UNG-FN, Študijsko leto 2020/21
3. letnik - 1. stopnja Fizika in astrofizika
Continuum Mechanics written exam
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Exercise 1. The velocity field of a fluid, in a rectangular Cartesian reference system, has the following spatial description :

$$v_1 = -\frac{1}{2}\alpha x_1 - f(r)x_2, \quad v_2 = -\frac{1}{2}\alpha x_2 + f(r)x_1, \quad v_3 = \alpha x_3$$

where $r = (x_1^2 + x_2^2)^{1/2}$, and α is a positive constant and $f(r)$ is a real function of r.

A) (5 pt) Calculate the vorticity vector ω.

B) (6 pt) Under which condition is the motion of the fluid irrotational ? Find the functions f which make the motion irrotational.

C) (5 pt) For a velocity field as in point B) calculate the circulation round the circle Γ defined by $x_1^2 + x_2^2 = r^2$ and $x_3 = constant$.

Exercise 2. (6 pt) A wind with velocity 30 m/s is blowing against a vertical wall, orthogonal with respect to the wind velocity. Determine the increase of the pressure on the surface of the wall due to the wind. Assume the air is an ideal fluid with density 1.25 $\mathrm{Kg/m^3}$ and $p_0 = 100$ kPa is the atmospheric pressure.

Exercise 3. The motion of a viscous fluid can be described by means of a velocity potential

$$\Phi = \frac{\alpha}{2}(x_1^2 - x_2^2)$$

where α is a positive constant and x_i are expressed in a Cartesian rectangular coordinate system.

A) (2 pts) Find the velocity field.

B) (4 pts) Find the stress tensor σ_{ij}.

C) (8 pts) Keeping into account (A) obtain the pressure distribution from the Navier-Stokes equations.

Exercise 4. (6 pts) A velocity field, in Cartesian rectangular coordinates, is given as:

$$v_1 = 0, \quad v_2 = \frac{\alpha x_2}{\alpha + \beta t}, \quad v_3 = \alpha x_3$$

where α, β are non zero real numbers. Find the pathline for a particle that was at point (X_1, X_2, X_3) at time $t = t_0$.

Exercise 5. (6 pts) The velocity field of an incompressible fluid in cylindrical coordinates is:

$$v_r = v(r, \theta), \quad v_\theta = 0, \quad v_z = 0$$

Show that

$$v_r = \frac{f(\theta)}{r}$$

where $f(\theta)$ is any function of θ.