UNG-FN, Študijsko leto 2019/20
3. letnik - 1. stopnja Fizika in astrofizika
Continuum Mechanics written exam
15/6/2020
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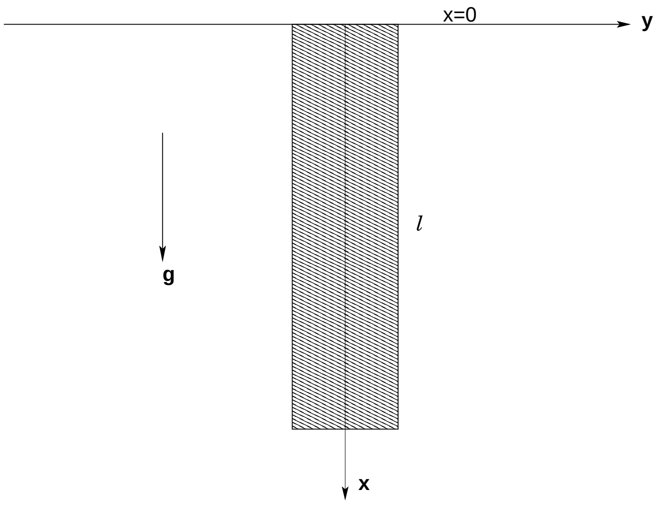
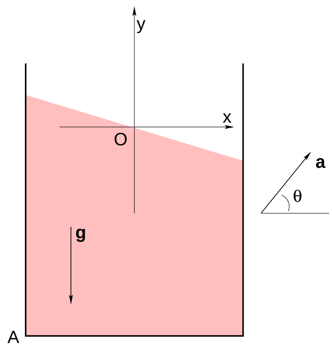


Exercise 1. A bar length ℓ, Young modulus E, Poisson ratio σ, and a square cross section, hangs vertically under gravity force from the ceiling. Let the x axis coincide with the axis of the bar and point downward, and let the upper surface to be fixed at the ceiling.

A) find the stress field in the bar.

B) find the displacement field components u_y and u_z.



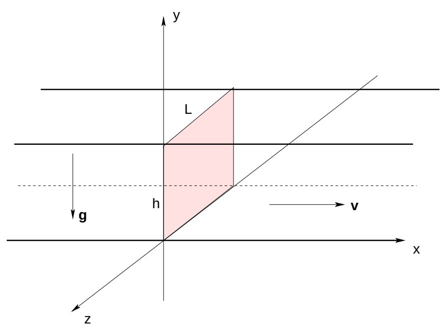
Exercise 2. A glass of gasoline as in figure moves with a constant acceleration **a** in the direction shown in the picture. The diameter of the glass is $2r_o$ and the height of the liquid at rest is h. Take the atmospheric pressure to be p_a.

A) Write the equations of motion.

B) Show that the free surface is flat, and find its angle of inclination (w.r.t x axis).

C) Find the pressure at the point $A = (-r_0, -h)$

D) What is the pressure in A if $\theta = \pi/2$?



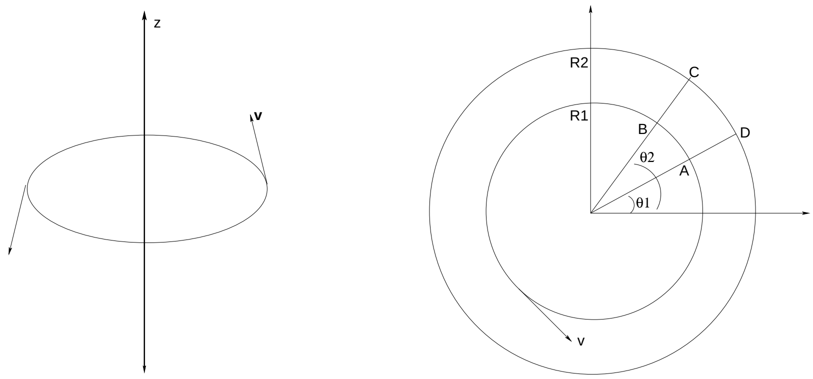
Exercise 3. Determine the flow in a pipe (axis x) with a rectangular cross-section of width L and height h in a gravitational field $\mathbf{g} = (0, -g, 0)$.

A) What are the boundary conditions and the simplified Navier-Stokes equation for this pipe when $L >> h$?

B) Find the velocity field $\mathbf{v}$

C) Find the friction at $y = 0$.

D) What is the discharge of the pipe ?



Exercise 4. An ideal vortex (axis z) has a velocity field $\mathbf{v} = \frac{\Gamma}{2\pi r}\mathbf{e}_\theta$ where Γ is the circulation r is the distance of the point from the vortex axis and $\mathbf{e}_\theta$ is the unit vector tangent to the streamline.

A) Calculate the circulation
$$\int_{ABCD} \mathbf{v} \cdot \mathbf{d}l$$
along the closed path ABCD as in the picture.

B) use the Stokes theorem to calculate the vorticity at a point non lying on the vortex axis. From the value of the vorticity infer what kind of flow is it.

C) Compare the result in (A) with the case of a fluid in a rotating vessel (axis z) with angular speed $\mathbf{\Omega} = (0, 0, \Omega)$ and velocity $\mathbf{v} = \mathbf{\Omega} \times \mathbf{r}$. What is the value of the circulation $\int_{ABCD} \mathbf{v} \cdot \mathbf{d}l$?