Linear algebra: Homework Week 4, 23/2/2019

Ex. (1).
Say which are the (non trivial) invariant subspaces of the linear map $f : \mathbb{R}^3 \to \mathbb{R}^3$ of which the matrix with respect to the canonical basis is:

$$\begin{pmatrix} \cos\theta & -\sin\theta & 0 \\ \sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

where $0 < \theta < \pi$.

Ex. (2).
Let $\phi \in \mathrm{End}(\mathbb{R}^2)$ be defined as:

$$\phi(x, y) = (5x - y, 8x - y)$$

with respect to the canonical basis $\mathcal{E}$ of $\mathbb{R}^2$.

- (0.1 pt) Write the associated matrix $M_\phi^{\mathcal{E},\mathcal{E}}$
- (0.2 pt) Write the characteristic polynomial $p_\phi(t)$, $t \in \mathbb{R}$.
- (0.5 pt) Find the eigenvalues and eigenvectors of ϕ .
- (0.2 pt) Show by explicitly computing that

  $$\Delta = Q^{-1} \cdot M_\phi^{\mathcal{E},\mathcal{E}} \cdot Q$$

  Δ is a diagonal matrix, provided that $Q = M_{id}^{\mathcal{E},\mathcal{B}}$ is the matrix of change of basis between $\mathcal{E}$ and the basis of eigenvectors $\mathcal{B}$.