Linear algebra: Homework Week 3, 23/1/2019

Ex. (1).
Calculate, using the cofactor expansion, the following determinant:

$$det \begin{pmatrix} 2 & 0 & 3 & -1 \\ 4 & 1 & 2 & 0 \\ 0 & 3 & 0 & 1 \\ 2 & 0 & 2 & -1 \end{pmatrix}$$

Ex. (2).
Calculate, using the cofactor expansion, the following determinant:

$$det \begin{pmatrix} 2 & -3 & -1 \\ 2 & -1 & -3 \\ 1 & -3 & -1 \end{pmatrix}$$

Alternatively, use first a reduction by rows procedure so to transform the matrix in to an upper triangular matrix, and then calculate its determinant (compare with the result from the first method).

Ex. (3).
For the following quadratic form:

$$Q = 2x^2 + 2xy + 2y^2$$

write (w.r.t canonical basis) the matrix A of the associated bilinear form $b : \mathbb{R}^2 \times \mathbb{R}^2 \to \mathbb{R}$; show that b is not degenerate ; and calculate the isotropic cone of b (hint put $t = x/y$).

Ex. (4).
Find the solutions of the following system of linear equations:

$$\Sigma : \begin{cases} x + y + z & = 0 \\ 2x - y + az & = 2 \\ x - 2ay = 4 \end{cases}$$

For which values of the real parameter a, is the system solvable ?