Reduced Order Modelling for Data Assimilation in Parametrized Optimal Control Framework

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PARAMETRIZED OPTIMAL FLOW CONTROL PROBLEMS are suited for several fields of applied mathematics, such as:

- data assimilation inverse problems
- forecasting models

PDEs, DATA, CONTROL (physics/boundary)

• Functional minimization (Non-linear Time-Dependent)

Optimal solution

the parameter changes...

STRATEGY (FROM HIGH-FIDELITY TO REDUCED MODEL)

Problem: solve several parametric instances of

\[
\min_{y \in \mathcal{X}, u \in \mathcal{U}} \frac{1}{2} \| y - y_0 \|^2_Y + \frac{\alpha}{2} \| u \|^2_U \quad \text{subject to} \quad \mathcal{E}(y, u; \mu) = 0
\]

Space-Time (\( \dim = N_h + N_t \)) Lagrangian Formulation

(\( \dim = 3N \))

- General Framework (linear, non-linear, time-dependent)
- Three equations
- High - Dimensionality

Proper Orthogonal Decomposition (POD) for Space-Time Optimal Control Problems [1]

APPLICATION: CONTROL FOR COASTAL WATER HEIGHT (SWE) [2]

Minimisation of

\[
\frac{1}{2} \int_0^T (h - h_0(\mu))^2 \, dx \, dt + \frac{1}{2} \int_0^T (v - v_0(\mu))^2 \, dx \, dt + \frac{\alpha}{2} \int_0^T u^2 \, dx \, dt
\]

constrained to

\[
\begin{align*}
\rho + \rho \Delta v + \rho (v \cdot \nabla v) - \nabla q - a &= 0 & \text{in } \Omega \times [0, T], \\
\mathbf{n} \cdot (q \mathbf{n}) &= 0 & \text{on } \partial \Omega \times [0, T], \\
v &= v_0 & \text{on } \partial \Omega \times [0, T], \\
h &= h_0 & \text{on } \partial \Omega \times [0, T], \\
v &= 0 & \text{on } \partial \Omega \times [0, T].
\end{align*}
\]

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References


WHY NEURAL NETWORKS?

1) To correct data which are difficult to interpret, scattered.
2) Faster online reduced solver
3) Analysis (pre-process and post-process phase)

OTHER APPLICATIONS

Pollutant control in the Gulf of Trieste (Uncertainty Quantification) [3].

Driving bifurcations phenomena through optimal control (F. Pichi Poster on ROM and NN for Bifurcations) [4].

Water height solution for \( \mu = (0.1, 0.5, 1) \)

\( \Omega = [0, 10] \times [0, 10] \)

\( \alpha = 10^{-5} \)

(\( \Delta t = 0.1 \))

Errors \( \sim 1e-4 \), Speedup \( \sim 30 \)

ROM vs FE dim = 270 vs 94'016.