Exercise 1. Given the following displacement field in an isotropic linearly elastic solid:

$$u_1 = kx_2x_3 \quad u_2 = kx_1x_3 \quad u_3 = kx_1x_2 \tag{1}$$

with $k = 10^{-4}$, find:

A) the stress and strain components.

B) the volume change and say what type of deformation Eq. 1 does represent.

Exercise 2. An isotropic elastic sphere ($E_\gamma = 207$ GPa, $\mu = 79.2$ GPa) of 5 cm radius is under the uniform stress field:

$$\sigma_{ik} = \begin{pmatrix} 7 & 2 & 0 \\ 2 & -3 & 0 \\ 0 & 0 & 0 \end{pmatrix} MPa \tag{2}$$

Find the change of volume of the sphere [hint take $\lambda = \frac{\mu(E_\gamma - 2\mu)}{3\mu - E_\gamma}$]

Exercise 3. Given the two-dimensional flow

$$v_1 = kx_2, \quad v_2 = 0$$

A) find the streamline passing through the point (α_1, α_2)

B) find the pathline equation $\mathbf{x} = \mathbf{x}(t)$ for a fluid particle that is at (X_1, X_2) at $t = 0$.

Exercise 4. Given the velocity field of a linearly viscous fluid with viscosity η,

$$v_1 = \alpha x_1, \quad v_2 = -\alpha x_2, \quad v_3 = 0$$

A) show that the velocity field is irrotational

B) find the stress tensor

C) find the acceleration field $\mathbf{a} = \frac{d\mathbf{v}}{dt}$