Introduction

When dealing with Parametrized Partial Differential Equations the computational cost required by a large number of solutions for each new value of the involved parameters may be unaffordably large. To mitigate that, different methods have been studied in order to find solutions in a more efficient way (11).

In particular, in our work we exploit a POD approach to obtain the basis functions used to project the original problem and to reconstruct an approximate solution manifold. Everything is developed in a Finite Volume framework (2) where we try to manage segregated approaches also at reduced level.

1) Segregated approaches, FOM level

Incompressible Navier-Stokes equations:
\[
\begin{align*}
\frac{\partial u}{\partial t} + \nabla u &+ \sum_{j=1}^{N} \nabla p_j = \rho \nabla^2 u \\
\nabla \cdot (\rho u) &= 0
\end{align*}
\]

Since the equations are coupled in principle it is not possible to solve them separately; they should be treated as a system and iterative block solvers are needed for nonlinear systems. Unluckily these methods require the storage of big matrices at each step that makes them very inefficient for big meshes. A good compromise between efficiency and quality of the solution is represented by the segregated methods employed in SIMPLE and PISO strategies solvers: momentum and continuity equations are solved iteratively and one at a time exchanging information at the end of each iteration.

2) SIMPLE algorithm

We based our work on the SIMPLE algorithm, one of the most spreaded segregated approaches:

- 1. guess beginning pressure and velocity fields \( P^0 \) and \( u^0 \) respectively, set \( P = P^0 \) and \( u = u^0 \);
- 2. project and solve the momentum equation;
- 3. project and solve the pressure equation;
- 4. reconstruct the full velocity field;
- 5. update fluxes to have a divergence-free velocity solution;
- 6. if convergence check is not satisfied repeat from point 2, otherwise stop.

3) Segregated approaches, ROM level

In order to be as coherent as possible with respect to the FOM model, we decided to simulate the SIMPLE algorithm also at the ROM level so that it is possible to project the same equations used to obtain the snapshots:

1. guess beginning discretized pressure and velocity fields \( P' \) and \( u' \) respectively, set \( P = P' \) and \( u = u' \), reconstruct the full velocity and pressure fields;
2. project and solve the momentum equation;
3. reconstruct the full velocity field;
4. project and solve the pressure equation;
5. reconstruct the full pressure field;
6. update fluxes to have a divergence-free velocity solution;
7. if convergence check is not satisfied repeat from point 2, otherwise stop.

4) Applications: parametrized viscosity problem

We have applied the ROM SIMPLE algorithm to an incompressible Navier-Stokes problem where the geometry is represented by a back step and the parameter we have chosen is the viscosity \( \mu \in [0.01, 1] \). 50 snapshots have been solved to apply the POD.

5) Applications: parametrized geometry problem

One of the impress works in the group is relative to geometrical parametrization and hyper-reduction (3). In this case the angle of attack of an airfoil has been parametrized. We fixed \( \alpha \in [-5,5] \) and solved 100 snapshots to project the problem. To compare the different snapshots it has been necessary to move the mesh in the right way; starting from a reference one, the modified one is obtained by the use of radial basis functions interpolation. In this way it is possible to compare the solutions: all the meshes have the same number of cells.

As we can see from the figures before, the FOM (left) and ROM (right) velocity solutions are pretty much the same. Also FOM (left) and ROM (right) pressure solutions are comparable.

6) Applications: compressible flows

Future work in the group will be devoted to compressible flows. In this case continuum Navier-Stokes equations are needed:

\[
\begin{align*}
\frac{\partial u}{\partial t} + \nabla \cdot (\rho u) &= 0 \\
\frac{\partial p}{\partial t} + \nabla \cdot (\rho u) - \nabla \cdot [\mu \nabla (u^2)] &= \rho g - \nabla \cdot (P + \frac{\mu}{2} \nabla u) - \nabla \cdot \left( \frac{\rho u}{2} \nabla (u^2) \right) \\
\frac{\partial E}{\partial t} + \nabla \cdot (\rho E u) &= \rho g - \nabla \cdot (P + \frac{\mu}{2} \nabla u)
\end{align*}
\]

In this last figure it is shown the velocity field obtained by a compressible FOM SIMPLE algorithm where state and energy equations have been added with respect to the incompressible model. The work for the reduced part is still in progress.

References


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