UNG-FN, Študijsko leto 2019/20
1. letnik - 1. stopnja Fizika in astrofizika
Linear Algebra written exam
2/7/2020
Lecturers: L. Giacomazzi, Z. Benher.

Questions and exercise 1 are recommended. [30 pt = 10.]

Q. 1. [2 pt] Say what kind of conic is represented by the points (x, y) of the affine plane such that

$$x^2 + y^2 - 4x - 6y = -4$$

Q. 2. [3 pt] In the affine space $\mathbb{A}^3$ a point $A = (3, -1, 2)$ is given. Provide the distance between A and the plane $\pi : \quad x + y + z = 5$

Q. 3. [2 pt] A bilinear form $g : \mathbb{R}^2 \times \mathbb{R}^2 \to \mathbb{R}$ is given of which the matrix w.r.t the canonical basis is

$$G = \begin{pmatrix} 2 & 0 \\ 0 & -2 \end{pmatrix}.$$

Say if g is a scalar product. Justify your answer.

Exercise 1. Let V be a vector space and $\mathcal{B} = \{v_1, v_2, v_3, v_4\}$ a basis of V. An endomorphism $\phi : V \to V$ is such that:
$\phi(v_1) = v_1+3v_2$, $\phi(v_2) = 3v_1-v_2$, $\phi(v_3) = 2v_3+v_4$, and $\phi(v_4) = v_3+2v_4$

i) [2 pt] Write the matrix $A = M_\phi^{\mathcal{B},\mathcal{B}}$

ii) [5 pt] Write the characteristic polynomial $p_\phi(\lambda)$

iii) [5 pt] Find the eigenvalues λ_j and eigenvectors ξ_j of ϕ.

iv) [2 pt] Write the eigenvectors and the eigenvalues of $\phi^2 = \phi \circ \phi$

Exercise 2. Given the linear application $f : \mathbb{R}^3 \to \mathbb{R}^3$ which with respect to canonical basis $\mathcal{E}_3$ is defined as:

$$f(xe_1 + ye_2 + ze_3) = (x - 2z)e_1 + (2x - y - 5z)e_2 + 3(y + z)e_3$$

i) [2 pt] Write the matrix $A = M_f^{\mathcal{E}_3,\mathcal{E}_3}$

ii) [3 pt] Find the dimension and a basis of $Im(f)$

iii) [4 pt] find the dimension and a basis of $ker(f)^{\perp}$.

iv) [4 pt] Write the matrix $A' = M_f^{\mathcal{B},\mathcal{B}}$ where $\mathcal{B} = \{v_1, v_2, v_3\}$ and $v_1 = e_1 + e_2$, $v_2 = e_2 - e_3$, $v_3 = e_1 + e_2 + e_3$.

Exercise 3. [12 pt] Given the system of linear equations:

$$\Sigma_t : \quad \begin{cases} 2x_1 + x_3 & = 5 \\ -tx_1 + (t-1)x_2 + x_3 & = 1 \\ (t-1)x_2 + tx_3 & = 1 \end{cases}$$

determine the solutions of Σ_t for t varying in the real numbers $\mathbb{R}$.