Linear algebra: Homework Week 2, 15/1/2019

Ex. (1).
Find the rank of the following matrix by reducing it by rows:

$$A = \begin{pmatrix} 0 & 1 & 2 & 1 \\ 1 & 1 & 1 & 0 \\ 0 & -1 & 1 & 1 \\ 1 & 1 & 4 & 2 \end{pmatrix}$$

Ex. (2).
Find by reducing by rows the rank of the following matrix:

$$A = \begin{pmatrix} 0 & 1 & 2 & 1 & 0 \\ 1 & 2 & 2 & 1 & 1 \\ 1 & 1 & a & 0 & 1 \\ 0 & a & 2a & a^2 & 0 \end{pmatrix}$$

where a is a real parameter. For which value of a, the $rk(A)$ is equal to 3 ?

Ex. (3).
Say if the linear application

$$f : \mathbb{R}^3 \to \mathbb{R}^3$$

$$(x, y, z) \to (x + 2y, y + z, 2z - x)$$

is injective and if it is surjective. Find a basis of $ker(f)$ and a basis of $Im(f)$.

Ex. (4).
Let $f : \mathbb{R}^3 \to \mathbb{R}^3$ a linear application such that its associated matrix is:

$$M_f = \begin{pmatrix} 1 & 2 & 3 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{pmatrix}$$

with respect to canonical basis. Find the rank of M_f, and basis of $ker(f)$ and $Im(f)$.