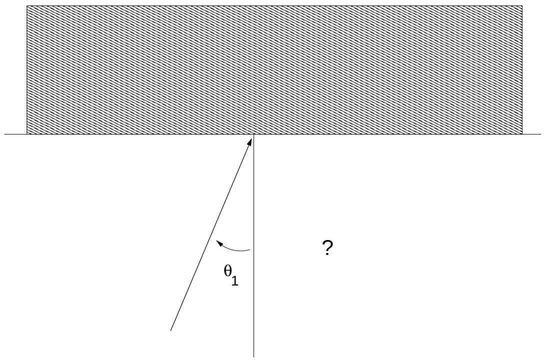
UNG-FN, Študijsko leto 2019/20
3. letnik - 1. stopnja Fizika in astrofizika
Continuum Mechanics written exam
4/9/2020
Lecturers L. Giacomazzi, A. Dixon



Exercise 1. A transverse elastic wave incidents on a traction-free plane boundary as shown in the picture. If the Poisson's ratio is $\sigma = 1/3$ determine

A) (5 pt) the angles of reflection of the reflected waves for an incident angle $\theta_1 = 20°$

B) (3 pt) find the value of the critical angle θ_{1c} and say what happens for $\theta_1 > \theta_{1c}$

Exercise 2. Given the following deformation field (in Cartesian orthogonal coordinates)

$$\xi_1 = 3x_3 \quad \xi_2 = -x_1 \quad \xi_3 = -2x_2$$

A) (4 pt) Determine the deformation gradient $G = \frac{\partial \xi_i}{\partial x_j}$ and provide the ratio of the deformed volume to the initial volume.

B) (6 pt) Determine the Lagrangean strain tensor u^L

$$u_{ij}^L = \frac{1}{2}\left[\frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i} + \frac{\partial u_k}{\partial x_i}\frac{\partial u_k}{\partial x_j}\right]$$

C) (3 pt) Determine the strain tensor valid for small deformations

$$u^*_{ij} = \frac{1}{2}\left[\frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i}\right]$$

and discuss its applicability in the present case.

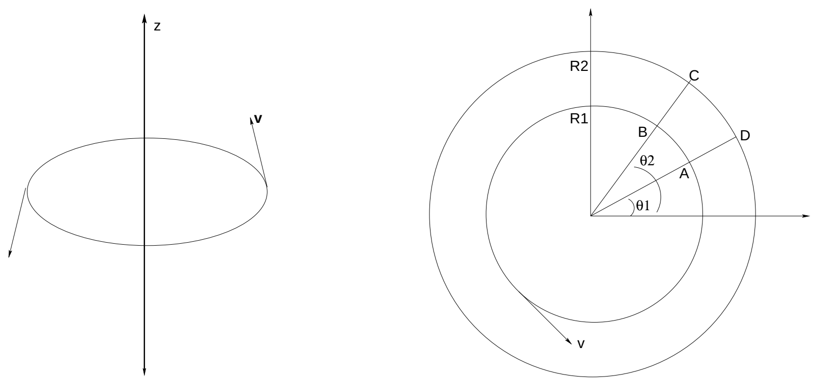
D) (5 pt) Show that $u^L = \frac{1}{2}\left({}^t GG - \mathbb{I}\right)$ where $\mathbb{I}$ is the unit matrix.

Exercise 3. Consider the following velocity field in cylindrical coordinates for an incompressible fluid

$$v_r = v(r), \quad v_\theta = 0, \quad v_z = 0$$

A) (7 pt) Show that $v_r = A/r$, where A is a constant so that the equation of conservation of mass is satisfied.

B) (5 pt) If the rate of mass flow through the circular cylindrical surface of radius r and unit length in the z-direction is Q_m, determine the constant A in terms of Q_m [justify your answer].



Exercise 4. An ideal vortex (axis z) has a velocity field $\mathbf{v} = \frac{\Gamma}{2\pi r}\mathbf{e}_\theta$ where Γ is the circulation r is the distance of the point from the vortex axis and $\mathbf{e}_\theta$ is the unit vector tangent to the streamline.

A) (6 pt) Calculate the circulation

$$\int_{ABCD} \mathbf{v} \cdot \mathbf{d}l$$

along the closed path ABCD as in the picture.

B) (6 pt) use the Stokes theorem to calculate the vorticity at a point non lying on the vortex axis. From the value of the vorticity infer what kind of flow is it.

C) (6 pt) Compare the result in (A) with the case of a fluid in a rotating vessel (axis z) with angular speed $\mathbf{\Omega} = (0, 0, \Omega)$ and velocity $\mathbf{v} = \mathbf{\Omega} \times \mathbf{r}$. What is the value of the circulation $\int_{ABCD} \mathbf{v} \cdot \mathbf{d}l$?