Universal Dynamics of a Localized Excitation after a global Interaction Quench

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Outline

- Introduction & Motivations:

  Out of equilibrium & Quantum Quenches

- Our Quench Protocol
- Hydrodynamics & KdV Reduction
- Results: Universal splitting

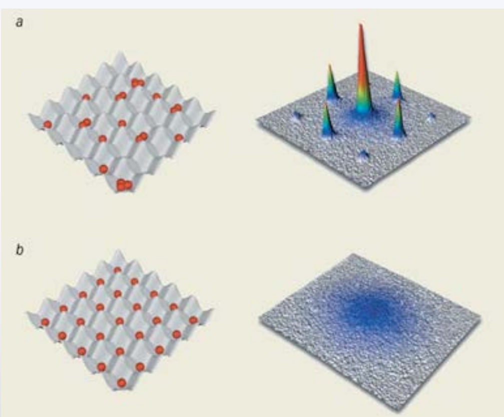
  for NLSE & Harmonic Calogero

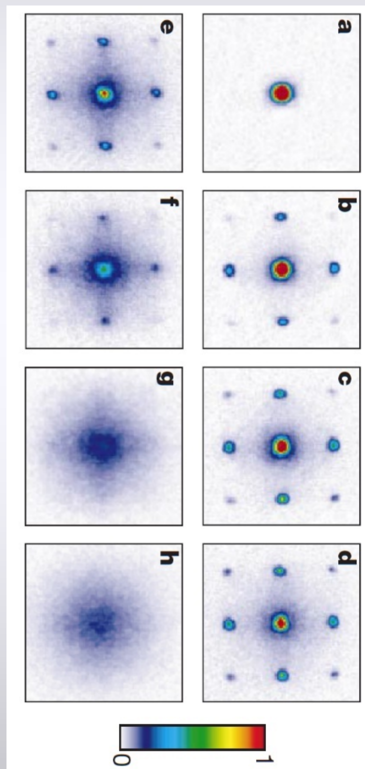
- Conclusions

Out-of-Equilibrium

- Experimental progresses challenge us with new questions:

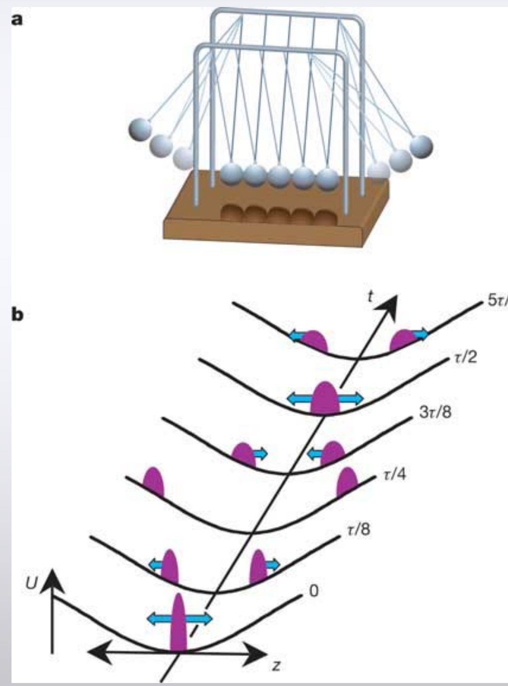
Transition from Superfluid to Mott Insulator

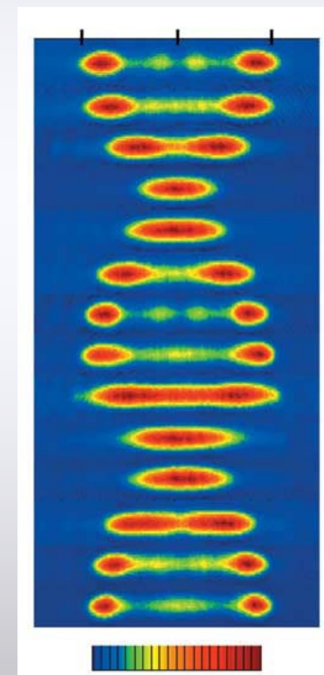




Greiner, Mandel, Esslinger, Haensch & Bloch, **Nature 415** (2002)

Quantum Newton's Cradle





Kinoshita, Wenger, & Weiss, **Nature 440** (2006)

Common questions

- Quest for recurring structures in out-of-equilibrium systems:

Equilibration?

Yes

- ❑ **Thermalization?**
- ❑ **Pre-Thermalization?**
- ❑ **GGE?**

No

- ❑ **Non-equilibrium State?**
- ❑ **GGE?**

- ❑ Aging
- ❑ (Dynamical) Quantum Phase Transitions
- ❑ Work Statistics

Out-of-Equilibrium Stat. Mech.?

- Reductionist Approach (universalities?)
- Different Set-ups to be considered
- Typical protocol: Quantum Quench
  - Initial condition: ground state of local Hamiltonian
  - Evolution: different Hamiltonian
- Extended excited states also considered (free fermions)

Bucciantini, Kormos, Calabrese, JPA **47** (2014)

Quantum Quench

- Initial condition: ground state of local Hamiltonian

  → low entanglement entropy

- Evolution: different Hamiltonian

  → entropy growth

- Late times: unitary evolution "scrambles" information

  → can describe system as effective mixed state

- Not much known for short times

Morawetz, PRB 90 (2004)
Chiocchetta et al., PRB 91 (2015)
Chiocchetta et al., PRB 94 (2016)

Unitary Dynamics

- Quantum dynamics $\rightarrow$ Unitary Evolution
- A pure sate evolve into a pure state
- However, locally, the asymptotic state can be effectively approximated by a mixed one:

$$\lim_{t\to\infty} \langle\Psi(t)|\mathcal{O}|\Psi(t)\rangle = \mathrm{Tr}\left[\rho_{\text{eff.}}\, \mathcal{O}\right]$$

- Out-of-equilibrium quantum systems act as their own bath
- Locality allows transition from quantum to classical

Quenching a Soliton

Our question:

What happens if you change the

interaction strength

in a system prepared

in a (moving) localized excitation?

Our Answer:

Short time dynamics is Universal

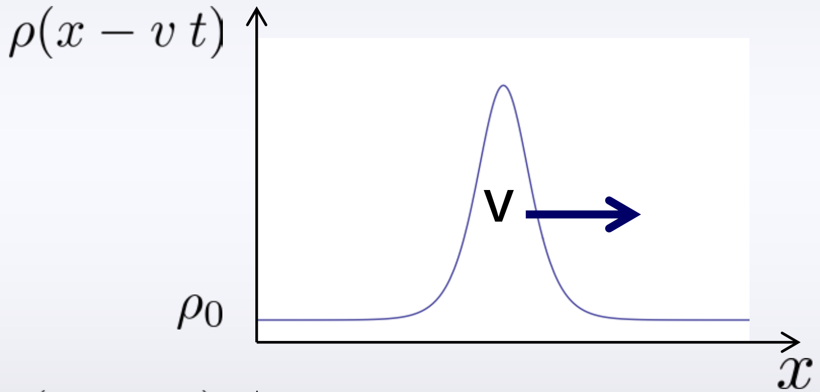
Our Protocol

- Instead of a ground state, let's start with a localized excited state in interacting system
- Let it evolve with a different Hamiltonian
- Universality emerges for short times!
- Previously: local quenches or extended excited states (in free systems) → long times

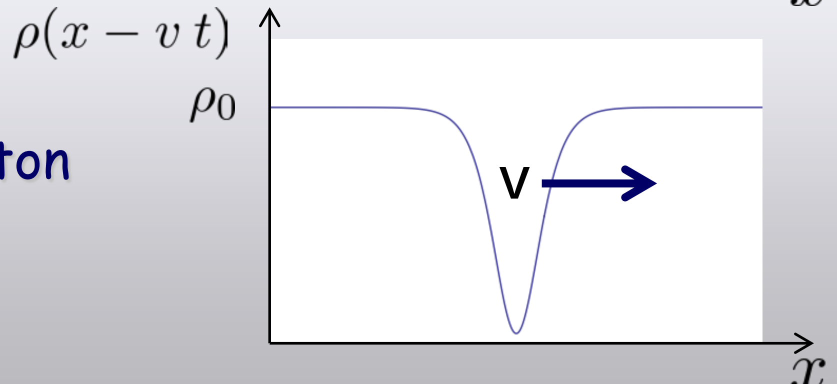
Our Set-Up

- Generally, localized excitations cannot be eigenstates of translational invariant Hamiltonians
- We consider a solitonic state (e.g. in cold atom systems)

Bright Soliton
(v > c)



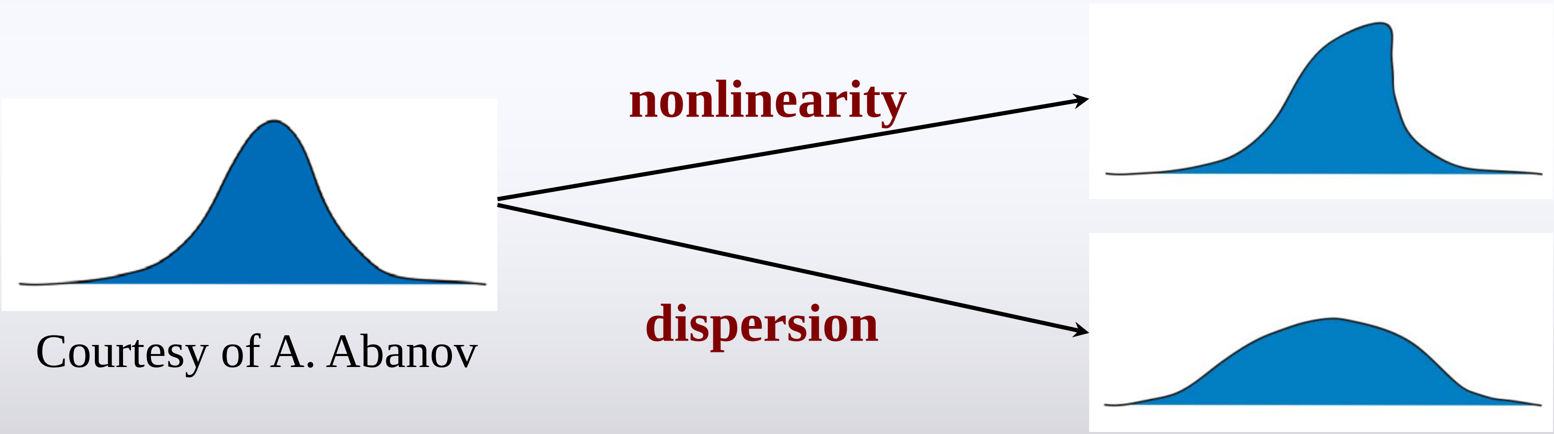
Dark (gray) Soliton
(v < c)



Solitons

- Soliton: "Localized excitation that propagates at constant velocity while maintaining its shape"
- Stable solutions of certain PDE

  → balancing of dispersive and non-linear terms



Courtesy of A. Abanov

- Multi-soliton solutions exist only for integrable systems
- Solitonic states are ubiquitous

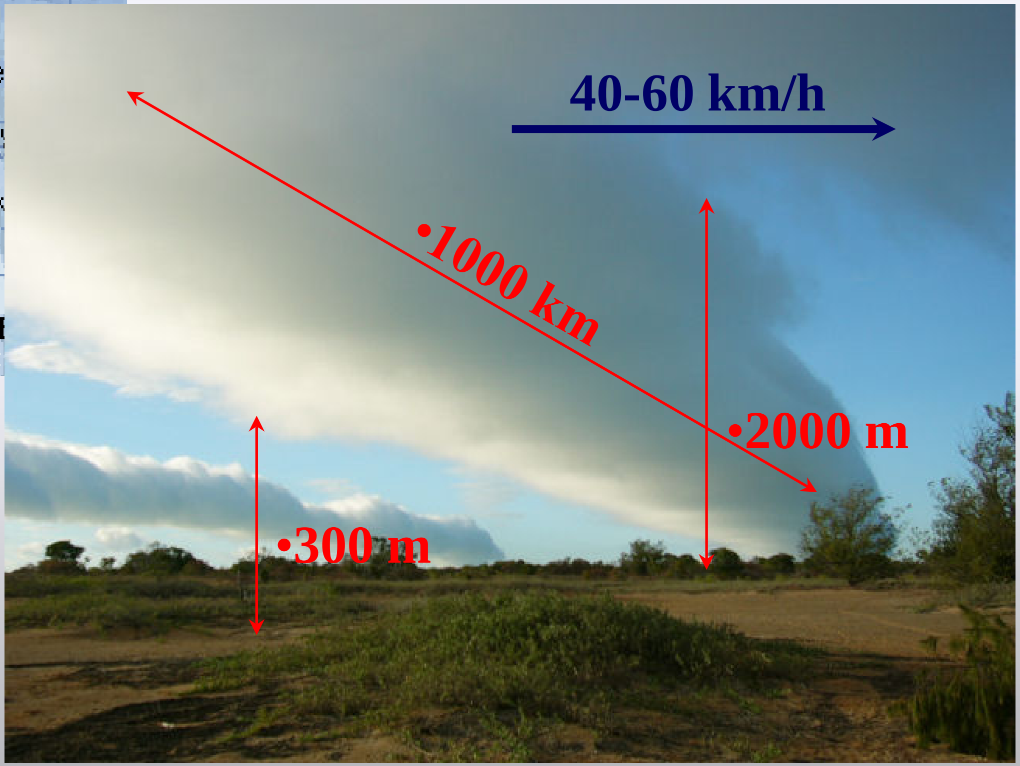
Soliton on Scott Russell Aqueduct

Dugald Duncan/Heriot-Watt University, Edinburgh

https://www.youtube.com/watch?v=SknvLa8qEu0

Morning Glory

Rolling Clouds in the Gulf of Carpentaria, Northern Australia

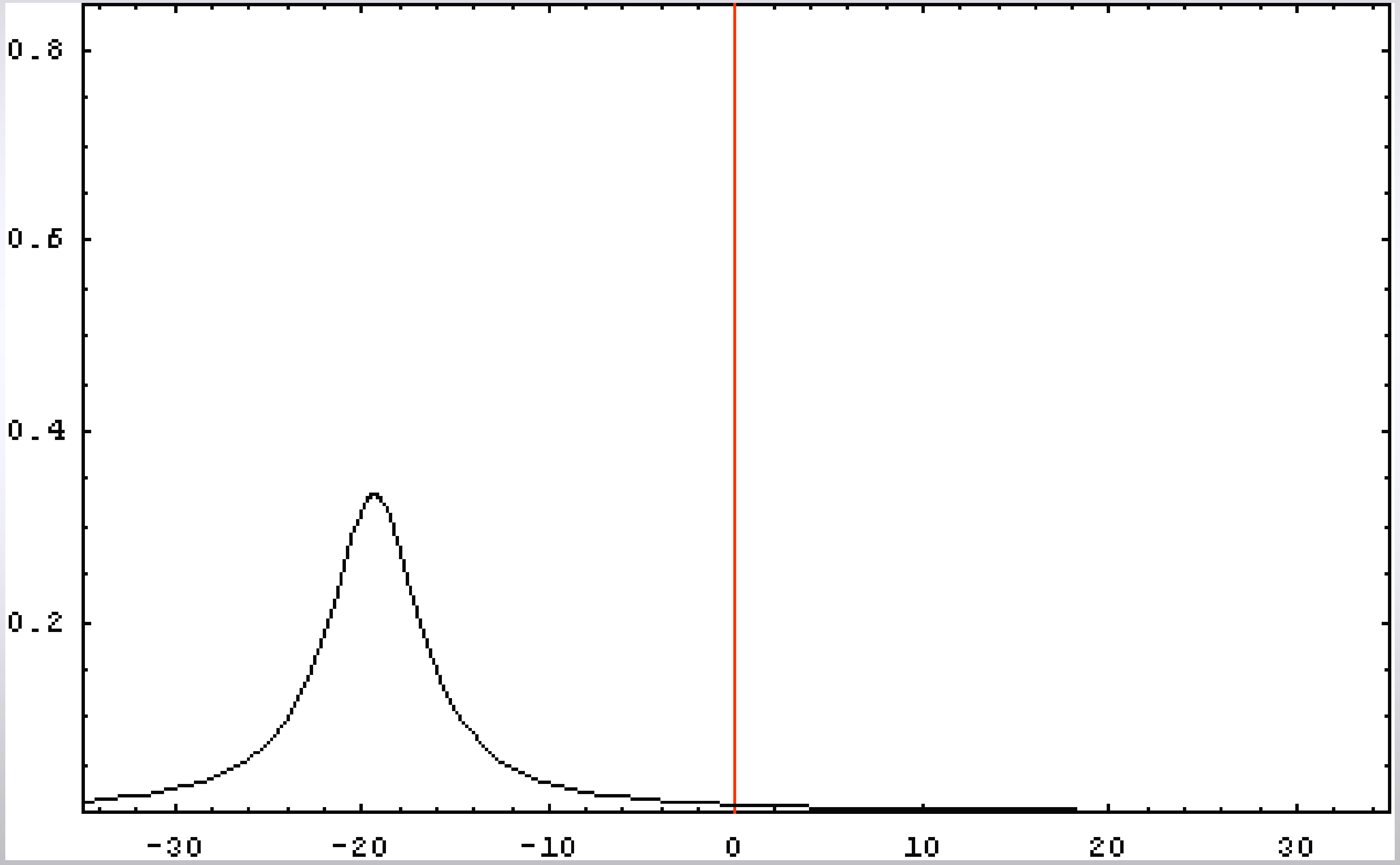


Morning Glory: solitons

Mick Petroff - Wikimedia Commons

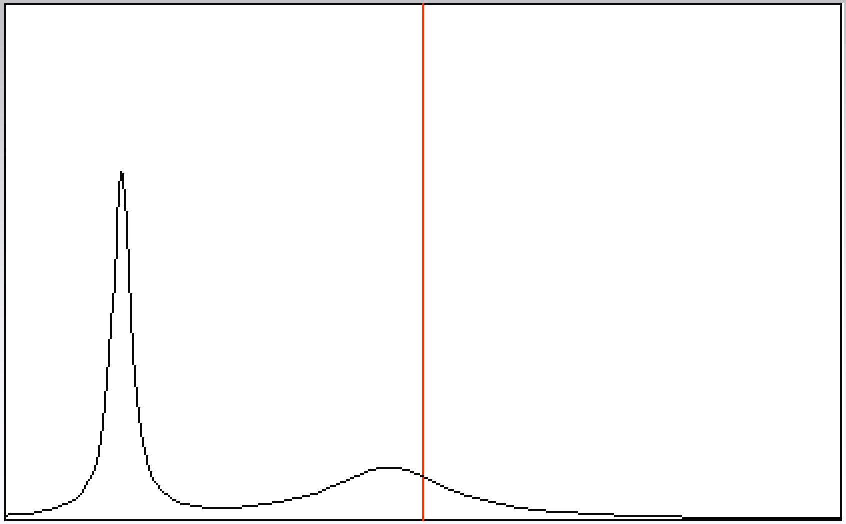
Soliton Dynamics

- Soliton-like solutions evolve without deformation

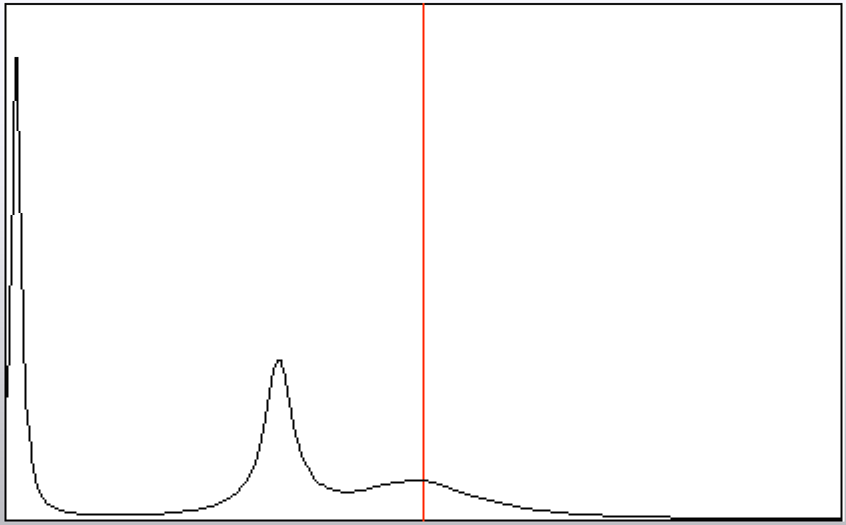


Courtesy of A. Abanov

Soliton Dynamics

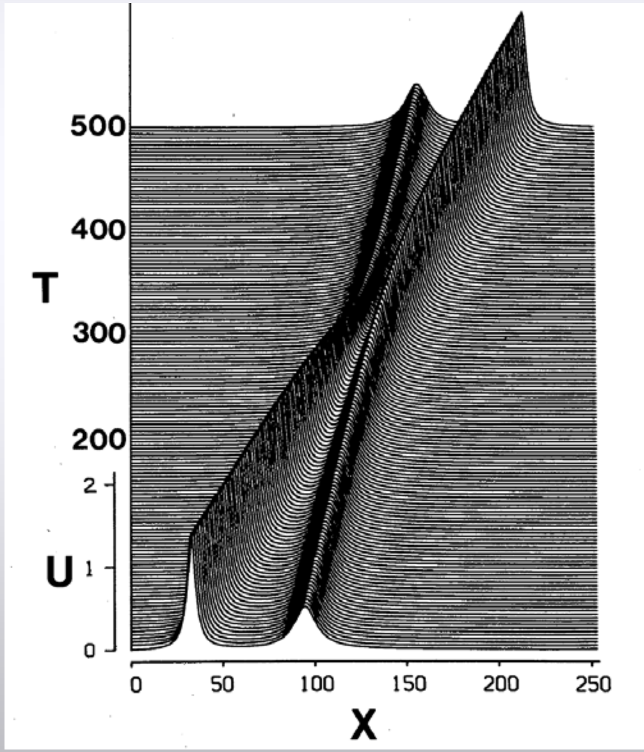


Courtesy of A. Abanov



Courtesy of A. Abanov

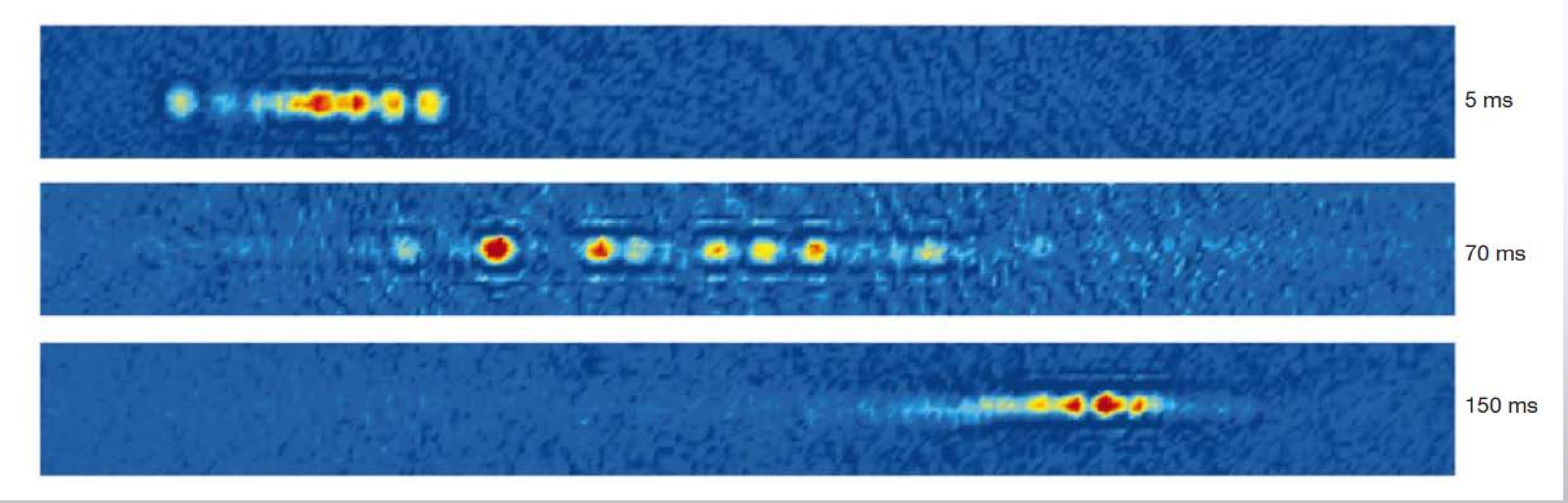
Multi-solitons exist only in integrable dynamics



D.R. Christie (1988)

Solitons in cold atomes

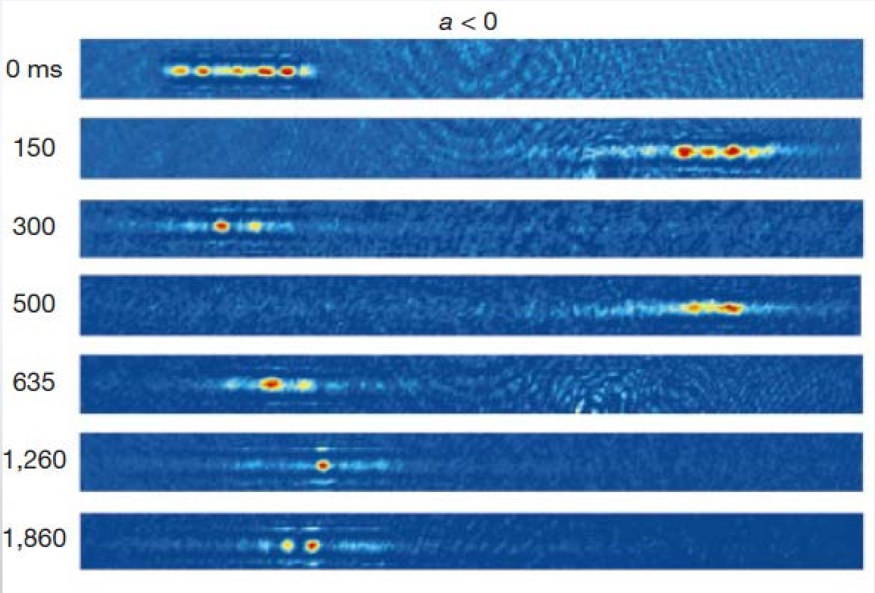
- A localized excitation cannot be eigenstate of translational invariant Hamiltonian
- Nonetheless, long-lived localized excitations are observed in cold atom systems:



Strecker, Partridge, Truscott, & Hulet, Nature **417** (2002)

Solitons in cold atoms

- Generated from ground state applying phase mask
- It is not clear how to describe them as quantum states
  - → Probably some sort of coherent state for interacting systems
- Emerge naturally in semi-classical hydrodynamic description
  - → Low-entanglement excitations!



Strecker et al., Nature 417 (2002)

Hydrodynamic Approach

- Existence of solitons (and many more experimental probes) indicates the validity of hydrodynamic description for cold atoms (f.i. Gross-Pitaevskii Eq.)
- Semi-classical description: only density & velocity
  $\rightarrow$ single-body reduced
- Valid for superfluids, weakly interacting systems…
  $\rightarrow$ low entanglement states

Our proposal

1. Excite a solitonic state & let it evolve

2. At some point, change interaction strength of underlying quantum Hamiltonian (change scattering length, sound velocity…)

3. Follow evolution immediately after the quench

- We use effective (semi-classical) hydrodynamics, not unitary evolution

Hydrodynamics

- We consider a one-component, Galilean invariant, isentropic, inviscid fluid:

$$H = \int dx \left[\frac{\rho v^2}{2} + \rho \epsilon(\rho) + A(\rho) \frac{(\partial_x \rho)^2}{4\rho} \right]$$

$$\dot{\rho} + \partial(\rho v) = 0$$

Continuity

$$\dot{v} + \partial \left(\frac{v^2}{2} + \omega(\rho) - A'(\rho)(\partial\sqrt{\rho})^2 - A(\rho) \frac{\partial^2 \sqrt{\rho}}{\sqrt{\rho}} \right) = 0$$

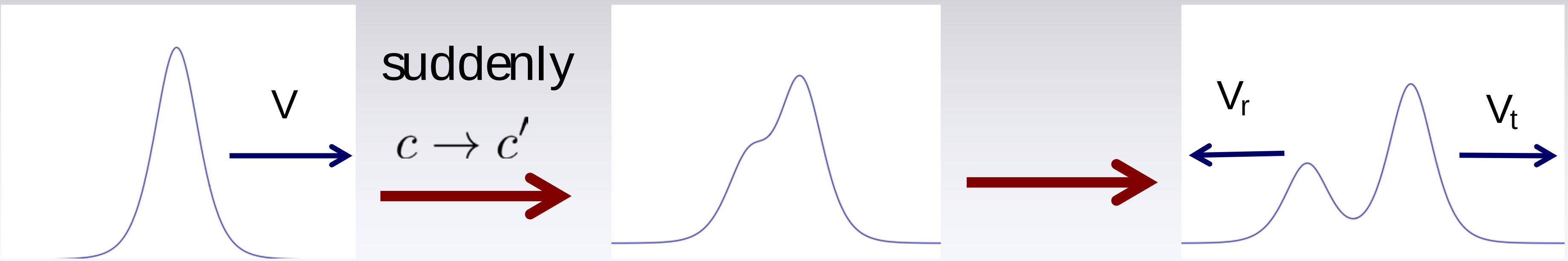
Euler

Enthalpy: $\omega = \partial_\rho \left[\rho \epsilon(\rho) \right]$

Quantum pressure

What to expect

- Short times (qualitatively like wave equation):



- Quench acts as external perturbation: soliton splits into transmitted and reflected component

- Longer Times: different scenarios (soliton trains + dispersive waves vs. dissipation)

Linearization

$$H = \int dx \left[\frac{\rho v^2}{2} + \rho\epsilon(\rho) + A(\rho)\frac{(\partial_x \rho)^2}{4\rho} \right]$$

$$\dot{\rho} + \partial(\rho v) = 0 \qquad \text{Continuity}$$

$$\dot{v} + \partial\left(\frac{v^2}{2} + \omega(\rho) - A'(\rho)(\partial\sqrt{\rho})^2 - A(\rho)\frac{\partial^2\sqrt{\rho}}{\sqrt{\rho}} \right) = 0 \qquad \text{Euler}$$

- Linearizing non-linear PDE: Bogolioubov modes (phonons, Luttinger Liquid…)

KdV Reduction

$$H = \int dx \left[\frac{\rho v^2}{2} + \rho\epsilon(\rho) + A(\rho)\frac{(\partial_x \rho)^2}{4\rho} \right]$$

- Non-linear behavior for small perturbations:

$$\begin{aligned} \rho(x,t) &= \rho_0 + \epsilon\, \rho^{(1)}(x,t) + \epsilon^2\, \rho^{(2)}(x,t) + \ldots \\ v(x,t) &= \epsilon\, v^{(1)}(x,t) + \epsilon^2\, v^{(2)}(x,t) + \ldots \end{aligned}$$

- Particular scaling of density, velocity, space & time

  $\rightarrow$ Korteweg-de Vries equation (KdV)

  (non-linear fixed point for local interactions)

Kulkarni & Abanov, PRA **86** (2012)

- KdV: wave on shallow water surfaces, chiral equation

KdV Reduction

- KdV scaling:

$$\begin{aligned} \rho(x,t) &= \rho_0 + \epsilon\,\rho^{(1)}(x,t) + \epsilon^2\,\rho^{(2)}(x,t) + \ldots \\ v(x,t) &= \epsilon\, v^{(1)}(x,t) + \epsilon^2\, v^{(2)}(x,t) + \ldots \end{aligned}$$

$$\dot\rho + \partial(\rho v) = 0$$

Continuity

$$\dot v + \partial\left(\frac{v^2}{2} + \omega(\rho) - A'(\rho)(\partial\sqrt{\rho})^2 - A(\rho)\frac{\partial^2\sqrt{\rho}}{\sqrt{\rho}}\right) = 0$$

Euler

$$u_\pm = \rho^{(1)} = \frac{c}{\omega_0'}\, v^{(1)}$$

Kulkarni, & Abanov, PRA **86** (2012)

$$\dot u_\pm \mp \partial_x\left[c u_\pm + \frac{\zeta}{2}u_\pm^2 - \alpha\partial_x^2 u_\pm\right] = 0$$

$$c \equiv \sqrt{\rho_0\omega_0'}$$

$$\zeta \equiv \frac{c}{\rho_0} + \frac{\partial c}{\partial\rho_0}$$

$$\alpha \equiv \frac{A(\rho_0)}{4c}$$

Example: Lieb-Liniger <=> NLSE

- Lieb-Liniger: $H_{\text{micro}}(c) = -\sum_{j=1}^{N} \frac{\partial^2}{\partial x_j^2} + 2g \sum_{j<l} \delta(x_j - x_l)$
- For weak interaction $\gamma \equiv \frac{m}{\hbar^2} \frac{g}{\rho_0} \ll 1$ collective description by Non-Linear Schrödinger Equation

$$H(c) = \int dx \left[\frac{\hbar^2}{2m} |\partial_x \Psi|^2 + \frac{g}{2} |\Psi|^4 \right]$$

- Reduce to canonical hydrodynamic form with ansatz

$$\Psi = \sqrt{\rho}\, e^{i \frac{m}{\hbar} \int^x v(x') dx'}$$

Example: NLSE

$$i\hbar\partial_t\Psi(x,t) = \left\{-\frac{\hbar^2}{2m}\partial_{xx} + c\,|\psi(x,t)|^2\right\}\psi(x,t)$$

$$\downarrow \quad \Psi = \sqrt{\rho}\, e^{i\frac{m}{\hbar}\int^x v(x')dx'}$$

$$\omega(\rho) = \frac{g}{m}\rho$$

$$A = \frac{\hbar^2}{2m^2}$$

$$\dot{\rho} + \partial(\rho v) = 0$$

$$\dot{v} + \partial\left(\frac{v^2}{2} + \omega(\rho) - A'(\rho)(\partial\sqrt{\rho})^2 - A(\rho)\frac{\partial^2\sqrt{\rho}}{\sqrt{\rho}}\right) = 0$$

$\downarrow$ **KdV Reduction:** $\delta\rho(x,t) = \rho_0 - \rho(x,t) \ll \rho_0$

$$\dot{u}_\pm \mp \partial_x\left[c u_\pm + \frac{\zeta}{2}u_\pm^2 - \alpha\partial_x^2 u_\pm\right] = 0\,, \qquad u_\pm = \delta\rho = \frac{c}{\omega_0'}\,\delta v$$

$$c = \sqrt{\frac{g\rho_0}{m}}\,,\zeta = \frac{3}{2}\frac{c}{\rho_0}\,,\alpha = \frac{\hbar^2}{8m^2\,c}$$

Our approach

1. Excite a shallow solitonic state & let it evolve

   $\rightarrow$ stable evolution due to initial Hamiltonian

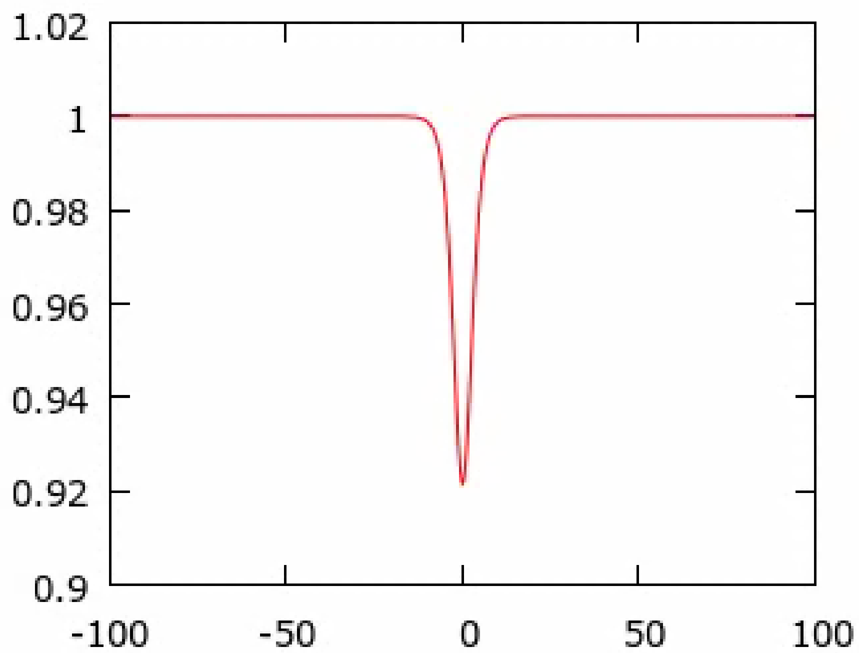
2. Change interaction strength of underlying quantum system

3. Describe post quench dynamics using KdV, with parameters from post-quench system

$\Rightarrow$ Universality for short time from KdV

Quench

- Interaction Quench: $H_0 = H(c_0) \to H = H(c)$
- Initial Soliton: $$s(x,t) = -U \cosh^{-2}\left[\frac{x \pm Vt}{W}\right]$$



NLSE: Gray Soliton

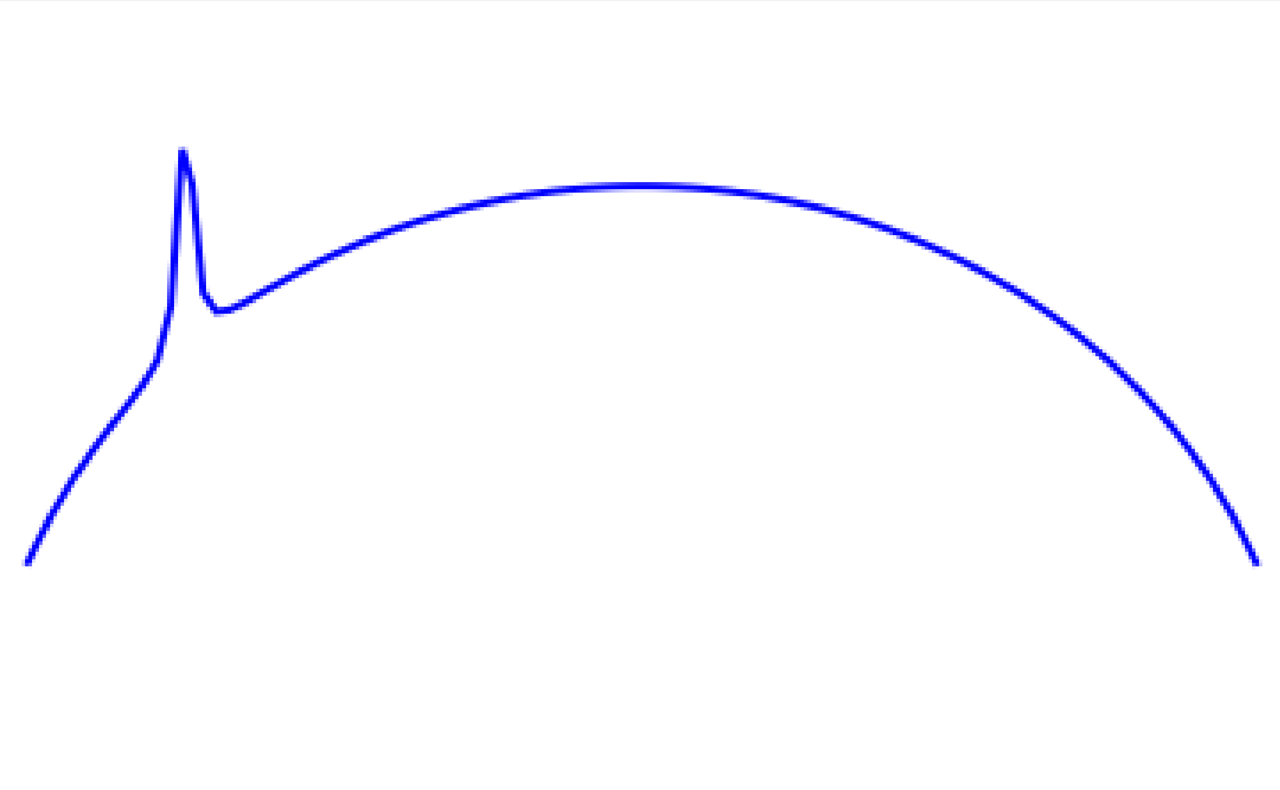
KdV

$$W = 2\sqrt{\frac{\alpha}{c - V}}$$

$$U = 3\,\frac{c - V}{\zeta}$$

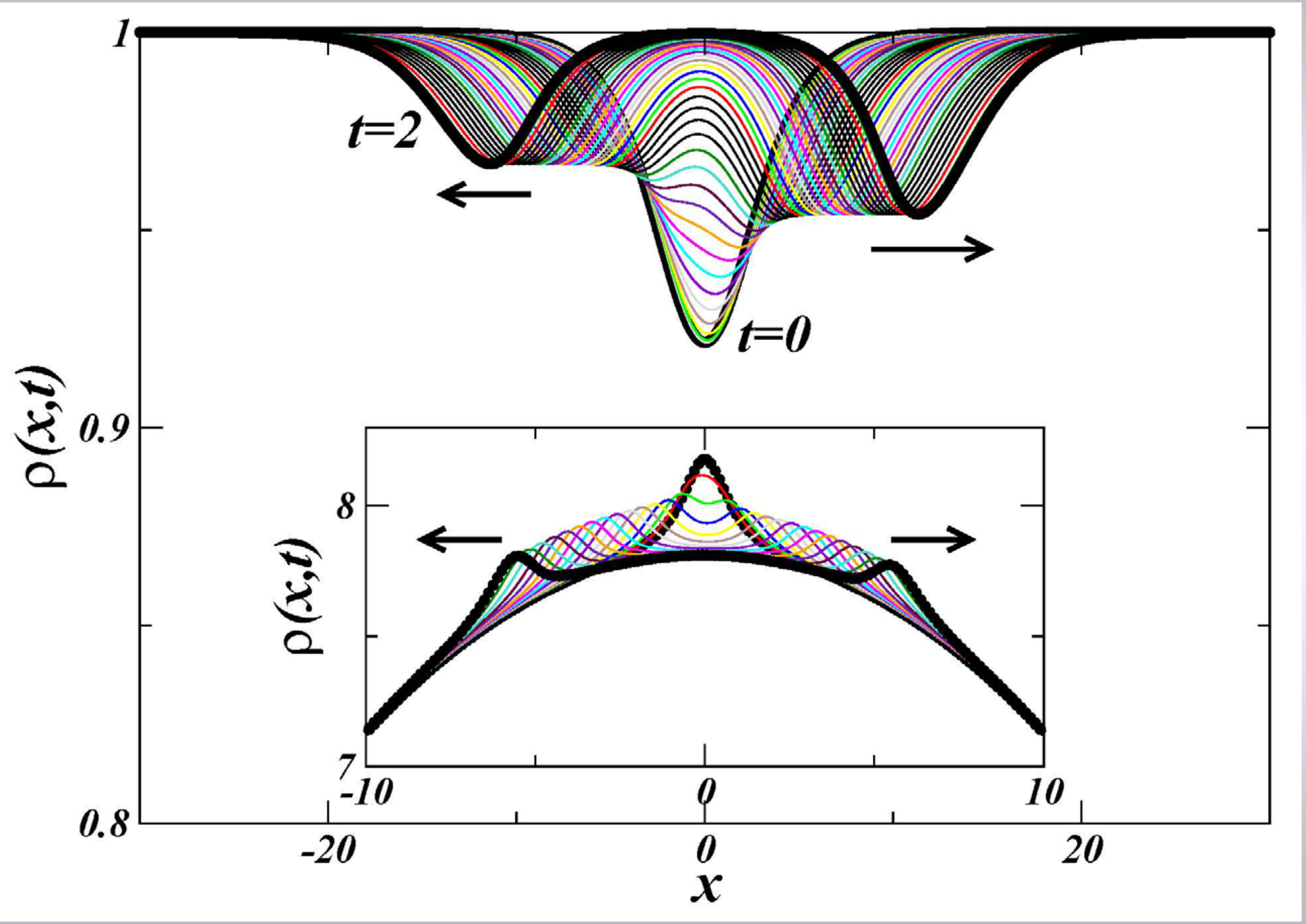
Quench

- Interaction Quench: $H_0 = H(c_0) \to H = H(c)$



Harmonic Calogero: Bright Soliton

Soliton Splitting



Soliton Splitting

- Quench acts as external perturbation: soliton splits into transmitted and reflected components

- Continuity and momentum conservation yield

$$u(x,t) = R\, s(x - V_r t) + T\, s(x - V_t t)$$

$$R(V, V_r, V_t) = \frac{V_t - V}{V_t - V_r} \qquad T(V, V_r, V_t) = \frac{V - V_r}{V_t - V_r}$$

- Here: just kinematics. Need (KdV) dynamics to fix $V_{r,t}$

Chiral Profiles

$$u(x,t) = R\, s(x - V_r t) + T\, s(x - V_t t)$$

- Using KdV we determined

$$\eta \equiv \frac{1 + \frac{\rho_0}{c'}\,\frac{\partial c'}{\partial \rho_0}}{1 + \frac{\rho_0}{c}\,\frac{\partial c}{\partial \rho_0}}$$

$$\begin{aligned} V_r &= -\left[c - \eta\, R\,(c - V)\right]\frac{c'}{c} \\ V_t &= \left[c - \eta\, T\,(c - V)\right]\frac{c'}{c} \\ R &= \frac{1}{2}\left[1 - \frac{c}{c'}\,\frac{V}{\eta\, V + (1-\eta)\, c}\right] \\ T &= \frac{1}{2}\left[1 + \frac{c}{c'}\,\frac{V}{\eta\, V + (1-\eta)\, c}\right] \end{aligned}$$

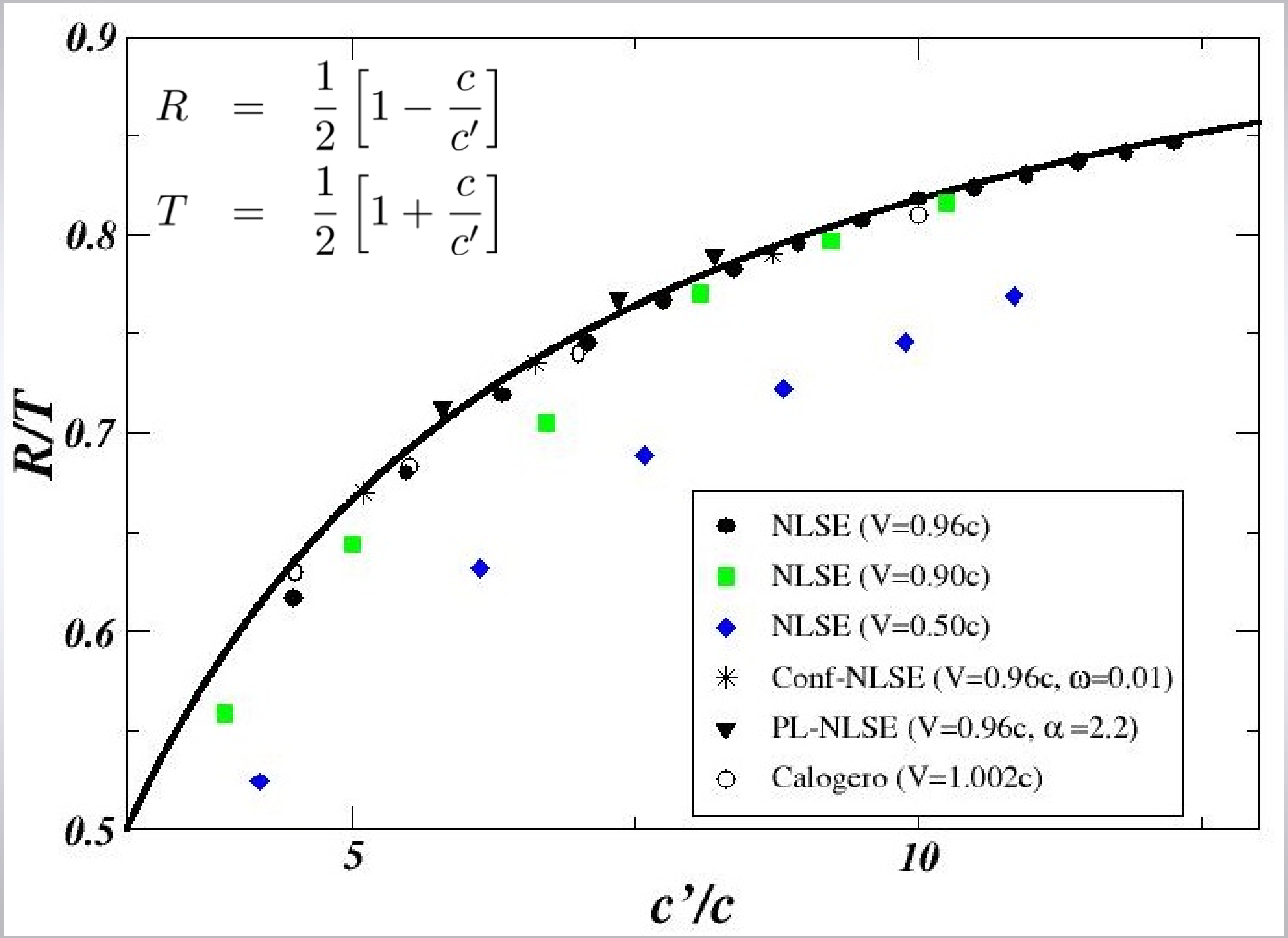
$$c \propto \rho_0^\theta \quad \Rightarrow \quad \eta = 1$$

$$\begin{aligned} V_r &= -\left(T\,c + R\,V\right)\frac{c'}{c} \\ V_t &= \left(R\,c + T\,V\right)\frac{c'}{c} \\ R &= \frac{1}{2}\left[1 - \frac{c}{c'}\right] \\ T &= \frac{1}{2}\left[1 + \frac{c}{c'}\right] \end{aligned}$$

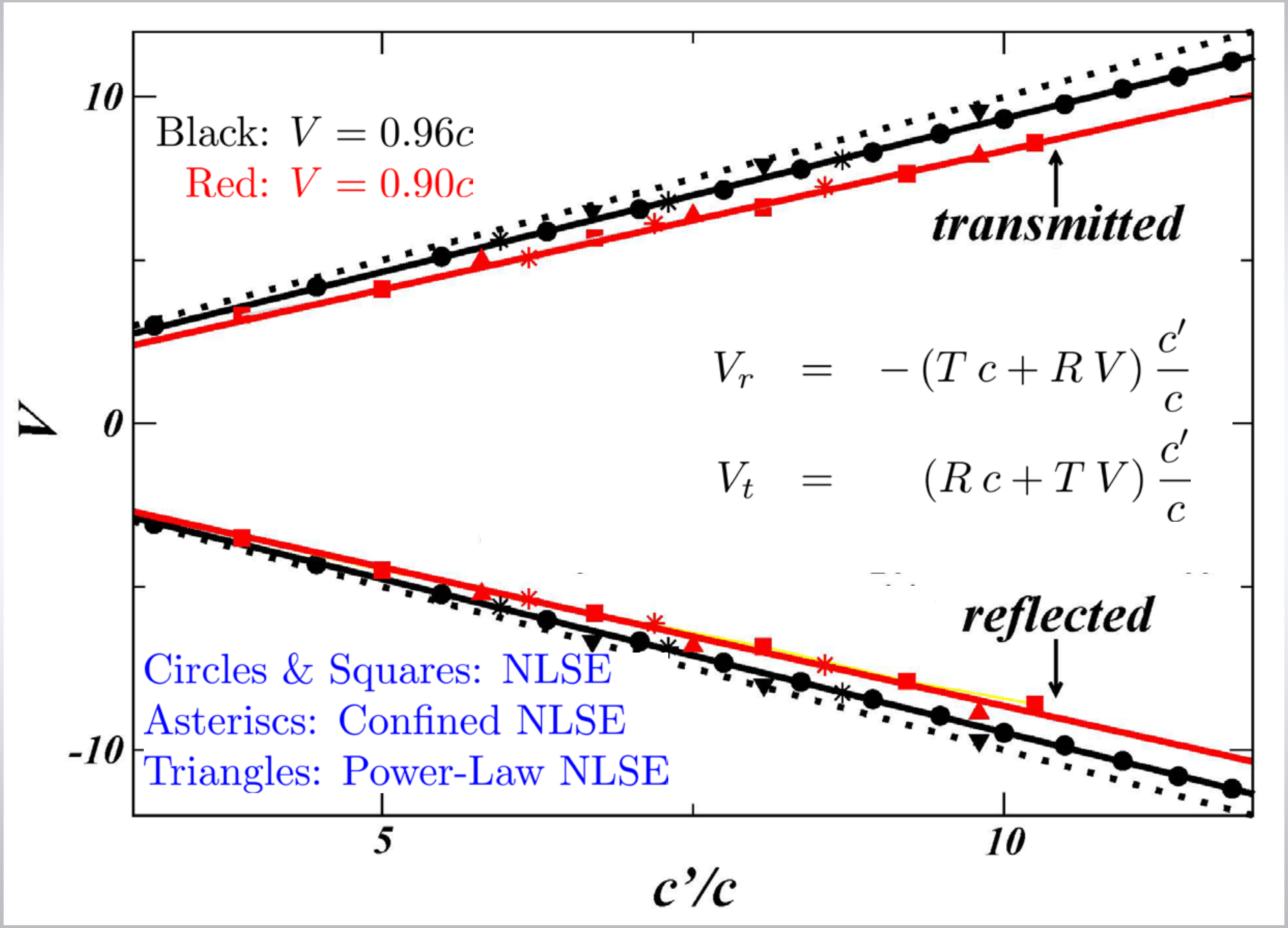
Same as linear problem, but non-trivial velocities!

- Completely UNIVERSAL !

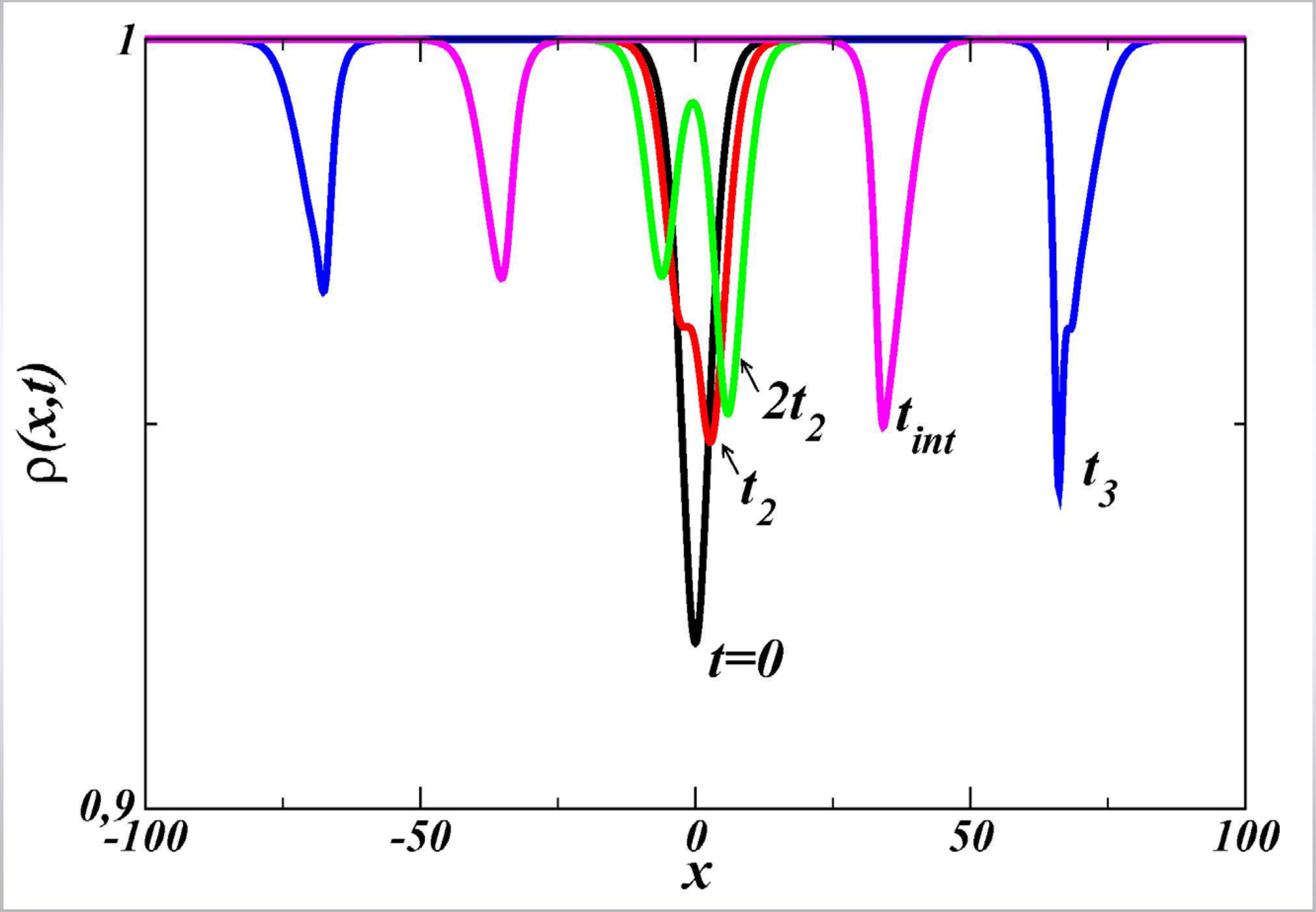
Numerical Checks: Amplitudes



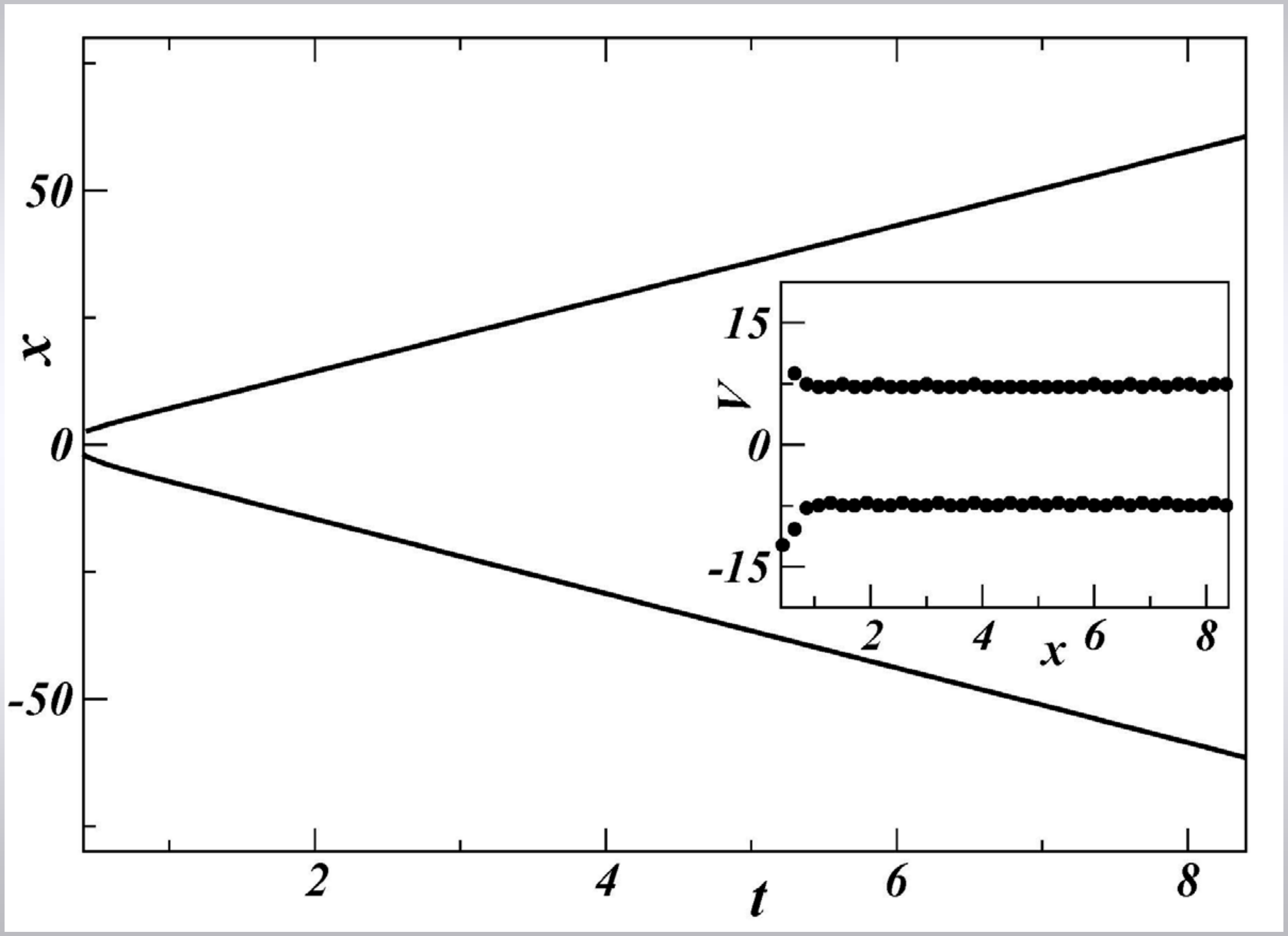
Numerical Checks: Velocities



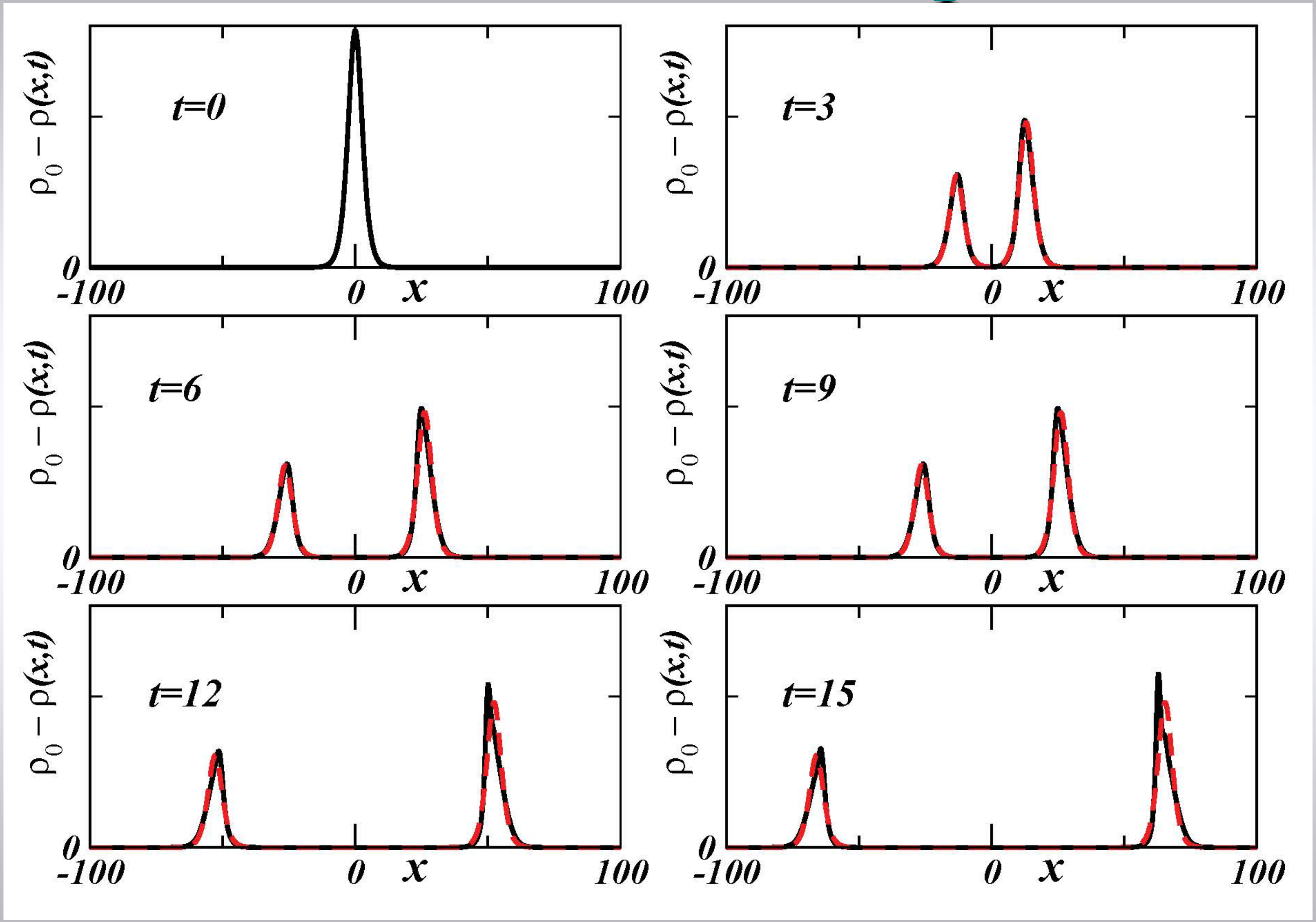
Numerical Simulations on NLSE



Stability of velocity



NLSE numerics vs KdV predictions



Peak Velocities

- We introduce reduced velocities:

$$V_r = -\left[c - \eta\, R\,(c - V)\right]\frac{c'}{c} \quad \nu \equiv \frac{c - |V|}{c} \quad \nu_r = \eta\,\frac{\nu}{2}\left[1 - \frac{c}{c'}\,\frac{1-\nu}{1-\eta\,\nu}\right]$$

$$V_t = \left[c - \eta\, T\,(c - V)\right]\frac{c'}{c} \longrightarrow \nu_t = \eta\,\frac{\nu}{2}\left[1 + \frac{c}{c'}\,\frac{1-\nu}{1-\eta\,\nu}\right]$$

- Transmitted and reflected profiles are not solitons of post-quench system → internal dynamics
- For instance: profile peaks and center of mass move at different velocities:

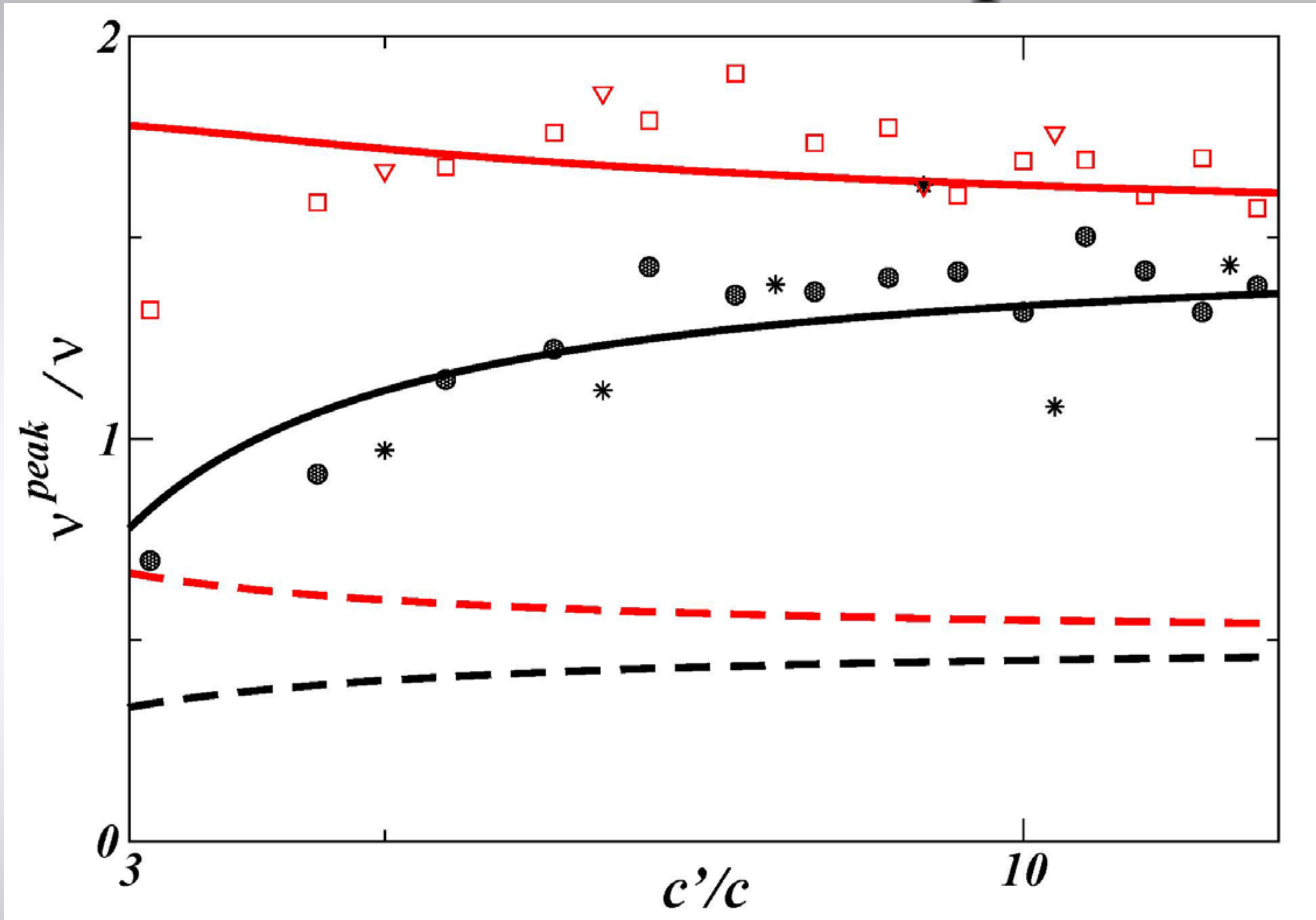
$$\boxed{\begin{aligned} \nu_r^{\text{Peak}} &= 3\,\nu_r - 2\frac{\eta}{\beta}\,\nu = \left[3 - \frac{2}{\beta_r}\right]\nu_r \\ \nu_t^{\text{Peak}} &= 3\,\nu_t - 2\frac{\eta}{\beta}\,\nu = \left[3 - \frac{2}{\beta_t}\right]\nu_t \end{aligned}}$$

$$\beta \equiv \frac{\zeta'}{\zeta}\,\frac{\alpha}{\alpha'}$$

$$\beta_t = T\beta$$

$$\beta_r = R\beta$$

NLSE numerics vs KdV predictions



Points: numerical reduced peak velocities for NLSE. Black are reflected (filled circles - $V = 0.96c$-, & stars - $V = 0.9c$-), red are transmitted (squares -$V = 0.96c$-, & *down* triangles -$V = 0.9c$-).

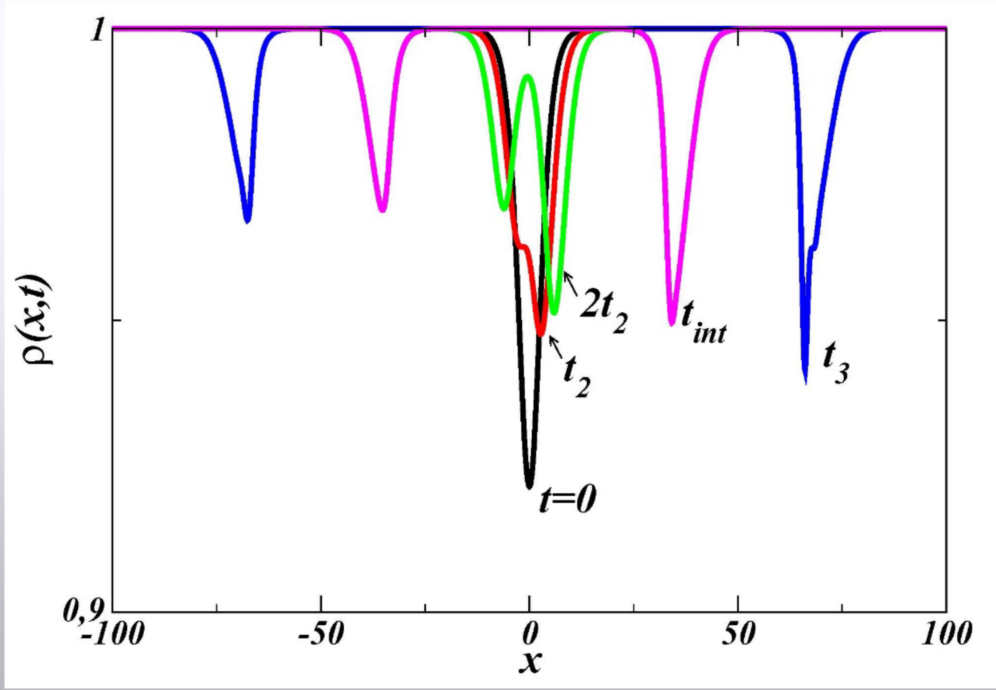
Lines: analytical curves for peak (solid) & bulk (dashed)

t_2

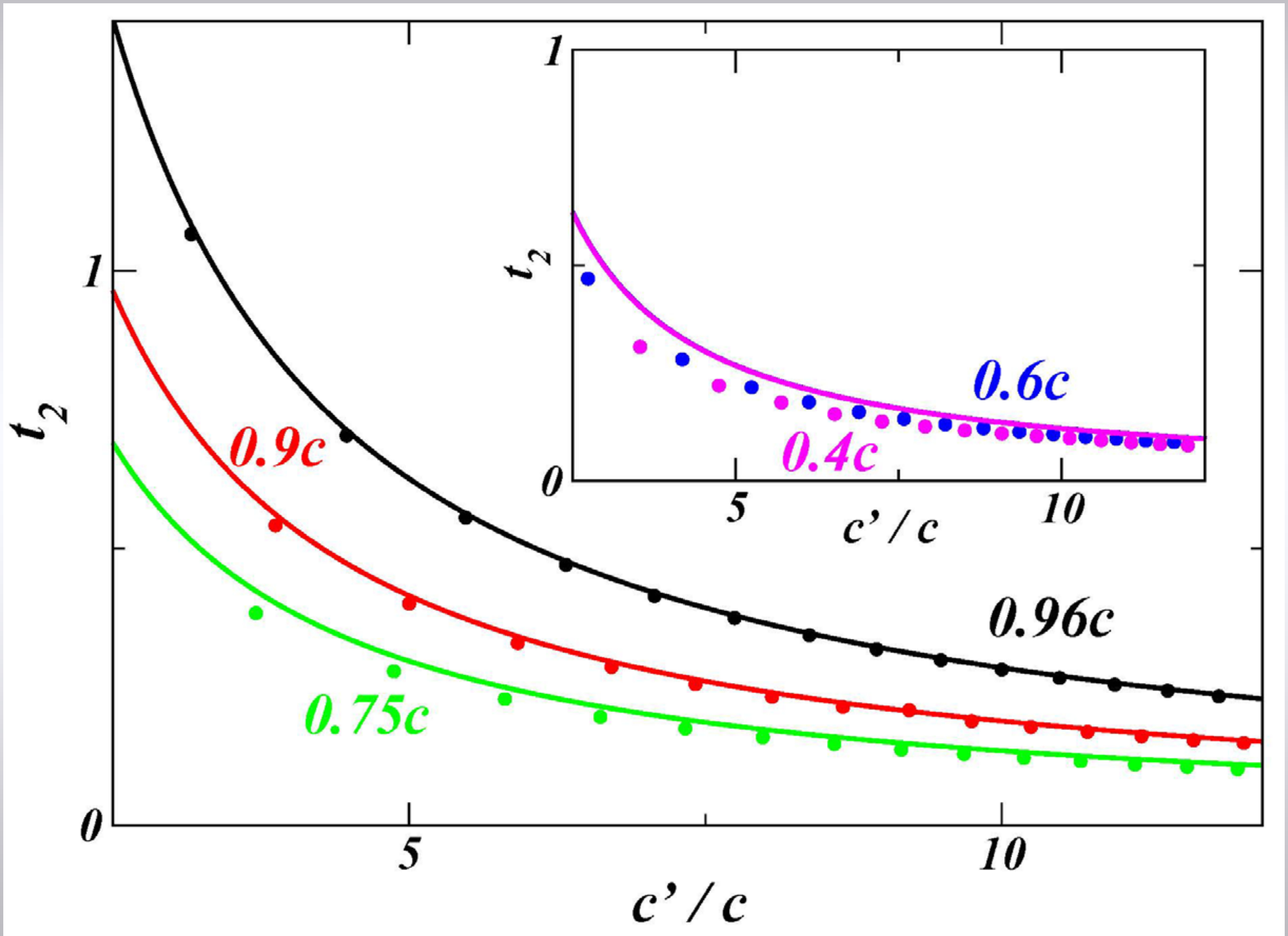
- We calculated the time at which the two profiles become discernible:

$$t_2 = \frac{W}{2(V_t - V_r)} \ln \frac{\sqrt{1+Q+Q^2} + \frac{\sqrt{3}}{2}(1+Q)}{\sqrt{1+Q+Q^2} - \frac{\sqrt{3}}{2}(1+Q)}$$

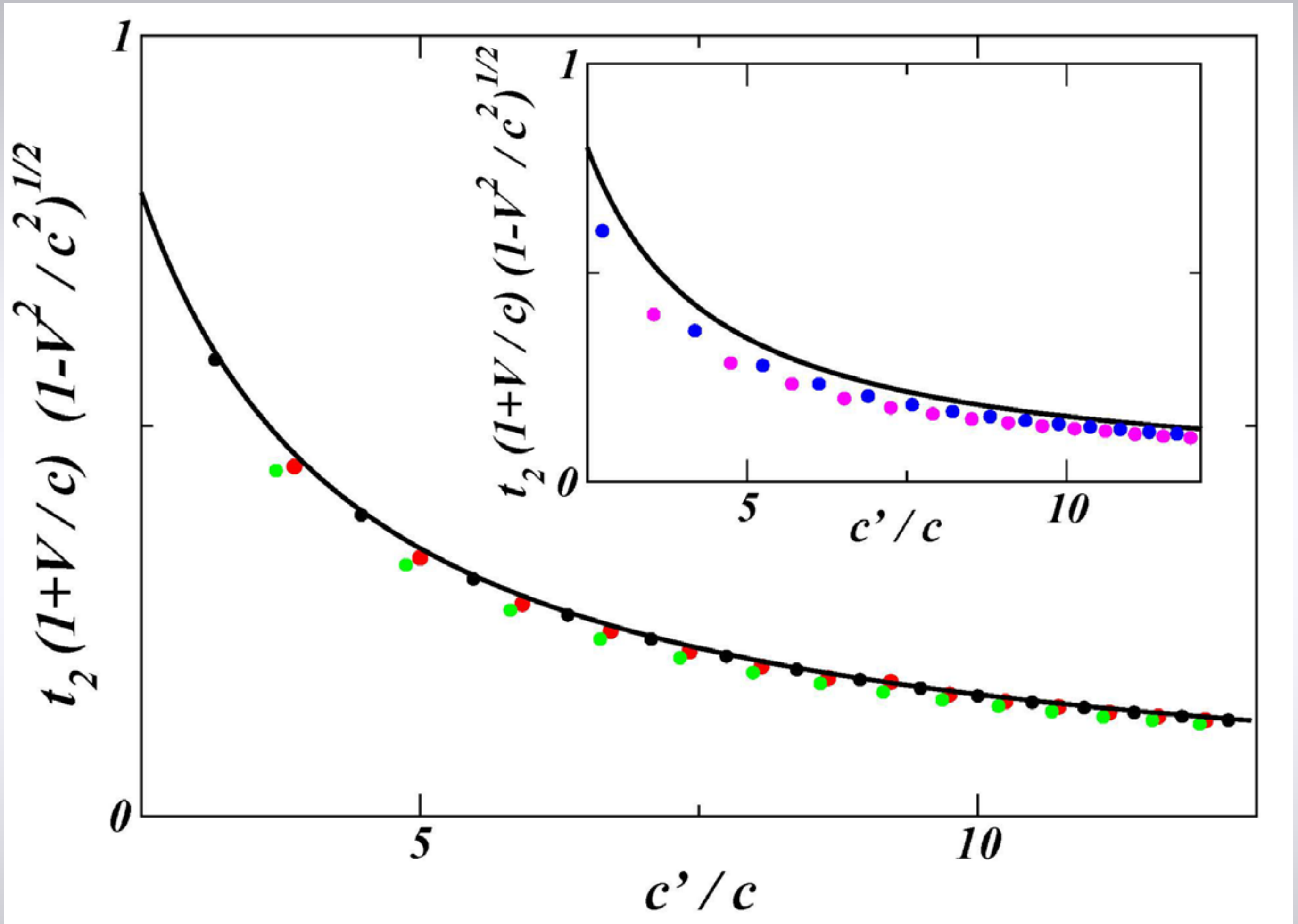
$$Q \equiv \left(\frac{c'}{c} + \sqrt{\frac{c'^2}{c^2} - 1} \right)^{2/3}$$



t_2

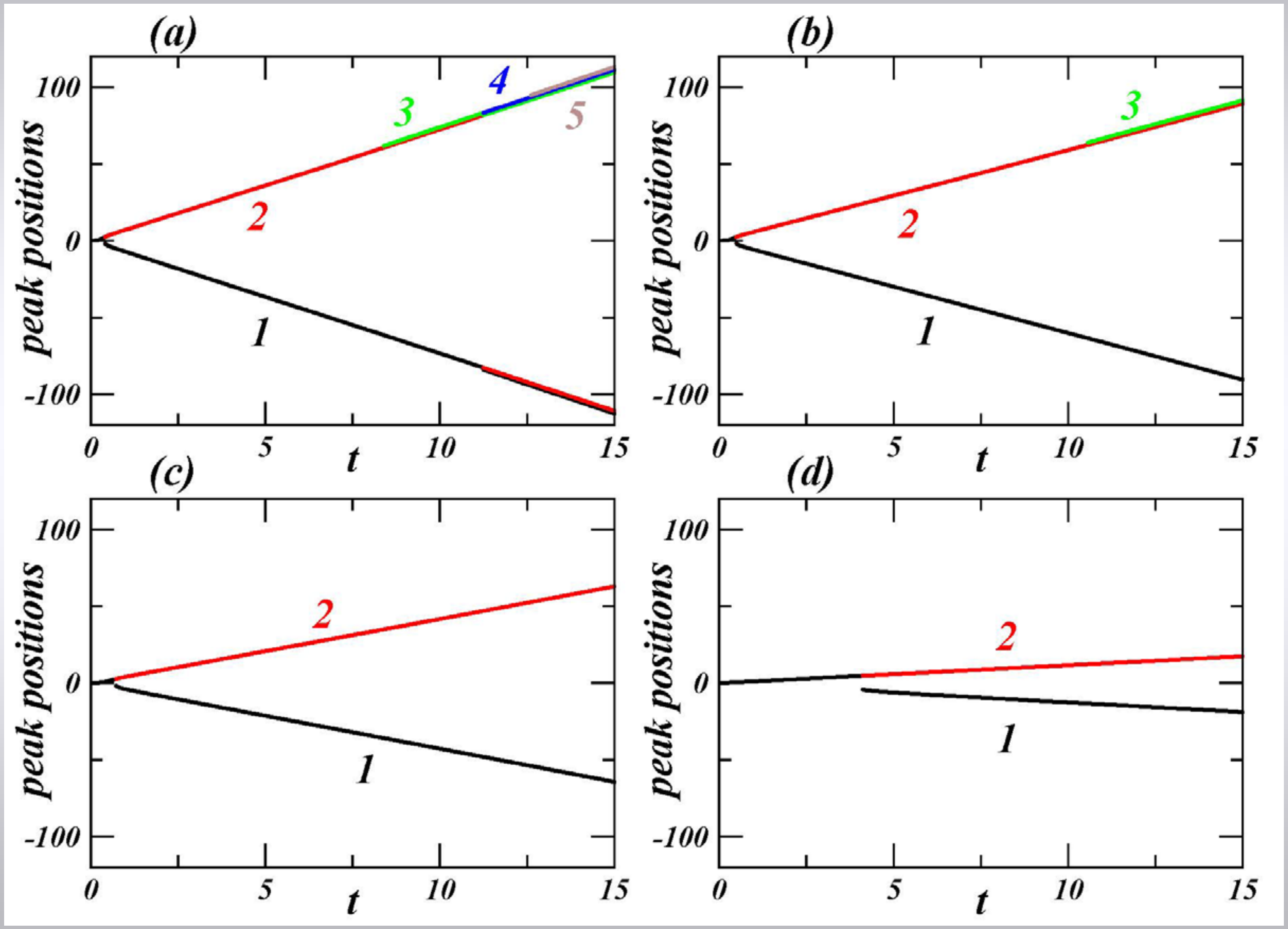


t_2



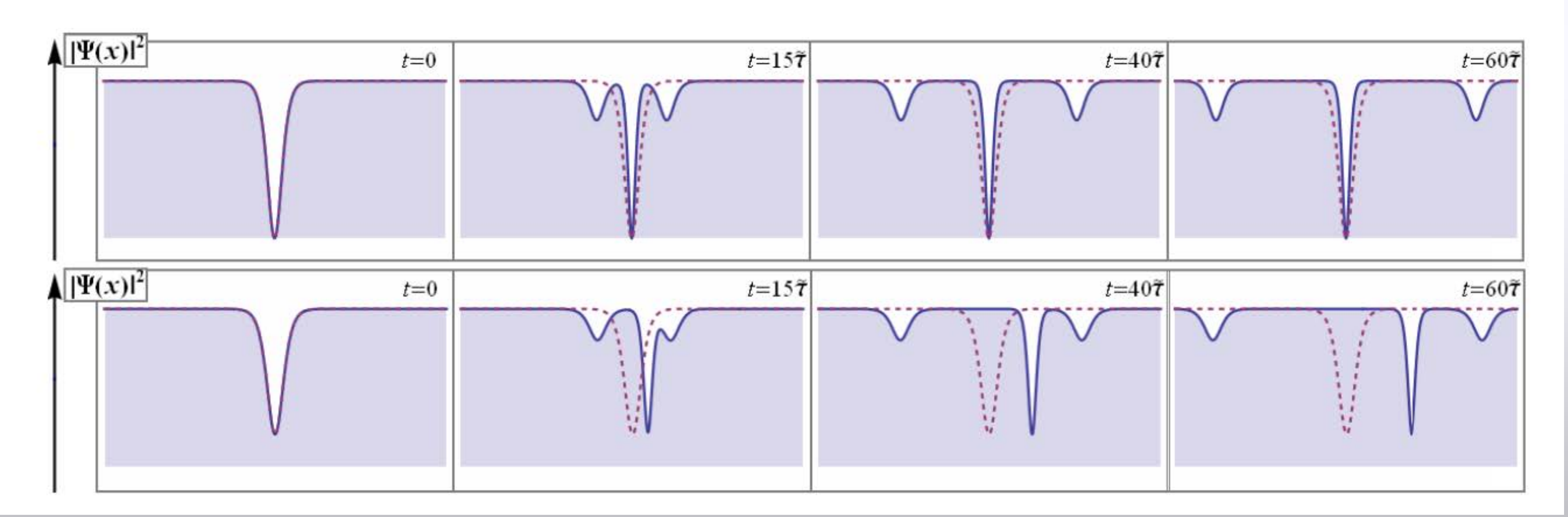
$$t_2\left(1+\frac{V}{c}\right)\sqrt{c^2-V^2}=\frac{1}{2c'}\ln\frac{\sqrt{1+Q+Q^2}+\frac{\sqrt{3}}{2}(1+Q)}{\sqrt{1+Q+Q^2}-\frac{\sqrt{3}}{2}(1+Q)}$$

Further Splitting



Large time asymptotics for NLSE

- Gamayun & al. considered same set-up
- Large time using integrability of NLSE (ISM)
- If $\dfrac{c'}{c} = \kappa$ integer $\Rightarrow 2\kappa - 1$ solitons (no dispersive waves)



Gamayun et al, PRA 91 (2015)

Harmonic Calogero

- Integrable in harmonic confinement!

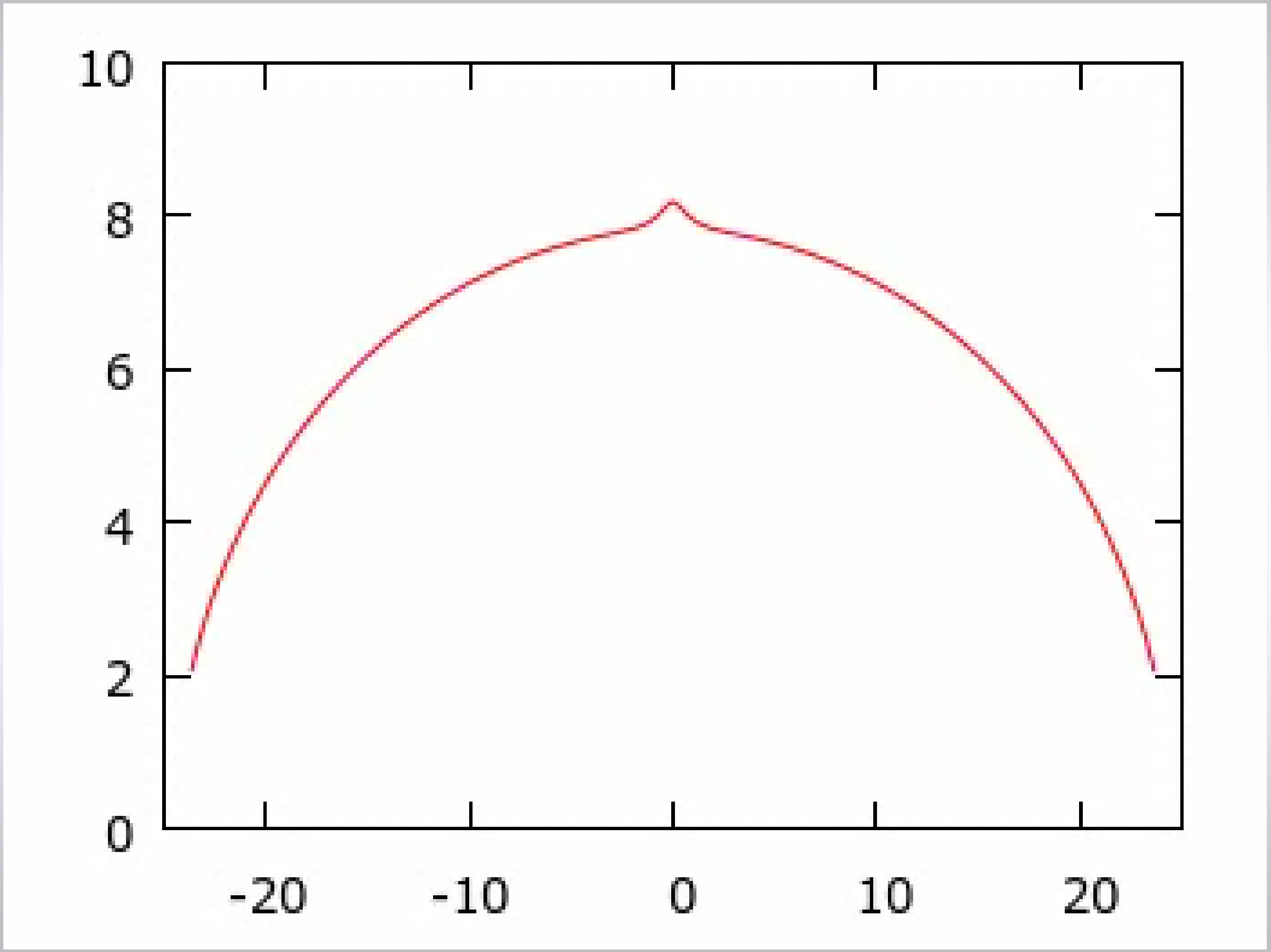
$$H = \frac{1}{2m}\sum_{j=1}^{N} p_j^2 + \frac{\hbar^2}{2m}\sum_{j\neq k}\frac{\lambda^2}{(x_j - x_k)^2} + \omega\sum_{j=1}^{N} x_j^2 \ ,$$

- Long(ish)-range model: hydrodynamics in Benjamin-Ono class (not KdV, different dispersion)

- Solitons have longer tails (Lorentzian)

- We simulate the model using microscopic Classical Newtonian evolution

Harmonic Calogero



Harmonic Calogero

$$H = \frac{1}{2m}\sum_{j=1}^{N} p_j^2 + \frac{\hbar^2}{2m}\sum_{j\neq k}\frac{\lambda^2}{(x_j - x_k)^2} + \omega\sum_{j=1}^{N} x_j^2 \ ,$$

- Quench protocol: change λ and ω so that background stays fixed (oscillations otherwise)

Rajabpour & Sotiriadis, PRA 89 (2014)

- For this case: $\omega' = \frac{\lambda'}{\lambda}\omega$

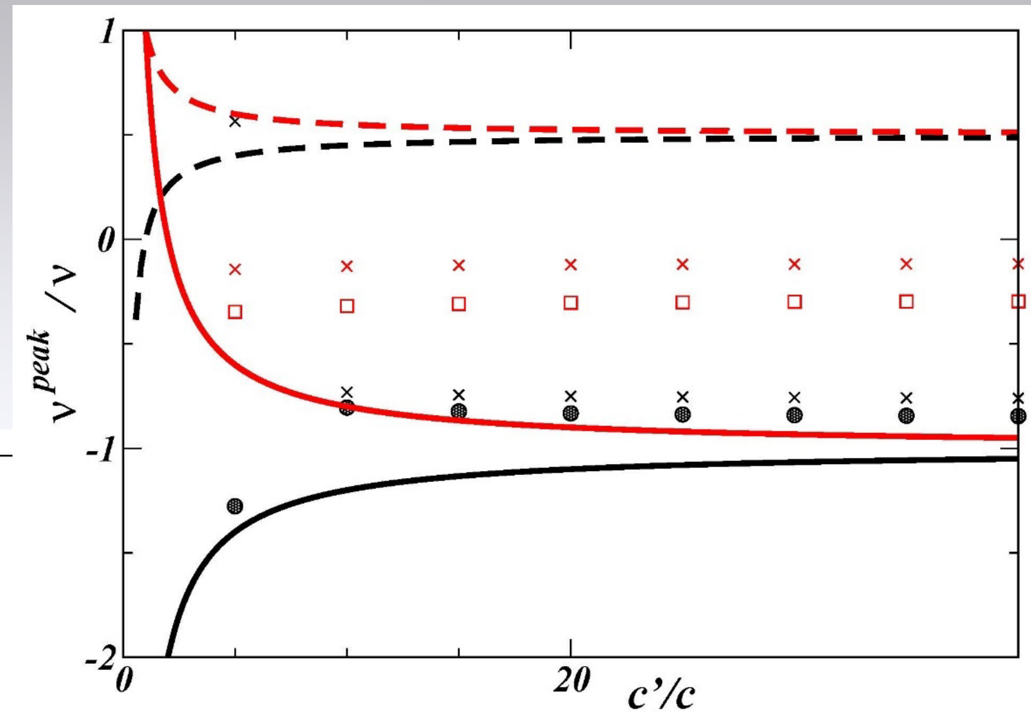
- Bulk velocities: same prediction as for KdV

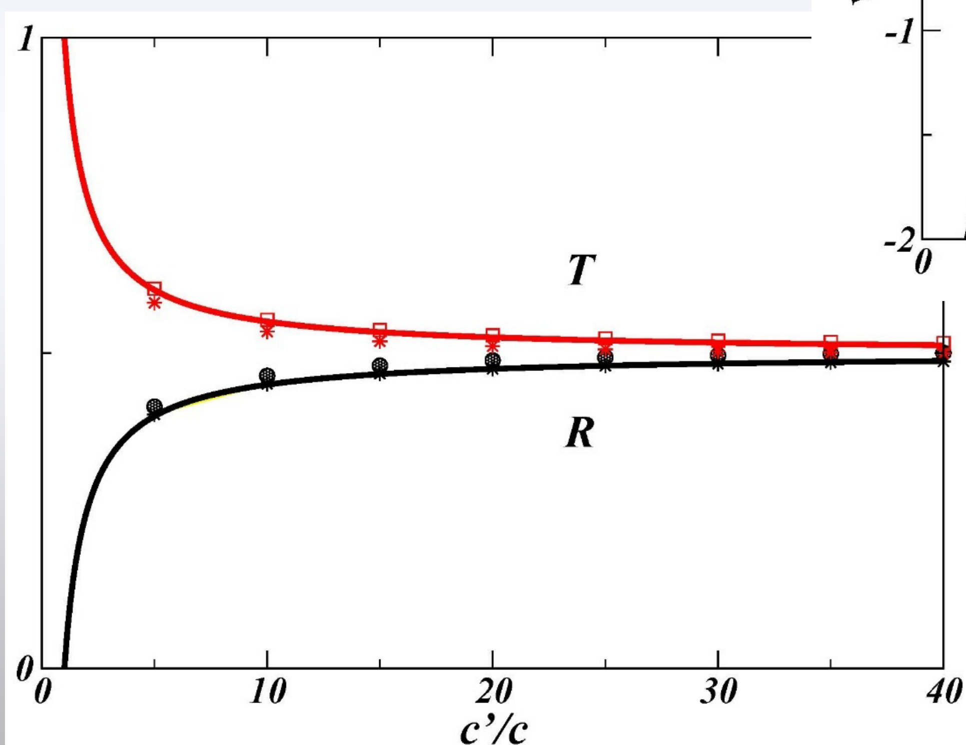
$$\begin{aligned} \nu_r &= \frac{\nu}{2}\left[1 - \frac{c}{c'}\right] \\ \nu_t &= \frac{\nu}{2}\left[1 + \frac{c}{c'}\right] \end{aligned}$$

- Peaks velocities:

$$\boxed{\begin{aligned} \nu_r^{\text{Peak}} &= \; = \left[4 - \frac{3}{R}\right]\nu_r \\ \nu_t^{\text{Peak}} &= \; = \left[4 - \frac{3}{T}\right]\nu_t \end{aligned}}$$

Harmonic Calogero





Points: numerical results for harm. Calogero.

Black are reflected (filled circles –V=1.04c- & stars –V=1.07c-), red are transmitted (squares –V=1.04c- & stars –V=1.07c-).

Lines: analytical curves for peak (solid) & bulk (dashed)

Conclusions

- We studied a quantum quench on localized excited state using an effective semi-classical hydrodynamics
- Universal dynamics for short time after quench: predicted shape and velocities of chiral profiles
- Great agreement with numerical simulations
- Experimentally feasible (bulk & peak velocities)
- Open questions: quantum nature of a soliton, microscopic unitary evolution, large time behavior

Thank you!

Quantum Quenches

- Take a system in its Ground State $|\Psi_0\rangle$
- Let it evolve according to different Hamiltonian $H \neq H_0$
- Unitary evolution: $|\Psi(t)\rangle = \sum_j \langle j|\Psi_0\rangle \, e^{iE_j t} \, |j\rangle$

$$\boxed{H|j\rangle = E_j \, |j\rangle}$$

Does the system reach a stationary state, in some sense?

Gibbs Ensemble

$$|\Psi(t)\rangle = \sum_j c_j \, e^{iE_j t} \, |j\rangle$$

- Restricted to local observables, most quantum quenches result in an effective stationary mixed state

$$\lim_{t\to\infty} \langle \Psi(t)|\mathcal{O}|\Psi(t)\rangle = \mathrm{Tr}\left[\rho_{\mathrm{eff.}} \, \mathcal{O}\right]$$

- Moreover, generally: $\rho_{\mathrm{eff}} = \dfrac{e^{-\beta_{\mathrm{eff}} H}}{\mathcal{Z}}$

(Gibbs distribution consequence of Eigenstate Thermalization Hypothesis)

Deutsch, PRA **43** (1991); Srednicki, PRE **50** (1994); Rigol, Dunjko, & Olshanii, Nature **452** (2009)…

Generalized Gibbs Ensemble

- If system has local conservation laws (f.i. integrabilty), these should be included → **G.G.E.**

$$\lim_{t\to\infty} \langle \Psi(t)|\mathcal{O}|\Psi(t)\rangle = \mathrm{Tr}\left[\rho_{\text{eff.}}\, \mathcal{O}\right]$$

$$\rho_{\text{eff}} = \frac{e^{-\sum_l \beta_l I_l}}{\mathcal{Z}}$$

- Open problem: find all local charges

Countless efforts from

- SISSA (Mussardo, Silva, Gambassi & collaborators);
- Pisa/SISSA (Calabrese & collaborators);
- Oxford (Cardy, Essler & collaborators);
- Amsterdam (Caux & collaborators);
- Many more (Polkovnikov, Mitra, Kehrein, Andrei, Prosen)…