UNG-FN, Študijsko leto 2019/20
1. letnik - 1. stopnja Fizika in astrofizika
Linear Algebra written exam
8/9/2020
Lecturers: L. Giacomazzi, Z. Benher.

Questions and exercise 1 are recommended. [30 pt = 10.]

Q. 1. [3 pt] A linear map $t : \mathbb{R}^{2,2} \to \mathbb{R}^{2,2}$ is defined as:

$$t\begin{pmatrix} a & b \\ c & d \end{pmatrix} = \begin{pmatrix} a & c \\ b & d \end{pmatrix}$$

Find the matrix $T = M_t^{\mathcal{B},\mathcal{B}}$ representing t with respect to the basis

$$\mathcal{B} = \left\{ \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix}, \begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix}, \begin{pmatrix} 0 & 0 \\ 1 & 0 \end{pmatrix}, \begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix} \right\}$$

Q. 2. [3 pt] In the real affine space $\mathbb{A}^3$ find the cartesian equation of the line passing through points $P = (1, -1, 1)$ and $Q = (0, 2 - 1)$.

Q. 3. [3 pt] A bilinear symmetric form $g : \mathbb{R}^2 \times \mathbb{R}^2 \to \mathbb{R}$ is given of which the matrix w.r.t the canonical basis is

$$G = \begin{pmatrix} 2 & 1 \\ 1 & 2 \end{pmatrix}.$$

If $W = \{v \in \mathbb{R}^2 \,|\, v = \lambda(1, 2) \quad \lambda \in \mathbb{R}\}$ find the g-orthogonal subspace $W^\perp$ of W.

Exercise 1. Let V be a real vector space and $\mathcal{B} = \{v_1, v_2, v_3\}$ a basis of V. An endomorphism $\phi : V \to V$ is such that:
$\phi(v_1) = v_1 + v_2 - v_3, \quad \phi(v_2) = -v_1 + 3v_2 - v_3, \quad \phi(v_3) = -v_1 + v_2 + v_3,$
and

i) [2 pt] Write the matrix $A = M_\phi^{\mathcal{B},\mathcal{B}}$

ii) [5 pt] Write the characteristic polynomial $p_\phi(\lambda)$

iii) [5 pt] Find the eigenvalues λ_j and eigenvectors ξ_j of ϕ.

iv) [2 pt] Write the matrix of $\phi^2 = \phi \circ \phi$ with respect to $\mathcal{B}$ and find the eigenvalues of ϕ^2.

Exercise 2. Given the linear application $f : V \to U$, where V, U are real vector spaces with basis $\mathcal{B} = \{v_1, v_2, v_3, v_4\}$ and $\mathcal{C} = \{w_1, w_2, w_3\}$, respectively, and f is defined as:

$$f(v_1) = w_1 - w_2 \quad f(v_2) = 2w_2 - 6w_3$$

$$f(v_3) = -2w_1 + 2w_2 \quad f(v_4) = w_2 - 3w_3$$

i) [2 pt] Write the matrix $A = M_f^{\mathcal{C},\mathcal{B}}$

ii) [4 pt] Find the dimension and a basis of $ker(f)$

iii) [4 pt] Find the dimension and a basis of $im(f)$.

iv) [3 pt] Say if the vector $w_1 + w_2 + w_3$ belongs to $im(f)$?

Exercise 3. [8 pt] Given the system of linear equations

$$\Sigma_t : \quad \begin{cases} (t-1)x_1 + 2x_2 - tx_3 & = 0 \\ 2x_1 - x_3 & = 0 \\ -(t+1)x_1 - tx_2 + (t+2)x_3 & = 0 \end{cases}$$

where t is a real number $t \in \mathbb{R}$. Let S_t be the set of solutions of the system Σ_t. Find the dimension of the subspace S_t as the parameter t varies in the real numbers $\mathbb{R}$.