Lectures on Linear Algebra, Week 2.
Test (18/1/2019): tick the correct answer.

Question 1. Let $\{v_1,....,v_n\}$ be vectors of a real vector space V and $n \in \mathbb{N}, n > 1$. If any $v \in V$ can be written as $v = a_1 v_1 + ... + a_n v_n$, where $a_i \in \mathbb{R}$, then:
(A) $\{v_1,....,v_n\}$ is a basis of V.
(B) $v_i \cdot v_j = k\delta_{ij}$, where $k \in \mathbb{R}$.
(C) V is a finitely generated vector space.

Question 2. Which one, among the following equations, describes a plane in $\mathbb{R}^3$?
(A) $y = xz + z_0$, where $z_0 \in \mathbb{R}$
(B) $y = x$
(C) $x_0 x + (y_0 + y)z + z_0 z = 0$, where $x_0, y_0, z_0 \in \mathbb{R}$

Question 3. If $A, B \in GL(n)$ then:
(A) $(AB)^{-1} = B^{-1}A^{-1}$
(B) $AB{=}BA$
(C) $(AB)^2 = AB$

Question 4. In $V = \mathbb{R}^2$ consider the bilinear symmetric form $g : V \times V \to \mathbb{R}$ defined on the canonical basis $\{e_1, e_2\}$ as: $g(e_1, e_1) = 2$, $g(e_1, e_2) = g(e_2, e_1) = 0$, $g(e_2, e_2) = 3$. Then, it is true that:
(A) $g(v, v) > 0 \ \forall v \in V$.
(B) The vectors $v_1, v_2 \in (V, g)$ defined by: $v_1 = (1/\sqrt{2}, 0), v_2 = (0, 1/\sqrt{3})$, are orthonormal.
(C) g is not positively definite.

Question 5. Given the matrix $A \in \mathbb{R}^{3,2}$, with $A = \begin{pmatrix} 1 & 2 \\ 0 & 1 \\ 2 & -2 \end{pmatrix}$, the rank of A is such that:
(A) $rk(A) = 1$
(B) $rk(A) = 3$
(C) $rk({}^tA) = rk(A)$

Question 6. Let V, W be two $\mathbb{R}$-vector spaces. A linear transformation $f : V \to W$ is injective if it is found that:
(A) $f(0_V) = 0_W$
(B) $ker(f) = \{0_V\}$
(C) $f(k_1 v_1 + k_2 v_2) = k_1 f(v_1) + k_2 f(v_2)$, $\forall\ k_1, k_2 \in \mathbb{R}$ and any $v_1,\ v_2 \in V$.

Question 7. Let $(V, \cdot)$ be an euclidean real vector space. If a basis $\mathcal{B} = \{v_1, ..., v_n\}$ is given on V such that $v, w \in V$ are written as $v = \sum_{j=1}^n a_j v_j$, $w = \sum_{j=1}^n b_j v_j$, then $v \cdot w$ is:
(A) $\sum_{k=1}^n a_k b_k$
(B) always positive if $v_i \cdot v_j \geq 0 \ \forall i, j$
(C) $\sum_{k=1}^n a_k b_k$, for example, when $\mathcal{B}$ has been obtained by using the Gram-Schmidt method.