Linear algebra: Homework Week 7, 15/3/2019

Ex. (1).
Let $\phi \in \mathrm{End}(\mathbb{R}^3)$ be defined in the canonical basis $\mathcal{E}$ as:

$$\phi(x, y, z) = (2x + y + z, x + 2y + z, x + y + 2z)$$

- (0.1pt) Write the matrix $A = M_\phi^{\mathcal{E},\mathcal{E}}$.
- (0.2pt) Write the characteristic polynomial $p_\phi(T)$.
- (0.4pt) Find the eigenvalues and a basis of eigenvectors of ϕ.
- (0.2pt) Find an orthonormal basis of eigenvectors $\mathcal{B}$. Show by explicitly computing the matrix product that $D = {}^tPAP$ is the diagonal matrix with the eigenvalues of ϕ on the diagonal, provided that $P = M^{\mathcal{E},\mathcal{B}}$ where $\mathcal{B}$ is the orthonormal basis of eigenvectors.
- (0.1pt) Show that P is orthogonal i.e. ${}^tPP = I_3$.

Ex. (2).
Given the matrix

$$A = \begin{pmatrix} 2 & 2 & 2 \\ 2 & -1 & 1 \\ 6a & 1 & a^2 \end{pmatrix}$$

where $a \in \mathbb{R}$, say for which values of a the matrix A is orthogonally diagonalisable.

Ex. (3).
Let $\phi \in \mathrm{End}(\mathbb{R}^3)$ be defined in the canonical basis $\mathcal{E}$ as:

$$\phi(x, y, z) = (2y + z, 2x + 3y + 2z, x + 2y)$$

- Write the matrix $A = M_\phi^{\mathcal{E},\mathcal{E}}$.
- Write the characteristic polynomial $p_\phi(T)$.
- Find the eigenvalues and a basis of eigenvectors of ϕ.