Linear algebra: Homework Week 6, 08/3/2019

Ex. (1).
Let $\phi \in \mathrm{End}(\mathbb{R}^3)$ be defined in the canonical basis as:

$$\phi(x,y,z) = \frac{1}{3}(2x - 2y + z, 2x + y - 2z, x + 2y + 2z)$$

check that ϕ is an isometry (with respect to the canonical scalar product).

Ex. (2).
Let $\phi \in \mathrm{End}(\mathbb{E}^2)$ be defined in the canonical basis as:

$$\phi(x,y) = (\frac{\sqrt{2}}{2}x - \frac{\sqrt{2}}{2}y, \frac{\sqrt{2}}{2}x + \frac{\sqrt{2}}{2}y)$$

- verify that ϕ is an isometry
- Given a basis $\mathcal{B} = \{(1,0),(1,1)\}$ of $\mathbb{R}^2$, write the matrix $M_\phi^{\mathcal{B},\mathcal{B}}$ of ϕ with respect to $\mathcal{B}$, and show it is not orthogonal.
- Given the vector $v = (2,1)$ in the canonical basis, calculate $||v||$ and $||\phi(v)||$ and show it is $||v|| = ||\phi(v)||$.

Ex. (3).
Let $\phi \in \mathrm{End}(\mathbb{E}^2)$ the endomorphism with associated matrix:

$$M_\phi^{\mathcal{B},\mathcal{B}} = \frac{1}{2}\begin{pmatrix} 5 & -5 \\ -1 & 5 \end{pmatrix}$$

where the basis is $\mathcal{B} = \{(1,0),(1,2)\}$. Check that ϕ is a self-adjoint endomorphism (hint: write the matrix of ϕ with respect to an orthonormal basis).

Ex. (4).
Let $\mathcal{D} = \frac{d}{dx}$ be the differentiation operator on the space of polynomials $\mathbb{R}[x]_{r\leq 2}$ equiped with the standard basis $\mathcal{E} = \{1, x, x^2\}$ and canonical scalar product $p \cdot q = \sum_{i=1}^{3} \alpha_i\beta_i$, where $p = \alpha_1 + \alpha_2 x + \alpha_3 x^2$ and $q = \beta_1 + \beta_2 x + \beta_3 x^2$

- is $\mathcal{D}$ self-adjoint ?
- Write the matrix $M_{\mathcal{D}}^{\mathcal{E},\mathcal{E}}$: is it symmetric ?