Ex. (1).
Let $\phi \in \text{End}(\mathbb{R}^3)$ be defined as:

$$\phi(x, y, z) = (3x - y, -x + 3y, x + y + 4z)$$

with respect to the canonical basis $\mathcal{E}$ of $\mathbb{R}^3$.

- (0.1 pt) Write the associated matrix $M_\phi^{\mathcal{E},\mathcal{E}}$

- (0.2 pt) Write the characteristic polynomial $p_\phi(t)$, $t \in \mathbb{R}$.

- (0.5 pt) Find the eigenvalues and eigenvectors of ϕ.

- (0.2 pt) Say if ϕ is simple (i.e. if M_ϕ is diagonalizable) and why (hint: compare algebraic and geometric multiplicities).

Ex. (2).
Say if the following matrices:

$$A = \begin{pmatrix} 2 & 0 & 0 \\ 1 & 1 & 1 \\ 0 & 0 & 1 \end{pmatrix} \quad \text{and} \quad B = \begin{pmatrix} -1 & -2 & 0 \\ 3 & 4 & 0 \\ 1 & 1 & 1 \end{pmatrix}$$

are similar ?

Ex. (3).
Find the Jordan normal form and the generalized eigenvectors of the following matrix:

$$A = \begin{pmatrix} 4 & 1 \\ -1 & 6 \end{pmatrix}$$