SISSA mathLab main references and open source software on mathematical modelling for hemodynamics/cardiovascular systems

Prof. G. Rozza's Group

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Mathematical modelling and **numerical simulations** of the **patient-specific configuration of the cardiovascular system** represents a research area of significant importance and in recent times it had a strong impulse due to the increasing demand from the medical community for quantitative investigations of cardiovascular diseases. High-fidelity numerical methods are commonly used for the solution of the equations governing the blood flow dynamics, where the parameters are geometric features, boundary conditions and/or physical properties. However, for applications which require repeated model evaluations over a wide range of parameter values, these methods are very expensive in terms of computational time and memory demand due to the large amount of degrees of freedom to be considered for a proper description of the flow system.

In such a framework, **reduced order models** are applied to enable fast computations varying the parameters, as often required in the clinical context. A further step forward in this scenario has been recently given by the development and diffusion of **artificial intelligence technologies** that could be used as an alternative and/or as a support tool to the classic model reduction framework thanks to their very high computational efficiency. This would be able in perspective to allow real time simulations to be accessed in hospitals and operating rooms through a simple "click".

In the following we provide a comprehensive list (together with a brief description of the contents) of Prof. Rozza's group publications in the framework of mathematical modelling for hemodynamics. Then we introduce open source software libraries developed by the group which have been used in these works and list the hospitals and universities we collaborate with.

Publications

A "**virtual surgery platform**" for patient-specific **coronary artery bypass grafts** has been developed in

- F. Ballarin, E. Faggiano, A. Manzoni, A. Quarteroni, G. Rozza, S. Ippolito, C. Antona, and R. Scrofani, *"Numerical modeling of hemodynamics scenarios of patient-specific coronary artery bypass grafts"*, Biomechanics and Modeling in Mechanobiology, 16(4), pp. 1373-1399, 2017. [doi]
- F. Ballarin, E. Faggiano, S. Ippolito, A. Manzoni, A. Quarteroni, G. Rozza, and R. Scrofani, "Fast simulations of patient-specific haemodynamics of coronary artery bypass grafts based on a POD-Galerkin method and a vascular shape parametrization", Journal of Computational Physics, 315, pp. 609-628, 2016. [doi]

and it allows to compare the effect of parametric changes on the fluid dynamics. Relevant parametric changes are those associated to transient patient's **conditions** (e.g. rest or stress, modeled by Reynolds numbers), **disease** (modeled by severity of stenosis) or **surgical choices** (modeled by geometrical properties of the grafts, e.g. diameters or angles). Model reduction by **POD-Galerkin** and a tailor-made **shape parametrization** allows to perform sensitivity analyses at a fraction of the computational cost that standard discretization techniques would have required.

On the same application of patient-specific coronary artery bypass grafts, a **data-driven ROM** based on **Artificial Neural Network** has been tested and validated in

- P. Siena, M. Girfoglio, and G. Rozza, "*Fast and accurate numerical simulations for the study of coronary artery bypass grafts by artificial neural network*", submitted, 2022. [preprint]
- P. Siena, M. Girfoglio, F. Ballarin, and G. Rozza, "*Data-driven reduced order modelling for patient-specific hemodynamics of coronary artery bypass grafts with physical and geometrical parameters*", submitted, 2022.

This is a collaboration with Politecnico di Milano, Catholic University, and Sacco Hospital.

Integration with **measured experimental (clinical) data** in a reduced optimal flow control and data assimilation framework has been sought in

- Z. Zainib, F. Ballarin, S. Fremes, P. Triverio, L. Jiménez-Juan, and G. Rozza, "*Reduced order methods for parametric optimal flow control in coronary bypass grafts, towards patient-specific data assimilation*", International Journal for Numerical Methods in Biomedical Engineering, 37(12), e3367, 2020. [preprint] [doi]
- Z. Zainib, "*Reduced order parametrized viscous optimal flow control problems and applications in coronary artery bypass grafts with patient-specific geometrical reconstruction and data assimilation*". Ph.D. thesis in Mathematical Analysis, Modelling, and Applications, SISSA, Italy, 2019. [download]

The mathematical formulation relies on an **optimal control framework** in which coefficients associated to outflow boundary are tuned so that the numerical simulation matches as best as possible the experimental results.

- E. Fevola, F. Ballarin, L. Jimènez-Juan, S. Fremes, S. Grivet-Talocia, G. Rozza, and P. Triverio, "*An optimal control approach to determine resistance-type boundary conditions from in-vivo data for cardiovascular simulations*", International Journal for Numerical Methods in Biomedical Engineering, 37(10), e3516, 2021. [preprint] [doi]

This is a collaboration with University of Toronto, Sunnybrook Hospital and Politecnico di Torino.

A combination of **model reduction** and **parameter space reduction** techniques has been presented in

- M. Tezzele, F. Ballarin, and G. Rozza, "*Combined parameter and model reduction of cardiovascular problems by means of active subspaces and POD-Galerkin methods*", in Mathematical and Numerical Modeling of the Cardiovascular System and Applications, D. Boffi, L. F. Pavarino, G. Rozza, S. Scacchi, and C. Vergara (eds.), Springer International Publishing, pp. 185-207, 2018. [preprint] [doi]

and applied to an idealized **carotid artery bifurcation**. Given a template healthy carotid artery, we introduce control points associated to a **geometrical parametrization** to represent the presence of a stenosis in the daughter branches. **Active subspaces** are employed in order to reduce the dimension of the parametric space (associated to the geometrical parametrization) based on a pressure drop criterion. Model reduction by **POD-Galerkin** methods benefits from this smaller parameter space both during the offline and online stage. For this studies we use PyGeM (see below)

The effect of a continuous flow **Left Ventricular Assist device** on the aortic flow is investigated by means of a model reduction framework based on **POD** with **interpolation** in:

- M. Girfoglio, F. Ballarin, G. Infantino, F. Nicolò, A. Montalto, G. Rozza, R. Scrofani, M. Comisso, and F. Musumeci, "*Non-intrusive PODI-ROM for patient-specific aortic blood flow in presence of a LVAD device*", submitted, 2022. [preprint]
    - M. Girfoglio, L. Scandurra, F. Ballarin, G. Infantino, F. Nicolò, A. Montalto, G. Rozza, R. Scrofani, M. Comisso, and F. Musumeci, "*Non-intrusive data-driven ROM framework for hemodynamics problems*", Acta Mechanica Sinica, 37, pp. 1183-1191, 2021 [preprint] [doi]

The goal has been to build a **physical parametric setting** with respect to the LVAD flow rate.
This is a collaboration with Sacco and San Camillo hospitals.

Some preliminary investigations in the context of **multiphysics problems** have been already carried out, especially for what concerns **fluid-structure interaction problems** in channels:

- M. Nonino, F. Ballarin, G. Rozza, and Y. Maday, "*Projection based semi--implicit partitioned Reduced Basis Method for non parametrized and parametrized Fluid--Structure Interaction problems*", submitted, 2022. [preprint]
- M. Nonino, F. Ballarin, and G. Rozza, "*A Monolithic and a Partitioned, Reduced Basis Method for Fluid-Structure Interaction Problems*", Fluids, 6(6), 229, 2021. [preprint] [doi]
- M. Nonino, F. Ballarin, G. Rozza, and Y. Maday, "*Overcoming slowly decaying Kolmogorov n-width by transport maps: application to model order reduction of fluid dynamics and fluid-structure interaction problems*", submitted, 2019. [preprint]
- F. Ballarin, G. Rozza, and Y. Maday, "*Reduced-order semi-implicit schemes for fluid-structure interaction problems*", in Model Reduction of Parametrized Systems, P. Benner, M. Ohlberger, A. Patera, G. Rozza, and K. Urban (eds.), Springer International Publishing, vol. 17, pp. 149-167, 2017. [preprint] [doi]
- F. Ballarin and G. Rozza, "*POD–Galerkin monolithic reduced order models for parametrized fluid-structure interaction problems*", International Journal for Numerical Methods in Fluids, 82(12), pp. 1010-1034, 2016. [doi]
- M. Girfoglio, A. Quaini and G. Rozza, "*Fluid-structure interaction simulations with a LES filtering approach in solids4Foam",* Communications in Applied and Industrial Mathematics, 12(1), 13-28, 2021. [preprint] [doi]

These methodological developments are motivated by the inherent multiphysics nature of the cardiovascular system, and are specifically tailored for a blood-vessel coupling. Further developments for blood-valve interaction will be scheduled in future.
This is a collaboration with University of Houston and Laboratorire Jacques Louis Lions, Sorbonne, Paris.

Reduced order methods for bifurcation of parametric flows, e.g. Coanda Effect in mitral valves, are presented in:

- G. Pitton, G. Rozza. *On the Application of Reduced Basis Methods to Bifurcation Problems in Incompressible Fluid Dynamics*. J. Scie.Comp., 2017, 73, p.157. [https://arxiv.org/abs/1801.00923]
- G. Pitton, A. Quaini, G. Rozza. *Computational reduction strategies for the detection of steady bifurcations in incompressible fluid-dynamics: applications to Coanda effect in cardiology*, J. Comp. Phys., 2017, 344, p.544. [https://arxiv.org/abs/1708.09718]
- M. Hess, A. Quaini, G. Rozza. *Reduced Basis Model Order Reduction for Navier-Stokes equations in domains with walls of varying curvature*, Int. J. CFD, 2019 [https://arxiv.org/abs/1901.03708]
- F. Pichi, F. Ballarin, G. Rozza, J.S. Hesthaven, *An artificial neural network approach to bifurcating phenomena in computational fluid dynamics, submitted*, 2021. [preprint]
- M. Khamlich, F. Pichi, G. Rozza. *Model order reduction for bifurcating phenomena in Fluid-Structure Interaction problems*, submitted, 2021. [preprint]

- M. Hess, A. Quaini, G. Rozza, *A comparison of reduced-order modeling approaches for PDEs with bifurcating solutions*, Electron. Trans. Numer. Anal. (ETNA), 56, 52-65, 2022. [preprint] [doi]

These works are carried out in collaboration with University of Houston and EPFL.

SISSA mathLab Open Source Software and Tools

RBniCS is an implementation in **FEniCS** of several reduced order modelling techniques (and, in particular, **certified reduced basis** method and **Proper Orthogonal Decomposition-Galerkin** methods) for parametrized problems. An object-oriented approach and an intuitive and versatile python interface greatly simplifies the definition of the mathematical problem. Several tutorials show the capabilities of the library. [gitlab] [github] [SISSA mathLab].

multiphenics is a python library that aims at providing tools in **FEniCS** for an easy prototyping of **multiphysics** problems on conforming meshes. In particular, it facilitates the definition of **subdomain/boundary restricted** variables and enables the definition of the problem by means of a **blockstructure**. Several tutorials show the capabilities of the library. [gitlab] [github] [SISSA mathLab].

PyGeM (Python Geometrical Morphing) is a package that allows you to deform a given geometry or mesh with different deformation techniques such as Free Form Deformation, Radial Basis Functions and Inverse Distance Weighting. [github] [SISSA mathLab].

F. Salmoiraghi, A. Scardigli, H. Telib, and G. Rozza, ***Free Form Deformation, mesh morphing and reduced order methods: enablers for efficient aerodynamic shape optimization,*** Int. J. CFD, 2018 [arxiv].

EZyRB is a python library for the Model Order Reduction based on **baricentric triangulation** for the selection of the parameter points and on **Proper Orthogonal Decomposition** for the selection of the modes. It is ideally suited for actual industrial problems, since its structure can interact with several simulation software simply providing the output file of the simulations. Up to now, it handles files in the vtk and mat formats. It has been used for the model order reduction of problems solved with **matlab** and **openFOAM**. [github] [SISSA mathLab].

ARGOS, the Advanced Reduced Groupware Online Simulation platform, is the online platform for the reduced order scientific computing framework ITHACA “In real Time Highly Advanced Computational Applications”.A gallery of applications and tutorials (from RBniCS) is available online and allows computation to be run from standard web browsers. In particular, the **ATLAS** project will collect all cardiovascular applications. [argos.sissa.it] [argos.sissa.it/atlas]. ATLAS is developed in cooperation with University of Pavia and their cooperation with Policlinico San Matteo and Policlinico San Donato.

The complete list of SISSA mathLab software is available on GitHub and on SISSA mathLab website.

Collaborations

SUNNYBROOK HEALTH SCIENCES CENTRE, TORONTO, CANADA & UNIVERSITY OF TORONTO, CANADA

Clinical collaboration on the study of patient-specific aortic coarctation cases and coronary artery bypass grafts configurations. Methodological development of reduced order modelling strategies for data assimilation based on experimental measurements.

OSPEDALE LUIGI SACCO, MILANO, ITALY

Clinical collaboration on the study of patient-specific coronary artery bypass grafts and left ventricular assist devices. Methodological development on reduced order modelling strategies for shape parametrization of bypass grafts.

AZIENDA OSPEDALIERA SAN CAMILLO FORLANINI, ROMA, ITALY

Clinical collaboration on the study of left ventricular assist devices. Methodological development of shape parametrization and non intrusive reduced order modelling techniques.

UNIVERSITY OF HOUSTON, TEXAS, U.S.

Methodological development of reduced order modelling techniques for problems with bifurcations.

I.R.C.C.S. Policlinico San Donato

UNIVERSITY OF PAVIA and POLICLINICO SAN DONATO, ITALY.

Developments of ATLAS and data driven non-intrusive model reduction techniques for parametric flows in haemodynamics.

UNIVERSITÀ DI PAVIA