Reduced Order Methods Applied to Nonlinear Time Dependent Optimal Flow Control Problems in Environmental Marine Sciences and Engineering

Maria Strazzullo1, Francesco Ballarin1, Renzo Mosetti2, Gianluigi Rozza1
1SISSA mathLab, Trieste, Italy
2OGS, National Institute of Oceanography and Experimental Geophysics, Trieste, Italy

Motivations in Environmental Sciences

Problems dealing with environmental marine sciences must be run many times for different physical and/or geometrical parametric configurations ($\mu \in \mathbb{R}^p$). In order to describe several natural phenomena, moreover, they are usually connected to very time consuming activities such as data assimilation and inverse problems governed by parametrized partial differential equations (PDE($\mu$)).

Reduced Order Modelling = fast and reliable tool needed in order to manage rapidly and efficiently different (potentially dangerous) situations thanks to numerical simulations.

Problem Formulation: Parametrized Optimal Flow Control Problems (OFCP($\mu$))

Given $\mu \in \mathcal{D}$, a space-time parameter, $Q = \Omega \times [0,T]$ find the state-control variable $x = (y,u) \in \mathcal{Y} = Y \times U$ (suitable Hilbert spaces) which solves the PDE($\mu$) $\mathcal{L}(y,u,\mu,\mathcal{D}) = 0$ and minimizes $\mathcal{J}(y,u,\mu) = \frac{1}{2} \| y - u(\mu) \|^2_{\Omega(t_0)} + \frac{\alpha}{2} \| u(\mu) \|^2_{\Omega(t_0)}$.

- $\mathcal{D}_{OPT} \subseteq \Omega$: observation domain,
- $\Omega_{CTRL} \subseteq \Omega$: control domain,
- $u(\mu) \in Y(\Omega_{CTRL})$: data, $\alpha \in [0,1]$ penalization parameter.

Solved with Lagrangian Approach ($y \in Y = \text{adjoint variable}$).

From Truth Problem to Reduced Order Model (ROM)

Truth Problem [1,2]: Spatial Discretization (dim $N_h$) + Time Discretization (dim $N_t$) $\rightarrow$ Unfeasible for many-query context and real-time context.

Goal: to achieve the accuracy of a high fidelity approximation but at greatly reduced cost of a low order model

Idea: $x(\mu) \rightarrow$ truth problem (dim $N_h$) + $\mathcal{O}(\mu) \rightarrow$ ROM (dim $N_{h,\mathcal{O}}$) + $\mathcal{O}(\mu) \rightarrow$ $\mathcal{O}(\mu)$

Algorithm: Apply Proper Orthogonal Decomposition (POD) to the solution of the solution $y(\mu), u(\mu) \in H^2_1(\Omega) \times L^2(\Omega)$ which minimizes (1) and solves

$$\min_{(y(\mu),u(\mu)) \in \mathcal{E}} \int_0^T \int_{\Omega} (y-y(\mu))^2 \, dx \, dt + \frac{\alpha}{2} \int_0^T \int_{\Omega} (u-u(\mu))^2 \, dx \, dt + \frac{\beta}{2} \int_0^T \int_{\Omega} |\nabla y|^2 \, dx \, dt$$

subject to

$$(y(\mu),u(\mu)) \in \mathcal{E}(\mu) \times \mathcal{E}(\mu) \times \mathcal{E}(\mu) \times \mathcal{E}(\mu) \times \mathcal{E}(\mu) \times \mathcal{E}(\mu) \times \mathcal{E}(\mu)$$

Results and Relative Errors $\sim 10^{-15}$

N Speedup
5 364
10 350
15 317

Pollutant Control in the Gulf of Trieste [3]

Motivations: monitor, manage and predict dangerous marine phenomena in a fast way in order to set up an environmental plan of action.

Aim: pollutant loss $y \in L^2_1(\Omega)$ under a safeguard threshold $y_l$.

Given $\mu \in [0,1] \times [-1,1] \times [-1,1]$, find $(y(\mu),u(\mu)) \in H^2_1(\Omega) \times L^2(\Omega)$ which minimizes (1) and solves

$$\min_{(y_u,u)} \int_0^T \int_{\Omega} (y-y(\mu))^2 \, dx \, dt + \frac{\alpha}{2} \int_0^T \int_{\Omega} (u-u(\mu))^2 \, dx \, dt + \frac{\beta}{2} \int_0^T \int_{\Omega} |\nabla y|^2 \, dx \, dt$$

subject to

$$(y(\mu),u(\mu)) \in \mathcal{E}(\mu) \times \mathcal{E}(\mu) \times \mathcal{E}(\mu) \times \mathcal{E}(\mu) \times \mathcal{E}(\mu) \times \mathcal{E}(\mu) \times \mathcal{E}(\mu)$$

Results and Relative Errors $\sim 10^{-9}$

N Speedup
5 6
10 1.6
15 1.2

Time Dependent Problem: River Bed [5]

Aim: recover $y_0 = [\mu^1(y^0 - y^0_0) + 2(y^0 - y^0_0)^2 + y^0_0, 0, y^0_0, y^0_0, 0, 0, 0, 0]$, $\mu \in [1/2, 0, 1/2, 2] \times [1, 2] \times [1, 3]$, find $(y(\mu),u(\mu),\mathcal{D}) \in L^2(0,T;H^2_1(\Omega))^2 \times L^2(0,T;L^2(\Omega))^2$ which solves

$$(y(\mu),u(\mu),\mathcal{D}) \in \mathcal{E}(\mu) \times \mathcal{E}(\mu) \times \mathcal{E}(\mu) \times \mathcal{E}(\mu) \times \mathcal{E}(\mu) \times \mathcal{E}(\mu) \times \mathcal{E}(\mu)$$

Results and Relative Errors $\sim 10^{-14}$

N Speedup
20 47579
25 34335
30 22477
35 17420

References


Acknowledgements

We acknowledge the support by European Union Funding for Research and Innovation – Horizon 2020 Program – in the framework of European Research Council Executive Agency: Consolidator Grant H2020 ERC CoG 2015 AROMA-CFD project 681447 “Advanced Reduced Order Methods with Applications in Computational Fluid Dynamics”. We also acknowledge the INdAM-GNCS project “Advanced intrusive and non-intrusive model order reduction techniques and applications”.