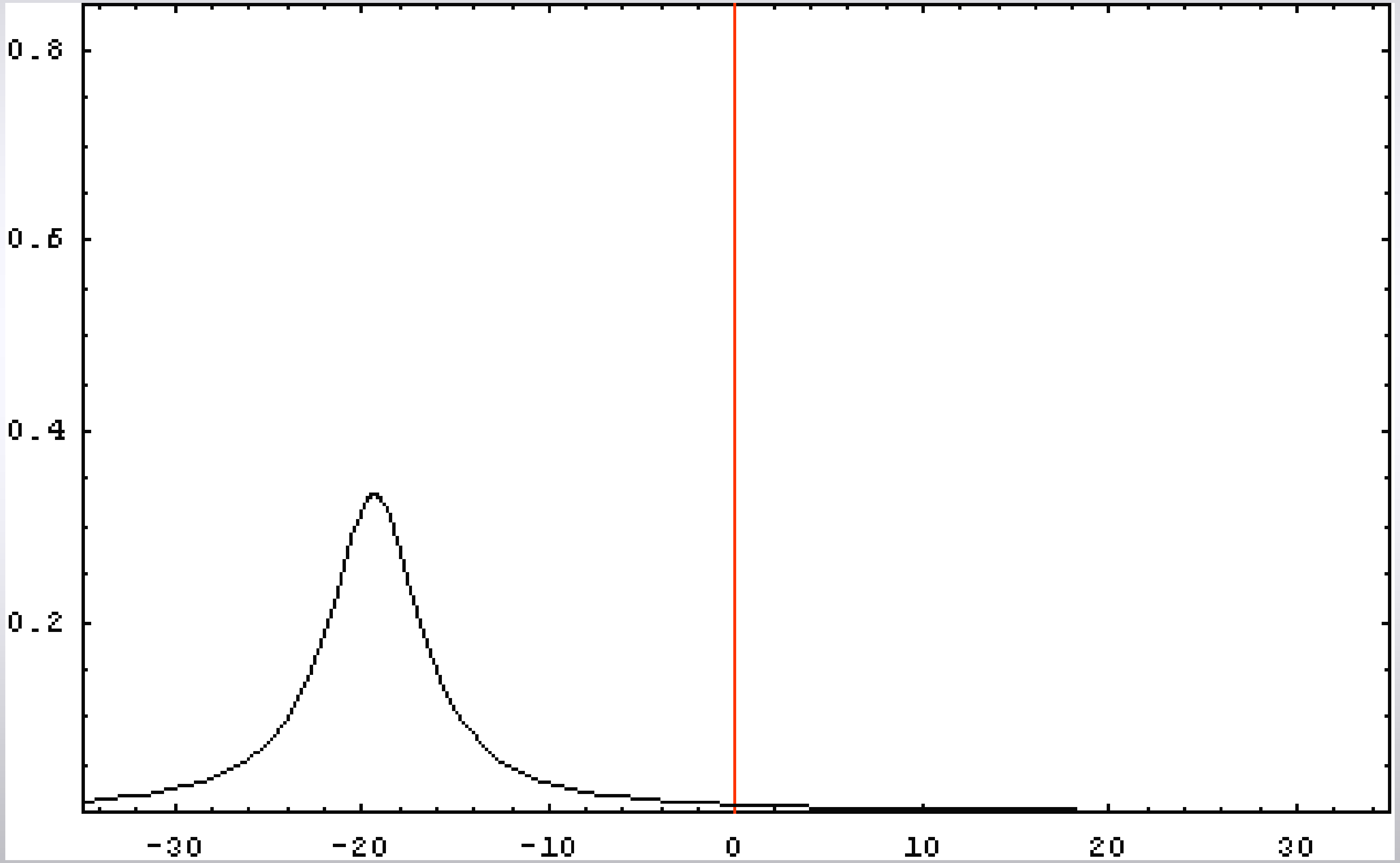


Morning Glory: solitons

Mick Petroff - Wikimedia Commons

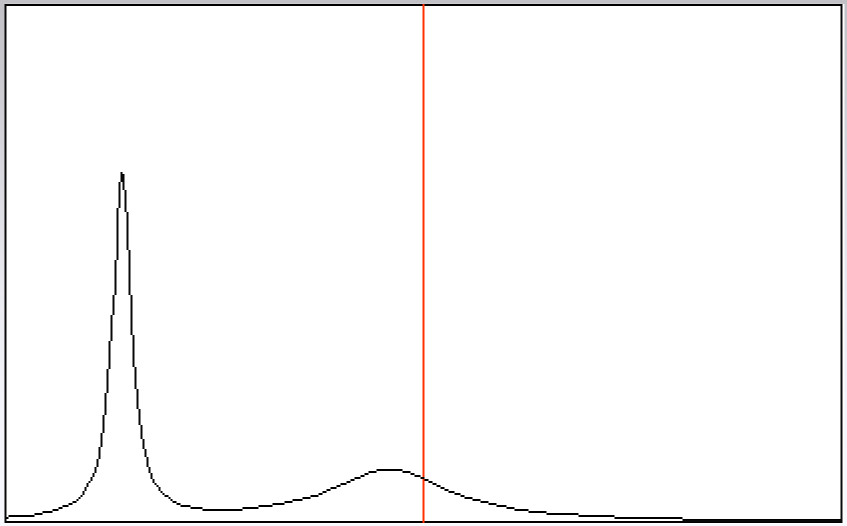
Soliton Dynamics

- Soliton-like solutions evolve without deformation

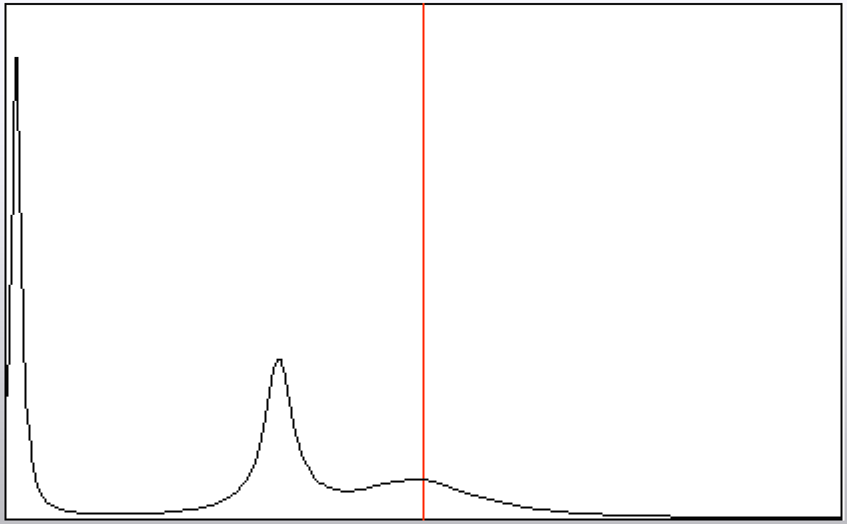


Courtesy of A. Abanov

Soliton Dynamics



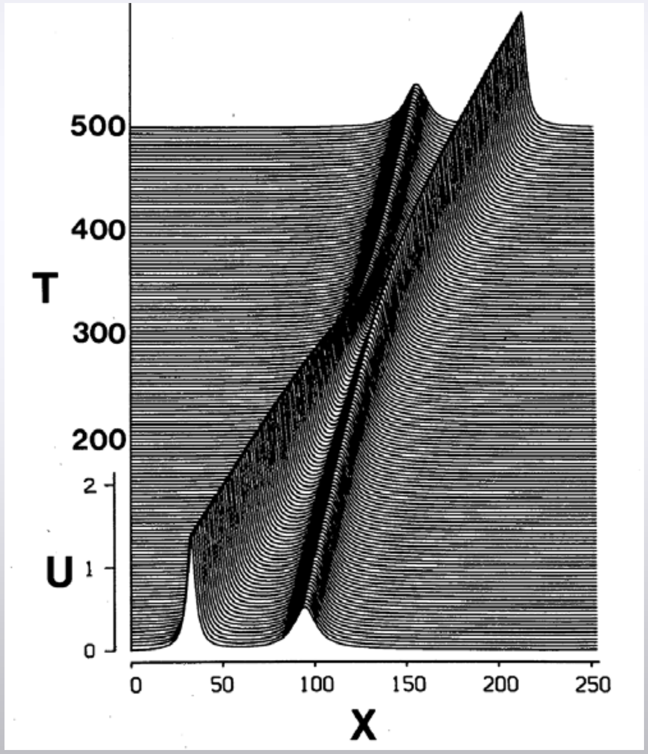
Courtesy of A. Abanov



Courtesy of A. Abanov

Multi-solitons exist only

in integrable dynamics



D.R. Christie (1988)

Solitons in cold atoms

- Generated from ground state applying phase mask
- It is not clear how to describe them as quantum states
  → Probably some sort of coherent state for interacting systems
- Emerge naturally from semi-classical hydrodyamic description
  → Low-entanglement excitations!

Solitons in cold atoms

VOLUME 83, NUMBER 25 PHYSICAL REVIEW LETTERS 20 DECEMBER 1999

Dark Solitons in Bose-Einstein Condensates

S. Burger, K. Bongs, S. Dettmer, W. Ertmer, and K. Sengstock
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[3]*Russian Research Center Kurchatov Institute, Kurchatov Square, 123182 Moscow, Russia*
(Received 27 October 1999)

Formation of a Matter-Wave Bright Soliton

L. Khaykovich,[1] F. Schreck,[1] G. Ferrari,[1,2] T. Bourdel,[1] J. Cubizolles,[1] L. D. Carr,[1] Y. Castin,[1] C. Salomon[1]*

17 MAY 2002 VOL 296 SCIENCE www.sciencemag.org

Heavy solitons in a fermionic superfluid

Tarik Yefsah[1], Ariel T. Sommer[1], Mark J. H. Ku[1], Lawrence W. Cheuk[1], Wenjie Ji[1], Waseem S. Bakr[1] & Martin W. Zwierlein[1]

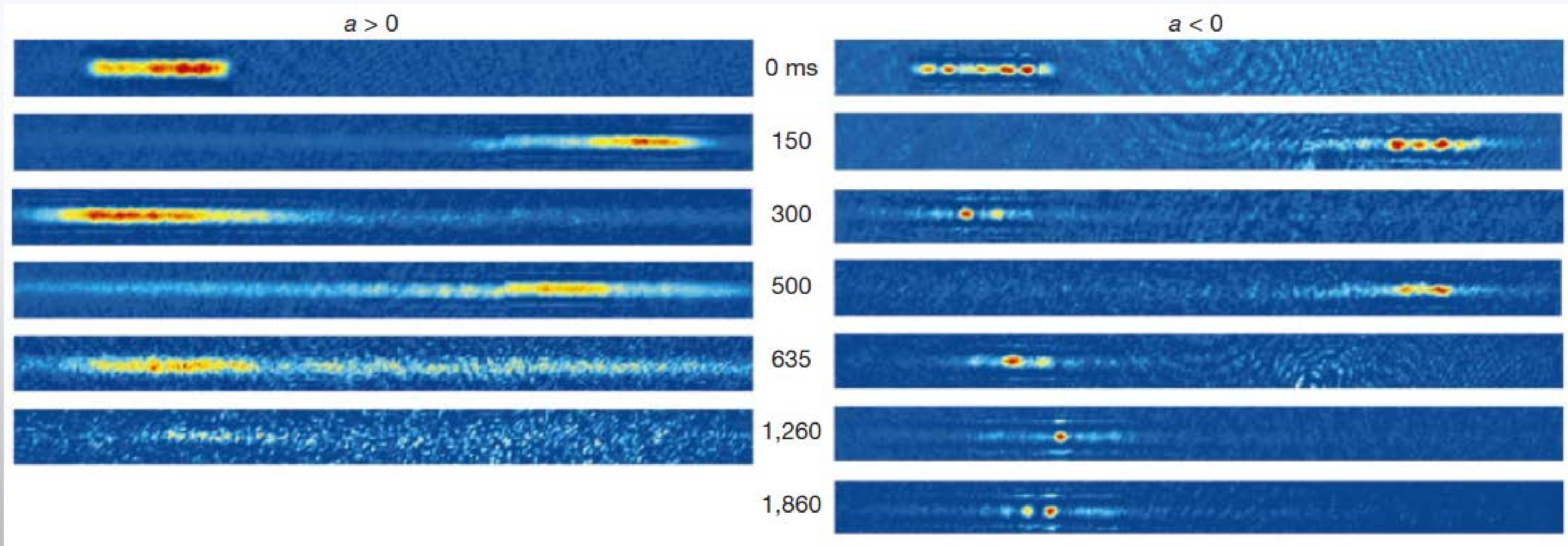
426 | NATURE | VOL 499 | 25 JULY 2013

Solitons in cold atoms

Formation and propagation of matter-wave soliton trains

Kevin E. Strecker*, Guthrie B. Partridge*, Andrew G. Truscott*† & Randall G. Hulet*

NATURE | VOL 417 | 9 MAY 2002 | www.nature.com



Hydrodynamic Approach

- Existence of solitons (and many more experimental probes) indicates the validity of hydrodynamic description for cold atoms (f.i. Gross-Pitaevskii Eq.)
- Semi-classical description: only density & velocity
  → single-body reduced
- Valid for superfluids, weakly interacting systems…
  → low entanglement states

Our proposal

1. Excite a solitonic state & let it evolve
2. At some point, change interaction strength of underlying quantum Hamiltonian (change scattering length, sound velocity…)
3. Follow evolution immediately after the quench

- Use effective (semi-classical) hydrodynamics, not unitary evolution

Hydrodynamcis

- We consider a one-component, Galilean invariant, isentropic, inviscid fluid:

$$H = \int dx \left[\frac{\rho v^2}{2} + \rho \epsilon(\rho) + A(\rho) \frac{(\partial_x \rho)^2}{4\rho} \right]$$

$$\dot{\rho} + \partial(\rho v) = 0$$ Continuity

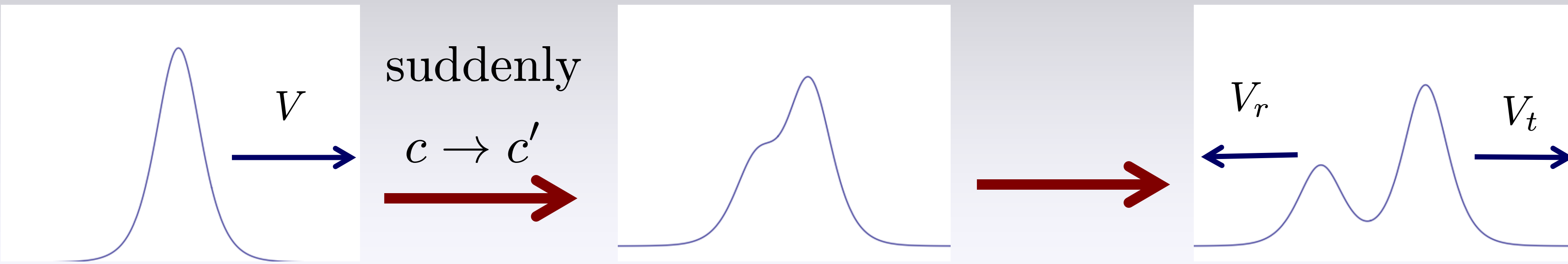
$$\dot{v} + \partial \left(\frac{v^2}{2} + \omega(\rho) - A'(\rho)(\partial\sqrt{\rho})^2 - A(\rho) \frac{\partial^2 \sqrt{\rho}}{\sqrt{\rho}} \right) = 0$$ Euler

Enthalpy: $\omega = \partial_\rho \left[\rho \epsilon(\rho) \right]$

Quantum pressure

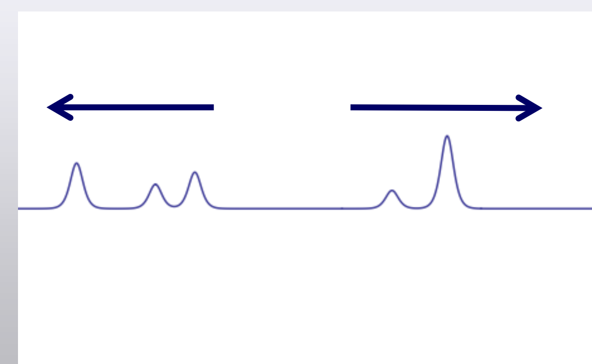
What to expect

- Short times (qualitatively like wave equation):



- Quench acts as external perturbation: soliton splits into transmitted and reflected component

- Longer Times: different scenarios (soliton trains + dispersive waves vs. dissipation)



Linearization

$$H = \int dx \left[\frac{\rho v^2}{2} + \rho\epsilon(\rho) + A(\rho)\frac{(\partial_x \rho)^2}{4\rho} \right]$$

$$\dot\rho + \partial(\rho v) = 0 \qquad \text{Continuity}$$

$$\dot v + \partial\left(\frac{v^2}{2} + \omega(\rho) - A'(\rho)(\partial\sqrt{\rho})^2 - A(\rho)\frac{\partial^2\sqrt{\rho}}{\sqrt{\rho}} \right) = 0 \qquad \text{Euler}$$

- Linearizing non-linear PDE: Bogolioubov modes (phonons, Luttinger Liquid…)

KdV Reduction

$$H = \int dx \left[\frac{\rho v^2}{2} + \rho \epsilon(\rho) + A(\rho) \frac{(\partial_x \rho)^2}{4\rho} \right]$$

- Non-linear behavior for small perturbations:

$$\begin{aligned} \rho(x,t) &= \rho_0 + \epsilon\, \rho^{(1)}(x,t) + \epsilon^2\, \rho^{(2)}(x,t) + \ldots \\ v(x,t) &= \epsilon\, v^{(1)}(x,t) + \epsilon^2\, v^{(2)}(x,t) + \ldots \end{aligned}$$

- Particular scaling of density, velocity, space & time

$\rightarrow$ Korteweg-de Vries equation (KdV)

(non-linear fixed point for local interactions)

Kulkarni & Abanov, PRA **86** (2012)

- KdV: wave on shallow water surfaces, chiral equation

KdV Reduction

- KdV scaling:
$$\begin{aligned} \rho(x,t) &= \rho_0 + \epsilon\,\rho^{(1)}(x,t) + \epsilon^2\,\rho^{(2)}(x,t) + \dots \\ v(x,t) &= \epsilon\, v^{(1)}(x,t) + \epsilon^2\, v^{(2)}(x,t) + \dots \end{aligned}$$

$$\dot\rho + \partial(\rho v) = 0$$
Continuity

$$\dot v + \partial\left(\frac{v^2}{2} + \omega(\rho) - A'(\rho)(\partial\sqrt{\rho})^2 - A(\rho)\frac{\partial^2\sqrt{\rho}}{\sqrt{\rho}}\right) = 0$$
Euler

$$u_\pm = \rho^{(1)} = \frac{c}{\omega_0'}\, v^{(1)}$$

Kulkarni, & Abanov, PRA **86** (2012)

$$\dot u_\pm \mp \partial_x\left[c u_\pm + \frac{\zeta}{2}u_\pm^2 - \alpha\partial_x^2 u_\pm\right] = 0$$

$$c \equiv \sqrt{\rho_0\omega_0'}$$

$$\zeta \equiv \frac{c}{\rho_0} + \frac{\partial c}{\partial \rho_0}$$

$$\alpha \equiv \frac{A(\rho_0)}{4c}$$

Example: Lieb-Liniger <=> NLSE

- Lieb-Liniger: $H_{\mathrm{micro}}(c) = -\sum_{j=1}^{N} \frac{\partial^2}{\partial x_j^2} + 2g \sum_{j<l} \delta(x_j - x_l)$

- For weak interaction $\gamma \equiv \frac{m}{\hbar^2} \frac{g}{\rho_0} \ll 1$ collective description by Non-Linear Schrödinger Equation

$$H(c) = \int dx \left[\frac{\hbar^2}{2m} \left|\partial_x \Psi\right|^2 + \frac{g}{2} |\Psi|^4 \right]$$

- Reduce to canonical hydrodynamic form with ansatz

$$\Psi = \sqrt{\rho}\, e^{i \frac{m}{\hbar} \int^x v(x') dx'}$$

Example: NLSE

$$i\hbar\partial_t \Psi(x,t) = \left\{ -\frac{\hbar^2}{2m}\partial_{xx} + c\left|\psi(x,t)\right|^2 \right\} \psi(x,t)$$

$$\Psi = \sqrt{\rho}\, e^{i\frac{m}{\hbar}\int^x v(x')dx'}$$

$$\omega(\rho) = \frac{g}{m}\rho$$

$$A = \frac{\hbar^2}{2m^2}$$

$$\dot{\rho} + \partial(\rho v) = 0$$

$$\dot{v} + \partial\left(\frac{v^2}{2} + \omega(\rho) - A'(\rho)(\partial\sqrt{\rho})^2 - A(\rho)\frac{\partial^2\sqrt{\rho}}{\sqrt{\rho}}\right) = 0$$

KdV Reduction: $\delta\rho(x,t) = \rho_0 - \rho(x,t) \ll \rho_0$

$$\dot{u}_\pm \mp \partial_x\left[c u_\pm + \frac{\zeta}{2}u_\pm^2 - \alpha\partial_x^2 u_\pm\right] = 0\,, \qquad u_\pm = \delta\rho = \frac{c}{\omega_0'}\delta v$$

$$c = \sqrt{\frac{g\rho_0}{m}}\,, \zeta = \frac{3}{2}\frac{c}{\rho_0}\,, \alpha = \frac{\hbar^2}{8m^2\, c}$$

Our approach

1. Excite a shallow solitonic state & let it evolve

   → Dynamic described by KdV

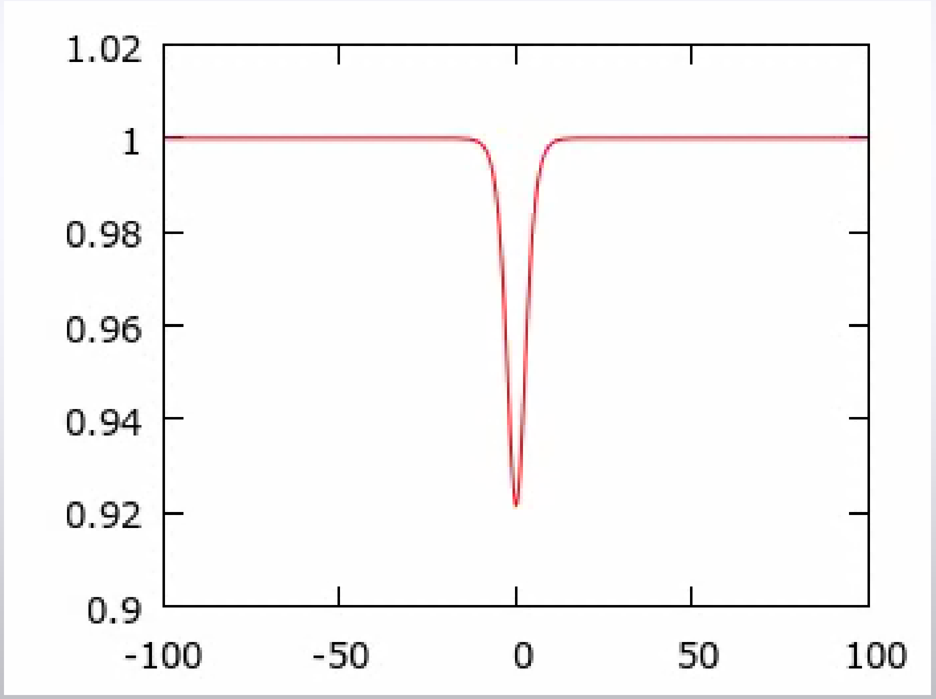
2. Change interaction strength of underlying quantum Hamiltonian

3. Describe post quench dynamics by new Kdv, with parameters modified by quench

- Universality for short time from KdV!

Quench

- Quantum Quench: $H_0 = H(c_0) \to H = H(c)$
- Initial KdV Soliton: $s(x,t) = -U \cosh^{-2}\left[\frac{x \pm Vt}{W}\right]$



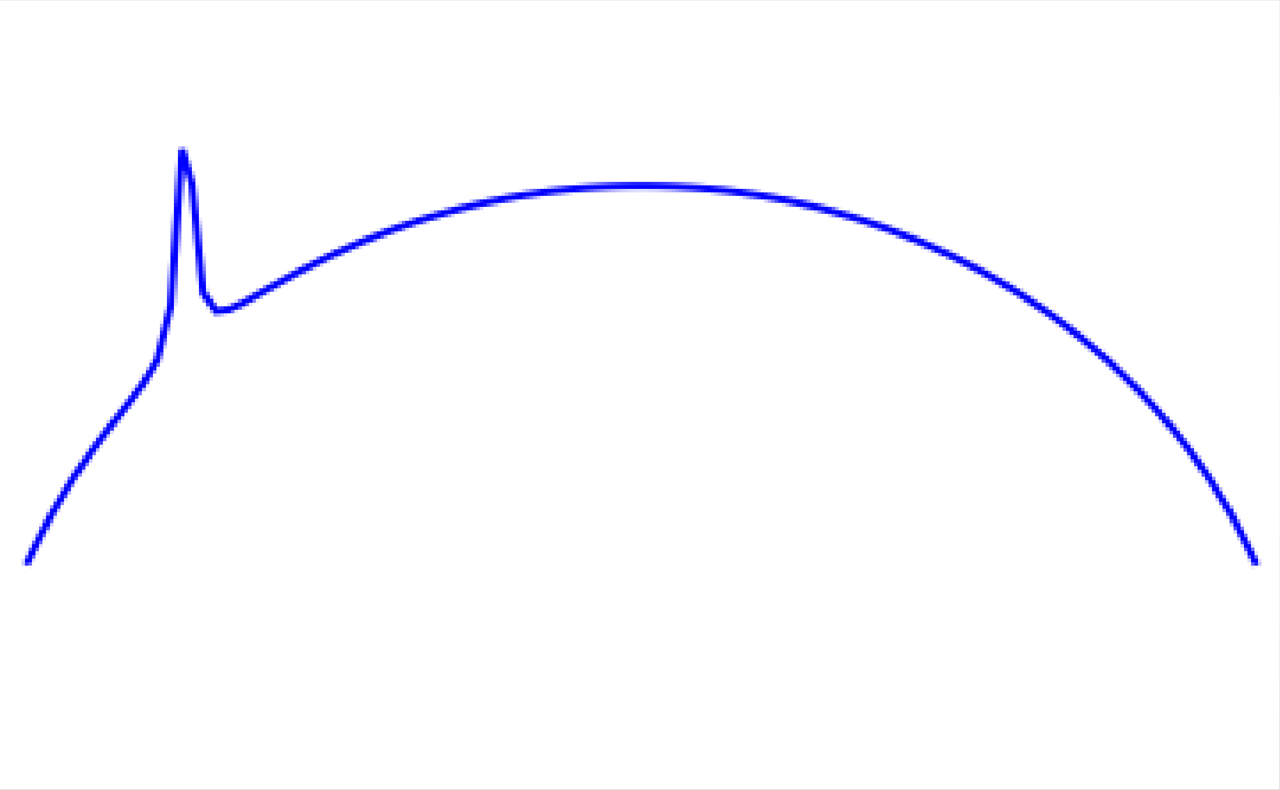
$$W = 2\sqrt{\frac{\alpha}{c-V}}$$

$$U = 3\,\frac{c-V}{\zeta}$$

NLSE: Gray Soliton

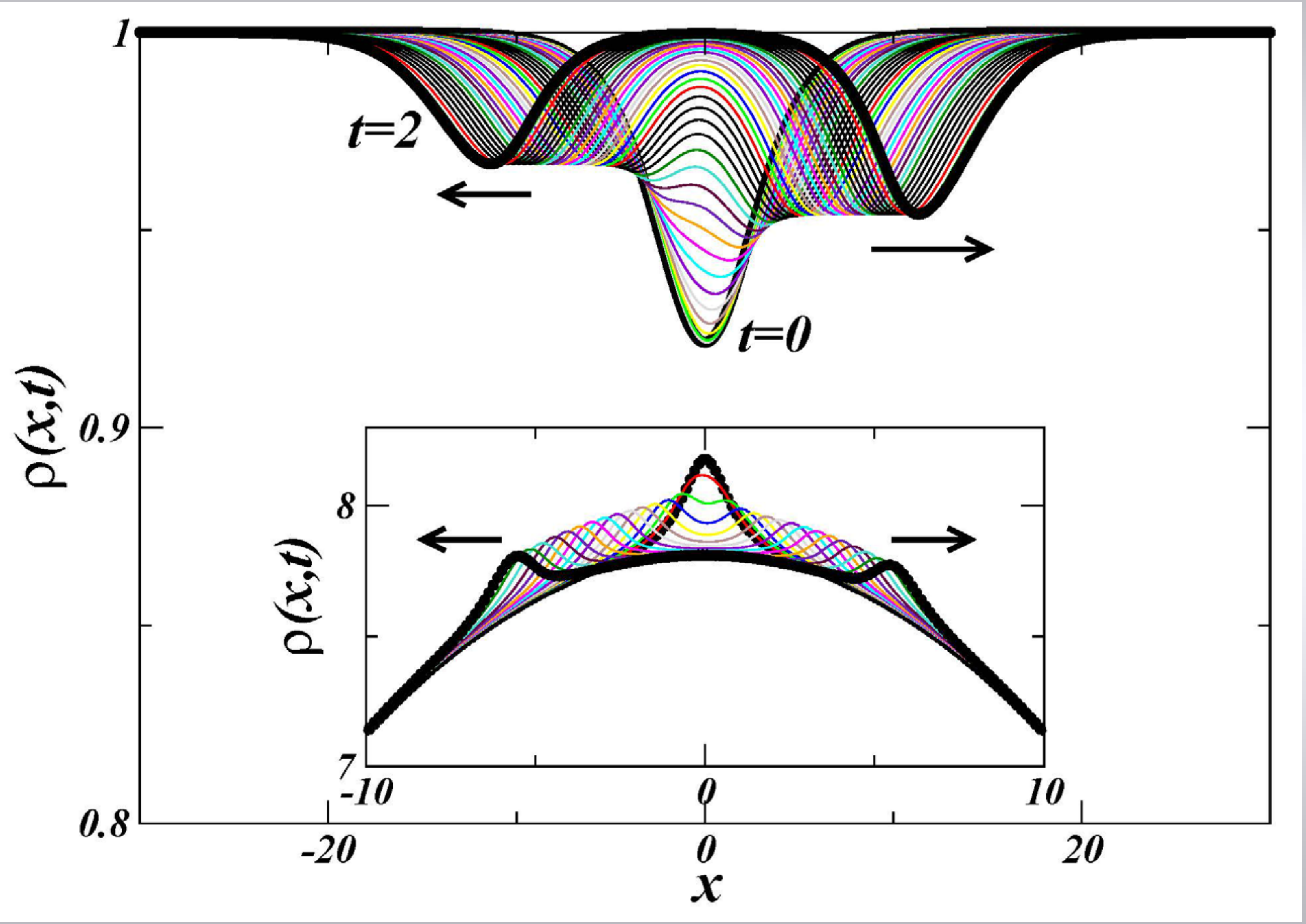
Quench

- Quantum Quench: $H_0 = H(c_0) \to H = H(c)$



Harmonic Calogero: Bright Soliton

Soliton Splitting



Soliton Splitting

- Quench acts as external perturbation: soliton splits into transmitted and reflected components

- Continuity and momentum conservation yield

$$u(x,t) = R\, s(x - V_r t) + T\, s(x - V_t t)$$

$$R(V, V_r, V_t) = \frac{V_t - V}{V_t - V_r} \qquad T(V, V_r, V_t) = \frac{V - V_r}{V_t - V_r}$$

- Here: just kinematics. Need (KdV) dynamics to fix $V_{r,t}$

Chiral Profiles

$$u(x,t) = R\, s(x - V_r t) + T\, s(x - V_t t)$$

- Using KdV we determined

$$\eta \equiv \frac{1 + \frac{\rho_0}{c'} \frac{\partial c'}{\partial \rho_0}}{1 + \frac{\rho_0}{c} \frac{\partial c}{\partial \rho_0}}$$

$$V_r = -\left[c - \eta\, R\, (c - V)\right] \frac{c'}{c}$$

$$V_t = \left[c - \eta\, T\, (c - V)\right] \frac{c'}{c}$$

$$R = \frac{1}{2}\left[1 - \frac{c}{c'}\, \frac{V}{\eta\, V + (1-\eta)\, c}\right]$$

$$T = \frac{1}{2}\left[1 + \frac{c}{c'}\, \frac{V}{\eta\, V + (1-\eta)\, c}\right]$$

$c \propto \rho_0^\theta \Rightarrow \eta = 1$ →

$$V_r = -\left(T\, c + R\, V\right) \frac{c'}{c}$$

$$V_t = \left(R\, c + T\, V\right) \frac{c'}{c}$$

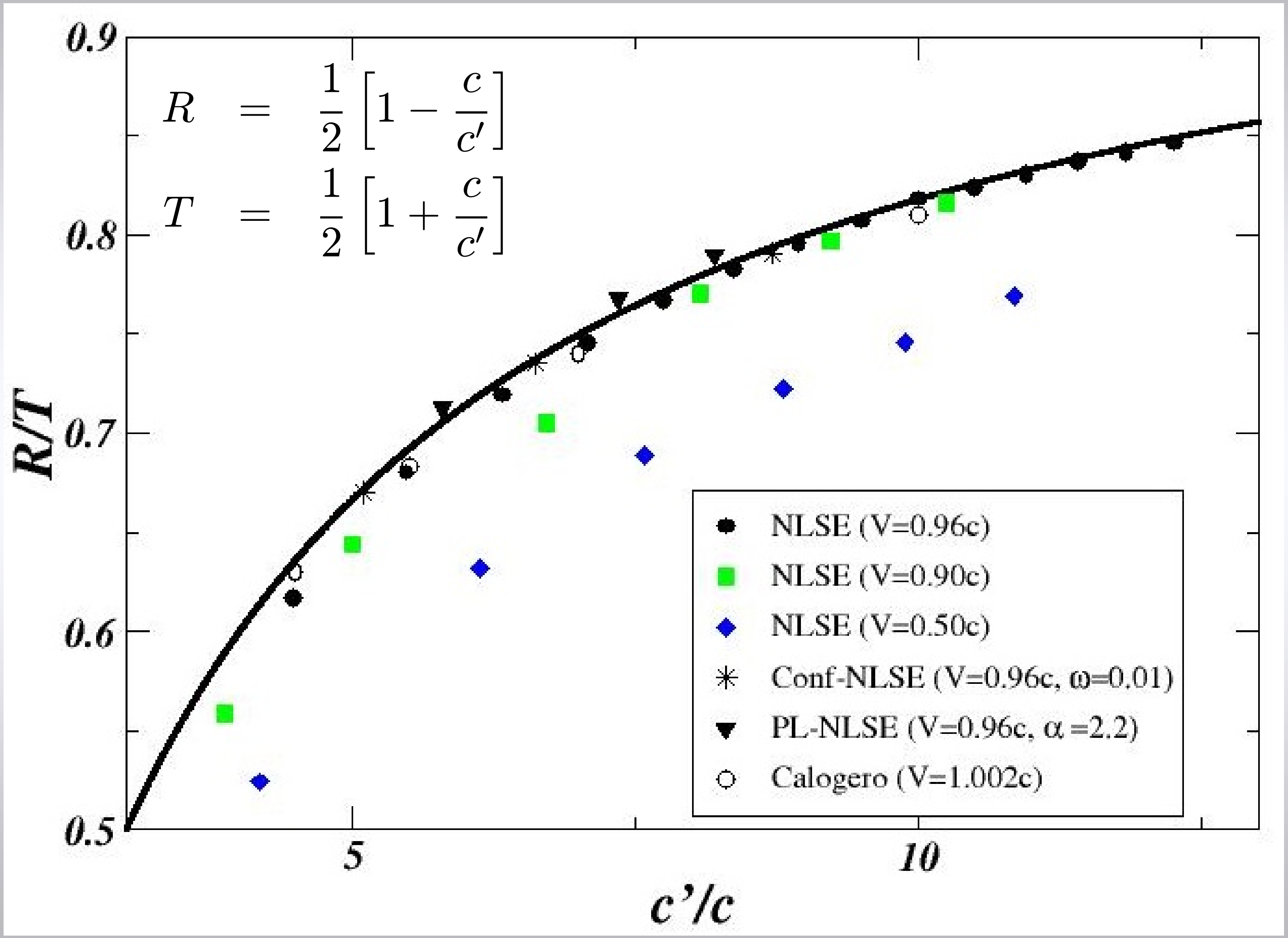
$$R = \frac{1}{2}\left[1 - \frac{c}{c'}\right]$$

$$T = \frac{1}{2}\left[1 + \frac{c}{c'}\right]$$

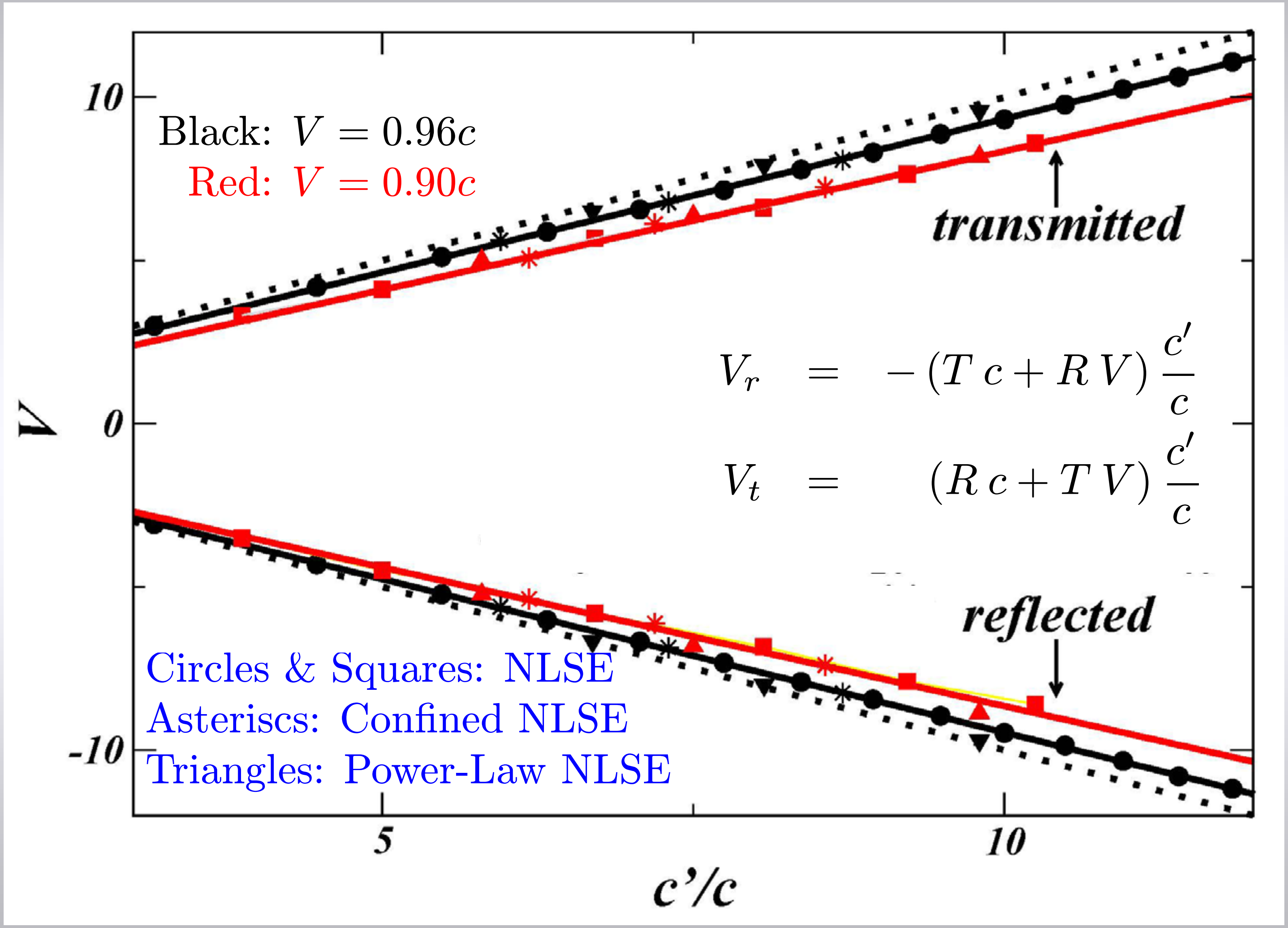
Same as linear problem, but non-trivial velocities!

- Completely UNIVERSAL !

Numerical Checks: Amplitudes



Numerical Checks: Velocities



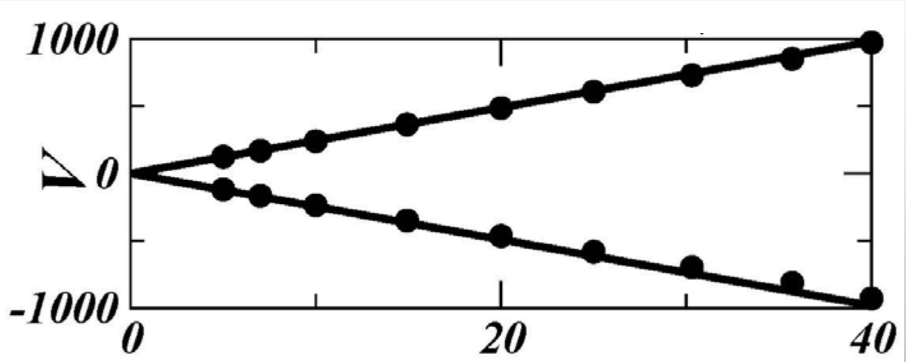
Harmonic Calogero

- Integrable in harmonic confinement!

$$H = \frac{1}{2m}\sum_{j=1}^{N} p_j^2 + \frac{\hbar^2}{2m}\sum_{j\neq k}\frac{\lambda^2}{(x_j - x_k)^2} + \omega\sum_{j=1}^{N} x_j^2 \ ,$$

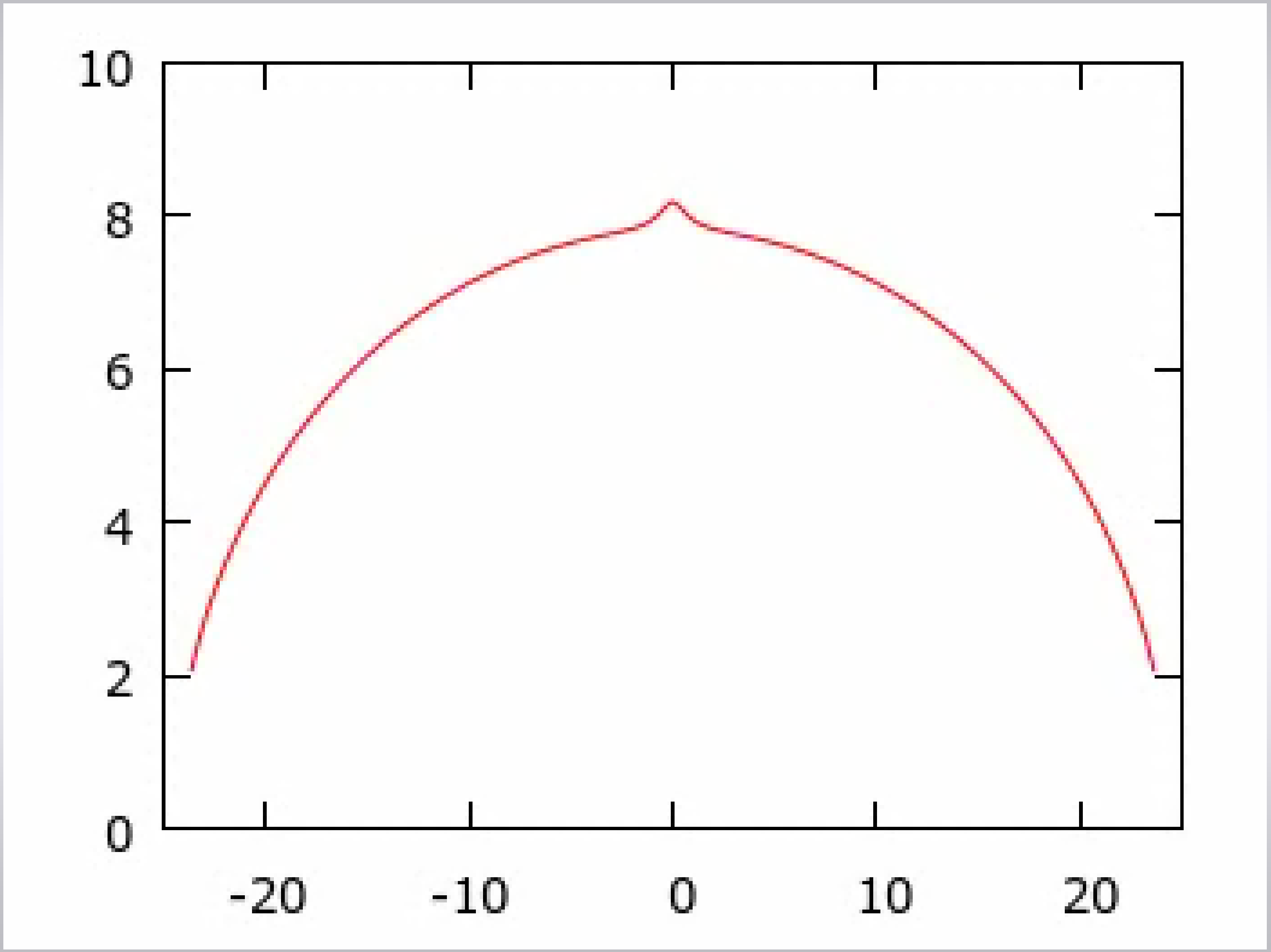
- Long(ish)-range model: hydrodynamics in Benjamin-Ono class (not KdV, different dispersion)

- Solitons have longer tails, but quench prediction still holds



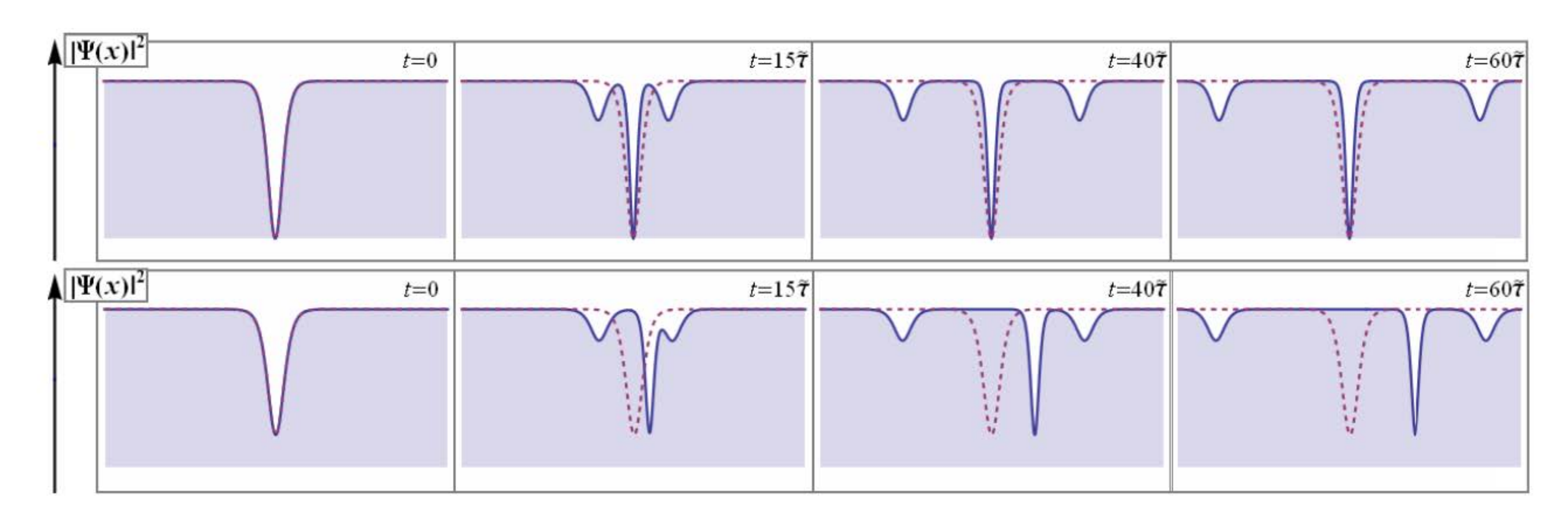
- Microscopic Classical Newtonian evolution

Harmonic Calogero



Large time asymptotics for NLSE

- Gamayun & al. considered same set-up
- Large time using integrability of NLSE (ISM)
- If $\frac{c'}{c} = \kappa$ integer $\Rightarrow$ $2\kappa - 1$ solitons (no dispersive waves)



Gamayun, Bezvershenko, and Cheianov, arXiv:1408.3312

Conclusions

- We studied a quantum quench on localized excited state using an effective semi-classical hydrodynamics
- Universal dynamics for short time after quench: predicted shape and velocities of chiral profiles
- Great agreement with numerical simulations
- Experimentally feasible!
- Open questions: quantum nature of a soliton, microscopic unitary evolution, large time behavior

Thank you!

…but wait, there is more!

Spontaneous Breaking of U(N) Symmetry in Invariant Matrix Models

Fabio Franchini

arXiv:1412.xxxx

Really done now,

Thank you!

Further Splitting