(T.A)

UNG-FN, Študijsko leto 2018/19
1. letnik - 1. stopnja Fizika in astrofizika

Linear Algebra Exam (18/06/2019):

FirstName/Ime:
LastName/Priimek:
Enrollment number:

Tick the correct answer of the given questions.
Threshold for the correction of the exercises: 5

Question 1. Which one of the following sets is not formed by linearly independent vectors of $\mathbb{R}^4$?
(A) $\{e_1 + e_2, e_3 + e_4, -2e_1 - 2e_2, e_3\}$.
(B) $\{e_1 + e_2, e_1 + e_3, e_1 - e_4, e_1\}$.
(C) $\{e_1 + e_2, e_1 + e_3, e_1 - e_4, e_2\}$.
(D) $\{e_1, e_2, e_3, e_4\}$.

Question 2. Given the following two vectors in $\mathbb{R}^3$, $\mathbf{a} = 2i - j$ and $\mathbf{b} = 3j + k$, where i, j, k are the basis unit vectors of an orthogonal cartesian coordinate system, their cross product $\mathbf{a} \times \mathbf{b}$ is:
(A) $-i - j + 6k$
(B) $-i - 2j + 6k$
(C) $-2i - j + 6k$

Question 3. Let ϕ be an endomorphism of $\mathbb{R}^3$ and $A = M_\phi^{\mathcal{E},\mathcal{E}}$ its matrix with respect to the canonical basis $\mathcal{E}$. Knowing that ϕ is simple and that there exists a matrix $P \in \mathsf{GL}(n)$ such that $A = P\,D\,P^{-1}$ where D is diagonal with λ_j diagonal elements, then:
(A) A^2 is in general not diagonalisable.
(B) A^2 is diagonalisable and an eigenvectors basis of A is still an eigenvectors basis of A^2.
(C) A^2 is diagonalisable with the same eigenvalues λ_j found for the matrix A.

Question 4. Le W_1, W_2 be two vector subspaces of $\mathbb{R}^3$. If $W_1 = \mathcal{L}(e_1 + e_2, e_1 - e_2)$ and $W_2 = \mathcal{L}(e_1 + e_2 + e_3)$ then:
(A) $W_1 \cap W_2 = e_1 + e_2$
(B) $W_1 + W_2 = \mathcal{L}(e_1, e_2)$.
(C) The sum $W_1 + W_2$ is direct and $W_1 \oplus W_2 = \mathbb{R}^3$

Question 5. The determinant of a square matrix $A = (a_{i,j}) \in \mathbb{R}^{n,n}$ can be written as:
(A) $det(A) = \sum_{\sigma \in S_n} \mathrm{sgn}(\sigma)\, a_{1,\sigma(1)} \cdots a_{n,\sigma(n)}$
(B) $det(A) = a_{1,\sigma(1)} \cdots a_{n,\sigma(n)}$
(C) $det(A) = a_{1,1} \cdots a_{n,n}$

Question 6. Which one of the following is not a property of the determinant of a square matrix ?
(A) A square matrix and its transpose have the same determinant.
(B) The determinant of a matrix does not change when adding, to the elements of a row, the elements of another row each one multiplied by a scalar λ.
(C) If two rows (or two columns) of a matrix are identical, then the determinant of the matrix is 1.

Question 7. The characteristic polynomial $p_A(T)$ of a matrix $A = M_\phi^{\mathcal{B},\mathcal{B}}$ of $\phi \in$End(V) in the basis $\mathcal{B} = \{v_1, ...v_n\}$ is:
(A) $p_A(T) = (-1)^n T^n + (-1)^{n-1} tr(A) T^{n-1} + ... + det(A)$
(B) $p_A(T) = (-1)^n T^n + (-1)^{n-1} det(A) T^{n-1} + ... + tr(A)$
(C) $p_A(T) = (-1)^n T^n + (-1)^{n-1} tr(A)^n T^{n-1} + ... + det(A)^n$

Question 8. In the euclidean space E^n let's consider the orthogonal projection P_W onto a subspace W of E^n. Then it is:
(A) $P_W^2 = I + P_W$
(B) $P_W^2 = I - P_W$
(C) $P_W^2 = P_W$

Question 9. In the affine plane, let's consider the line $r : \ \{(x, y) \mid ax + by + c = 0\}$. In which case does r represent a vector subspace:
(A) $c = -a$
(B) $c = 0$
(C) $c = -a - b$

Question 10. The dimension of $\mathbb{C}^2 = \mathbb{C} \times \mathbb{C}$, when considered as a real vector space, is:
(A) $dim_{\mathbb{R}}(\mathbb{C}^2) = 2$
(B) $dim_{\mathbb{R}}(\mathbb{C}^2) = 3$
(C) $dim_{\mathbb{R}}(\mathbb{C}^2) = 4$

Question 11. For which of the following pairs of vectors a, b does the equality holds in the triangle inequality $||a + b|| \leq ||a|| + ||b||$?
(A) $a = (1, 2)$ and $b = (-2, -3)$
(B) $a = (-1, -2)$ and $b = (3, 4)$
(C) $a = (2, 3)$ and $b = (1, \frac{3}{2})$

Question 12. Let's consider the following subset of the orthogonal matrices $O(n)$ in $\mathbb{R}^{n,n}$: SO(n)=$\{A \in$ O(n) $: det(A) = 1\}$. If $A, B \in$ SO(n) then:
(A) As multiplication of matrices is not commutative, if $C = AB$ belongs to SO(n), $D = BA$ may not belong to SO(n).
(B) The product $C = AB$ still belongs to SO(n).
(C) As SO(n) is a vector space any linear combination $C = \alpha A + \beta B$ still belongs to SO(n).

Question 13. Given the matrices $A = \begin{pmatrix} 2 & 1 & 3 \\ 4 & -1 & 2 \end{pmatrix}$ and $B = \begin{pmatrix} 1 & 2 \\ 0 & -1 \\ 1 & 1 \end{pmatrix}$ the product AB is:

(A) $\begin{pmatrix} 5 & 6 \\ 6 & 11 \end{pmatrix}$ (B) $\begin{pmatrix} 5 & 3 \\ 5 & 10 \end{pmatrix}$ (C) $\begin{pmatrix} 4 & 5 \\ 7 & 12 \end{pmatrix}$

Question 14. An endomorphism f of the polynomials vector space $V = \mathbb{R}[X]_2$ is defined as:

$$f(a + bX + cX^2) = 2a - b + 3cX^2.$$

The associated matrix with respect to the basis $\mathcal{B} = (1, X, X^2)$ is:

(A) $\begin{pmatrix} 2 & -1 \\ 0 & 3 \end{pmatrix}$ (B) $\begin{pmatrix} 2 & -1 & 3 \end{pmatrix}$ (C) $\begin{pmatrix} 2 & -1 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 3 \end{pmatrix}$

Question 15. A self-adjoint endomorphism ϕ of the euclidean space E^n has two distinct eigenvalues λ_1, λ_2, with $\lambda_1 \neq \lambda_2$ and v_1, v_2 eigenvectors, respectively. Then:

(A) $v_1 \in [\mathcal{L}(v_2)]^\perp$

(B) $(\lambda_1 - \lambda_2)v_1 \cdot v_2 = 1$

(C) $\phi(v_1) \cdot v_2 = \phi(\lambda_1 v_1 + \lambda_2 v_2) \cdot (v_1/\lambda_1 + v_2/\lambda_2)$