UNG-FN, Študijsko leto 2018/19

1. letnik - 1. stopnja Fizika in astrofizika

Linear Algebra Exam (02/09/2019)

FirstName/Ime:

LastName/Priimek:

Tick the correct answer of the given questions.

Threshold for the correction of the exercises: 5

Question 1. Which one of the following set is not a vector subspace of $\mathbb{R}^3$?

(A) $\{(x_1, x_2, x_3) \mid 3x_1 - 2x_2 + x_3 = 0\}$

(B) $\{(x_1, x_2, x_3) \mid 3x_1 + 4x_3 = 0, \quad 2x_1 - x_2 + x_3 = 0\}$

(C) $\{(x_1, x_2, x_3) \mid 3x_1 - 4x_2^2 = 0\}$

Question 2. The sign, $sgn(\sigma)$ [or parity, also indicated as $(-1)^\sigma$], of the following permutation σ:

$$\sigma = \begin{pmatrix} 1 & 2 & 3 \\ 3 & 2 & 1 \end{pmatrix}$$

is:

(A) +1 (B) -1 (C) +i

Question 3. Let V, W be two finitely generated real vector spaces. If f is an isomorphism between V and W, then:

(A) $V = W = \mathbb{R}^n$.

(B) $dim(V) = dim(W)$.

(C) $f(v) = f(w), \ \forall v, w \in V$.

Question 4. The determinant of a lower triangular matrix $A = (a_{ij}), \quad A \in \mathbb{R}^{n,n}$, is:

(A)

$$det(A) = \prod_{i=1}^{n} a_{ii}$$

(B)

$$det(A) = \prod_{i=1}^{n} a_{ni}$$

(C)

$$det(A) = \prod_{i=1}^{n} a_{in}$$

where $\prod_{i=1}^{n} a_{ii} = a_{11} \cdots a_{nn}$

Question 5. The trace of $D = AB$, where $A = (a_{ij})$, $\quad B = (b_{ij}) \quad A, B \in \mathbb{R}^{n,n}$ are two lower triangular matrices, is:
(A) $Tr(D) = \sum_{i=1}^{n} a_{ii} b_{ii}$.
(B) $Tr(D) = \sum_{i,j=1}^{n} a_{ij} b_{ij}$.
(B) $Tr(D) = Tr(A) \cdot Tr(B)$.

Question 6. Given the matrices $F = \begin{pmatrix} 2 & -1 & 3 \\ 4 & 1 & 2 \end{pmatrix}$ and $G = \begin{pmatrix} 1 & -1 \\ 0 & -1 \\ -2 & 1 \end{pmatrix}$ the product FG is:

(A) $\begin{pmatrix} -4 & 3 \\ 1 & -3 \end{pmatrix}$ (B) $\begin{pmatrix} -4 & 2 \\ 0 & -3 \end{pmatrix}$ (C) $\begin{pmatrix} -4 & 1 \\ 0 & -2 \end{pmatrix}$

Question 7. Let A be a $n \times n$ square matrix, $A = (C_1, ..., C_n)$ where $det(A) \neq 0$, and $k \in \mathbb{R} \backslash \{0\}$. Which one of the following statements is false ?
(A) $det(kC_1 + kC_2 + ... + kC_n, C_2, ..., C_n) = 0$
(B) $det(kC_1, kC_2, ..., kC_n) = k^n \cdot det(C_1, C_2, ..., C_n)$
(C) $det(C_1, C_2, ..., C_n) = -det(C_2, C_1, ..., C_n)$

Question 8. If $A, B \in GL(n)$ then:
(A) $det[(AB)^{-1}] = [det(B)]^{-1}[det(A)]^{-1}$
(B) $det(AB) = -det(BA)$
(C) $det(A + B) = det(A) + det(B)$

Question 9. Let V, W be two $\mathbb{R}$-vector spaces. If v_1, v_2 belongs to the kernel $ker(f)$ (or null space) of a linear transformation $f : V \to W$ then:
(A) $v_1 + v_2 \in ker(f)$
(B) $\alpha f(v_1) + \beta f(v_2)$ is a non-zero vector of $Im(f)$
(C) $\beta f(v_1) - \alpha f(v_2)$ is a non-zero vector of W

Question 10. Which one, among the following sets, describes a line in $\mathbb{A}^3$?
(A) $\{(x, y, z) \,|\, y = xz + z_0 \,, \, where \, z_0 \in \mathbb{R}\}$
(B) $\{(x, y, z) \,|\, 2y = x, \quad 2y - z = 1\}$
(C) $\{(x, y, z) \,|\, x_0 x + (y_0 + y)z + z_0 z = 0, \, where \, x_0, y_0, z_0 \in \mathbb{R}\}$

Question 11. In the euclidean space E^2 let's consider the orthogonal projection P_W onto the subspace $W = \{v \,|\, v = \lambda(1, 0), \lambda \in \mathbb{R}\}$ of E^2. Then it is:
(A) P_W has eigenvalues 1 and 2.
(B) $Im(P_W) = \{v \,|\, v = \lambda(1, 0)\}$ and $ker(P_W) = \{v \,|\, v = \lambda(0, 1)\}$
(C) $Im(P_W) = \{v \,|\, v = \lambda(0, 1)\}$ and $ker(P_W) = \{v \,|\, v = \lambda(1, 0)\}$

Question 12. Given the matrix A of a linear map $f : \mathbb{R}^n \to \mathbb{R}^n$, let's consider the eigenvalue problem:

$$Au = \lambda u.$$

If $\lambda = 0$ is an eigenvalue, then:
(A) the eigenvalue problem is ill-defined
(B) $u \in ker(f)$
(C) $u \in Im(f)$

Question 13. Let v, w be two vectors of the Euclidean vector space E^n. Which one of the following inequalities is false for any v, w ?
(A) $|v \cdot w| \leq ||v|| \, ||w||$
(B) $|v \cdot w| \leq 1$
(C) $|v| + |w| < |v + w|$

Question 14. The matrix (w.r.t. canonical basis) associated to the following quadratic form $Q(\mathbf{x}) = x_1^2 - 4x_2x_3 + 6x_1x_3 + 3x_2^2 + 4x_3^2$ is:

(A) $\begin{pmatrix} 1 & 0 & 3 \\ 0 & 3 & -2 \\ 3 & -2 & 4 \end{pmatrix}$ (B) $\begin{pmatrix} 1 & 0 & 6 \\ 0 & 3 & -4 \\ 6 & -4 & 4 \end{pmatrix}$ (C) $\begin{pmatrix} 1 & 0 & 1/6 \\ 0 & 3 & -1/4 \\ 1/6 & -1/4 & 4 \end{pmatrix}$

Question 15. Given the following polynomial $p(x) = -2x^3 + x^2 + 2x - 1$, which one among the following numbers is *a root* (or *a zero*) of $p(x)$?
(A) 0
(B) -2
(C) $\frac{1}{2}$
