Lectures on Linear Algebra, Week 4.
Multiple choice questions test.

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**Question 1.** Let $\phi : V \to W$ be a linear map between finitely generated vector spaces V, W. If $dim(Im(\phi)) \leq dim(W) < dim(V)$ then:
(A) $\phi$ could be surjective.
(B) $\phi$ is injective.
(C) $\phi$ could be an isomorphism.

**Question 2.** Let $\Sigma :\ Ax = b$, a sistem of linear equations where $A \in \mathbb{R}^{4,4}$ and $x, b \in \mathbb{R}^4$. If $rk(A) = 3$ and $rk(A|b) = 4$, from the Rouché Capelli's theorem it follows:
(A) There are $\infty^{4-3} = \infty^1$ solutions.
(B) $\Sigma$ is not solvable.
(C) The associated homogenous system $\Sigma_0 :\ Ax = 0$ has no solutions.

**Question 3.** The matrix (w.r.t. canonical basis) associated to the following quadratic form $Q(\mathbf{x}) = x_1^2 - 2x_1x_2 + 4x_1x_3 + 2x_2^2 + 3x_3^2$ is:

(A) $\begin{pmatrix} 1 & -1 & 2 \\ -1 & 2 & 0 \\ 2 & 0 & 3 \end{pmatrix}$ (B) $\begin{pmatrix} 1 & -2 & 4 \\ -2 & 2 & 0 \\ 4 & 0 & 3 \end{pmatrix}$ (C) $\begin{pmatrix} 1 & -1/2 & 1/4 \\ -1/2 & 2 & 0 \\ 1/4 & 0 & 3 \end{pmatrix}$

**Question 4.** Let $v_1$, $v_2$, $v_3$ vectors of $\mathbb{R}^3$, then the volume of the parallelepiped with sides given by $v_1$, $v_2$, $v_3$ is:
(A) $\frac{1}{3!}det(v_1, v_2, v_3)$
(B) $|det(v_1, v_2, v_3)|$
(C) $|det(\frac{v_1}{||v_1||}, \frac{v_2}{||v_2||}, \frac{v_3}{||v_3||})|$

**Question 5.** The isotropic cone of the following symmetric bilinear form $b : \mathbb{R}^2 \times \mathbb{R}^2 \to \mathbb{R}$, with $b((x_1, y_1), (x_2, y_2)) = 2x_1x_2 - 3y_1y_2$ is:
(A) $I_b = (0, 0)$
(B) $I_b = \{(x, y) \in \mathbb{R}^2 |\, x = \sqrt{3/2}y\}$
(C) $I_b = \{(x, y) \in \mathbb{R}^2 |\, x = \sqrt{3/2}y\} \cup \{(x, y) \in \mathbb{R}^2 |\, x = -\sqrt{3/2}y\}$

**Question 6.** Let $f$ a linear map between $\mathbb{R}^n$ and itself, and $A$ the associated matrix with respect to some basis $\mathcal{B}$, and $A'$ to $\mathcal{B}'$. If $H$ is the matrix of change of basis from $\mathcal{B}$ to $\mathcal{B}'$, then which one of the following is true ?
(A) $A = A'H^{-1}$
(B) $A = H^{-1}A'H$
(C) $A = H^{-1}A'H^{-1}$

**Question 7.** Let $A$ be a $n \times n$ square matrix, $A = (C_1, ..., C_n)$ where $\mathbb{R}^n \ni C_j \neq 0$, and $k \in \mathbb{R}\backslash\{0\}$. Which one of the following statements is true ?
(A) $det(C_1, C_2, ...., kC_1 + .... + kC_{n-1}) = 0$
(B) $det(kC_1, kC_2, ...., kC_n) = k \cdot det(C_1, C_2, ...., C_n)$
(C) $det(kC_1, C_2, ...., C_n) = \frac{1}{k!} \cdot det(C_1, C_2, ...., C_n)$

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**Question 8.** Let $g$ a bilinear map $g : \mathbb{R}^n \times \mathbb{R}^n \to \mathbb{R}$, and $A$ the associated matrix with respect to some basis $\mathcal{B}$, and $A'$ to $\mathcal{B}'$. If $H$ is the matrix of change of basis from $\mathcal{B}$ to $\mathcal{B}'$, then which one of the following is true ?
(A) $A = {}^tHA'H^{-1}$
(B) $A = H^{-1}A'H$
(C) $A = {}^tHA'H$