UNG-FN, Študijsko leto 2018/19
1. letnik - 1. stopnja Fizika in astrofizika

Linear Algebra Exam (03/07/2019)

FirstName/Ime:
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Tick the correct answer of the given questions.
Threshold for the correction of the exercises: 5

Question 1. Let $\Sigma : \; Ax = \xi$, a sistem of linear equations where $A \in \mathbb{R}^{5,5}$ and $x, \xi \in \mathbb{R}^5$. If $rk(A) = 2$ and $rk(A|\xi) = 3$, from the Rouché Capelli's theorem it follows:
(A) There is an infinite number of solutions.
(B) The associated homogenous system $\Sigma_0 : \; Ax = 0$ has no solutions.
(C) Σ is not solvable.

Question 2. The sign, $sgn(\sigma)$ [or parity, also indicated as $(-1)^\sigma$], of the following permutation σ:

$$\sigma = \begin{pmatrix} 1 & 2 & 3 \\ 2 & 3 & 1 \end{pmatrix}$$

is:
(A) +1 (B) -1 (C) +i

Question 3. Let V, W be two $\mathbb{R}$-vector spaces. The kernel $ker(f)$ (or null space) of a linear transformation $f : V \to W$ is defined as:
(A) $ker(f) = \{v \in V \,|\, f(v) = 0_W\}$
(B) $ker(f) = \{(v, w) \in V \times W \,|\, f(v) - w = 0_W\}$
(C) $ker(f) = \{v \in W \,|\, f(0_V) = v\}$

Question 4. Let V, W be two real vector spaces. An isomorphism between V and W is:
(A) any linear transformation between V and W.
(B) a bijective linear map between V and W.
(C) any linear map associated to a real symmetric matrix.

Question 5. Given the matrix $A \in \mathbb{R}^{n,n}$, let's consider the eigenvalue problem:

$$Au = \lambda u$$

When could $u = 0_{\mathbb{R}^n}$ be called an eigenvector ?
(A) Never, by definition
(B) Only for $\lambda = 0$
(C) Only for $\lambda \neq 0$.

Question 6. The matrix (w.r.t. canonical basis) associated to the following quadratic form $Q(\mathbf{x}) = x_1^2 - 4x_1x_2 + 4x_1x_3 + 3x_2^2 + 3x_3^2$ is:

(A) $\begin{pmatrix} 1 & -2 & 2 \\ -2 & 3 & 0 \\ 2 & 0 & 3 \end{pmatrix}$ (B) $\begin{pmatrix} 1 & -4 & 4 \\ -4 & 3 & 0 \\ 4 & 0 & 3 \end{pmatrix}$ (C) $\begin{pmatrix} 1 & -1/4 & 1/4 \\ -1/4 & 3 & 0 \\ 1/4 & 0 & 3 \end{pmatrix}$

Question 7. Let $f : V \to W$ be a linear map between two finitely generated $\mathbb{R}$-vector spaces V, W. It holds that:
(A) $dim(ker(f)) + dim(Im(f)) = dim(V)$
(B) $dim(Im(f)) = dim(W)$
(C) $dim(ker(f)) - dim(Im(f)) = dim(V)$

Question 8. In the euclidean space E^n let's consider the orthogonal projection P_W onto a subspace W of E^n. Then it is:
(A) $Im(P_W) = E^n$ and $ker(P_W) = 0_W$
(B) $Im(P_W) = W$ and $ker(P_W) = W^\perp$
(C) $Im(P_W) = W^\perp$ and $ker(P_W) = W$

Question 9. Which one, among the following equations, describes a plane in $\mathbb{A}^3$?
(A) $y = xz + z_0$, where $z_0 \in \mathbb{R}$
(B) $2y = x$
(C) $x_0x + (y_0 + y)z + z_0z = 0$, where $x_0, y_0, z_0 \in \mathbb{R}$

Question 10. Let $A = M^{\mathcal{B},\mathcal{E}}$ be the matrix of the change of basis from the canonical basis $\mathcal{E}$ to a basis $\mathcal{B} = \{b_1, ..., b_n\}$ of $\mathbb{R}^n$ and $v = \sum_{i=1}^n \xi_i b_i = \sum_{i=1}^n x_i e_i$. Then the components of v [for short $x = {}^t(x_1 ... x_n)$ and $\xi = {}^t(\xi_1 ... \xi_n)$] do transform as:
(A) $x = A\xi$
(B) $\xi = Ax$
(C) $b_i = Ae_i$, $\forall i = 1, ..., n$.

Question 11. Let v, w be two vectors of the Euclidean vector space E^n. Which one of the following inequalities is true for any v, w ?
(A) $|v \cdot w| \leq ||v|| \, ||w||$
(B) $|v \cdot w| \leq 1$
(C) $\frac{v \cdot w}{||v|| \, ||w||} \leq -1$

Question 12. Let's consider the endomorphism $\phi \in \text{End}(\mathbb{R}^2)$ which has

$$M_\phi^{\mathcal{E},\mathcal{E}} = \begin{pmatrix} 4 & 1 \\ 0 & 4 \end{pmatrix}.$$

Is ϕ simple (i.e. M_ϕ is diagonalizable) ?
(A) Yes, as $\lambda = 4$ is an eigenvalue with multiplicity 2.
(B) No, as the geometric multiplicity of $\lambda = 4$ does not match its algebraic multiplicity.
(C) No, because the eigenspace $V_{\lambda=4}$ has zero dimension $dim(V_{\lambda=4}) = 0$.

Question 13. Let A be a $n \times n$ square matrix, $A = (C_1, ..., C_n)$ where $\mathbb{R}^n \ni C_j \neq 0$, and $k \in \mathbb{R}\backslash\{0\}$. Which one of the following statements is true ?
(A) $det(kC_2 + ... + kC_n, C_2, ..., C_n) = 0$
(B) $det(kC_1, kC_2, ..., kC_n) = nk \cdot det(C_1, C_2, ..., C_n)$
(C) $det(C_1, C_2, ..., kC_n) = \frac{n}{k} \cdot det(C_1, C_2, ..., C_n)$

Question 14. Given the matrices $A = \begin{pmatrix} 2 & 1 & 3 \\ 4 & -1 & 2 \end{pmatrix}$ and $B = \begin{pmatrix} 1 & 1 \\ 0 & -1 \\ 2 & 1 \end{pmatrix}$ the product AB is:

(A) $\begin{pmatrix} 8 & 5 \\ 8 & 7 \end{pmatrix}$ (B) $\begin{pmatrix} 8 & 3 \\ 7 & 7 \end{pmatrix}$ (C) $\begin{pmatrix} 8 & 4 \\ 8 & 7 \end{pmatrix}$

Question 15. If $A, B \in GL(n)$ then:
(A) $(AB)^{-1} = A^{-1}B^{-1}$
(B) AB-BA=0
(C) $(AB)^{-1} = B^{-1}A^{-1}$
