A novel deep learning architecture for the model reduction of parametric time-dependent problems

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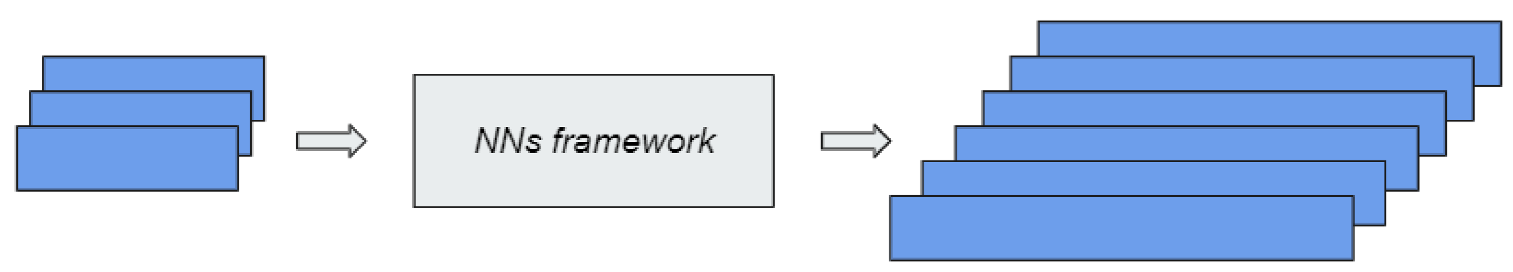
Introduction

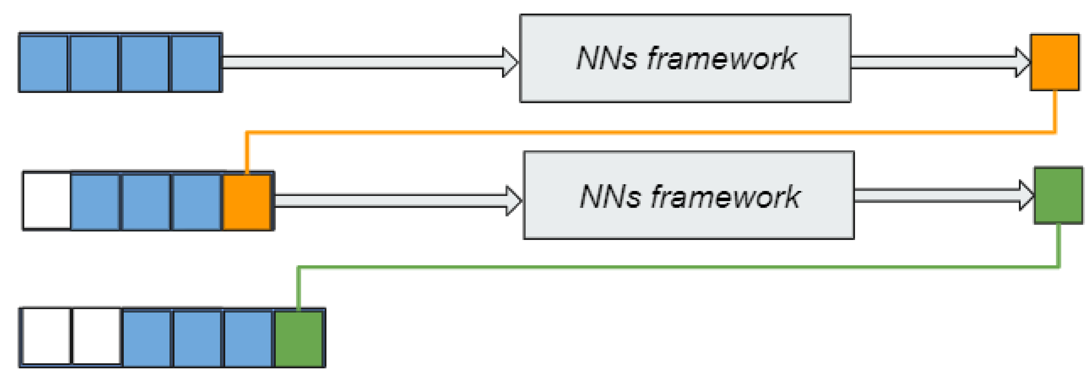
Deep learning-based reduced order models (DL-ROMs) have been recently proposed to overcome common limitations shared by conventional ROMs – built, e.g., exclusively through proper orthogonal decomposition (POD) – when applied to **nonlinear time-dependent parametrized PDEs**. In this work, thanks to a prior **dimensionality reduction** through POD, a **two-step DL-based prediction framework** has been implemented with the aim of providing long-term predictions with respect to the training window, for unseen parameter values. It exploits the advantages of **Long-Short Term Memory** (LSTM) layers combined with Convolutional ones, obtaining an architecture that consists of two parts: the first one aimed at providing a certain number of independent predictions for each new input parameter, and a second one trained to properly combine them in the correspondent exact evolution in time. In particular, the developed architecture has been tested for the reduction of the **incompressible Navier-Stokes equations in a laterally heated cavity**.

1 - Proposed Framework

The Framework is articulated in two Neural Network structures trained separately. In particular, the NNs used have been designed exploiting the sequential combination of 1D-Convolutional Layers and LSTM layers. Indeed, while the first filters the 1D input data, composed by a certain time-window of the time-series, the latter is capable of learning the long-term dependencies hidden in the evolution of the signal, which brings significant advantages in the future-time predictions.



Starting from exact numerical resolutions for short time-windows for few different parameters' values belonging to the training set, we aim to forecast the time-evolution of the signal of interest for arbitrarily long time periods, and for each general parameter belonging to the parameter-space considered.

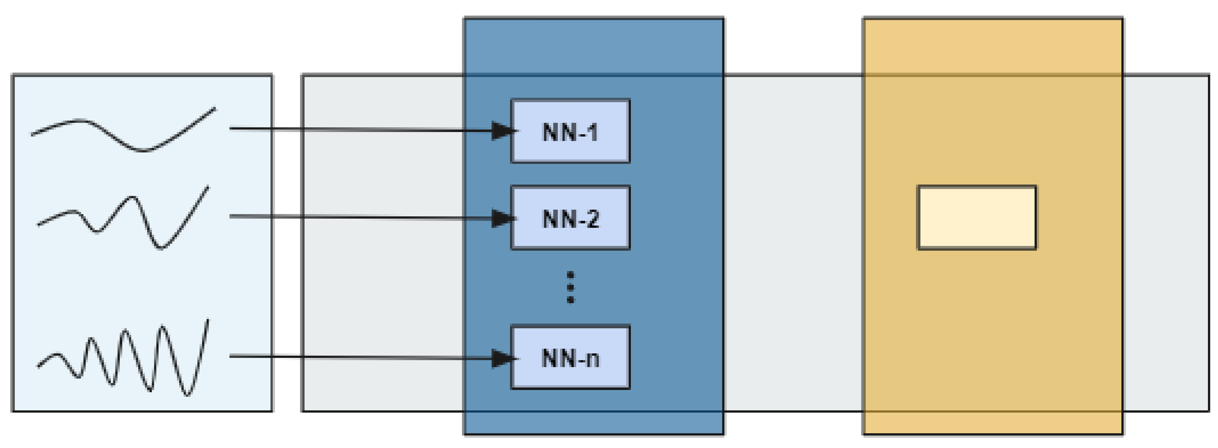


Therefore, the framework would result in an internal representation of the parametric system's dynamics, which can be reconstructed for an arbitrarily long period building a self-sustaining loop in time that, at each iteration, updates the time-window in input to the framework with the already obtained predictions.

2A - Training, phase I

The first-step Neural Networks are trained independently with different training-sets, each one composed by a computed solution $(g(t_i, a_j))$ for a different value of the parameters considered in the training $(a_j \in P_{training} \subset P_{space})$.

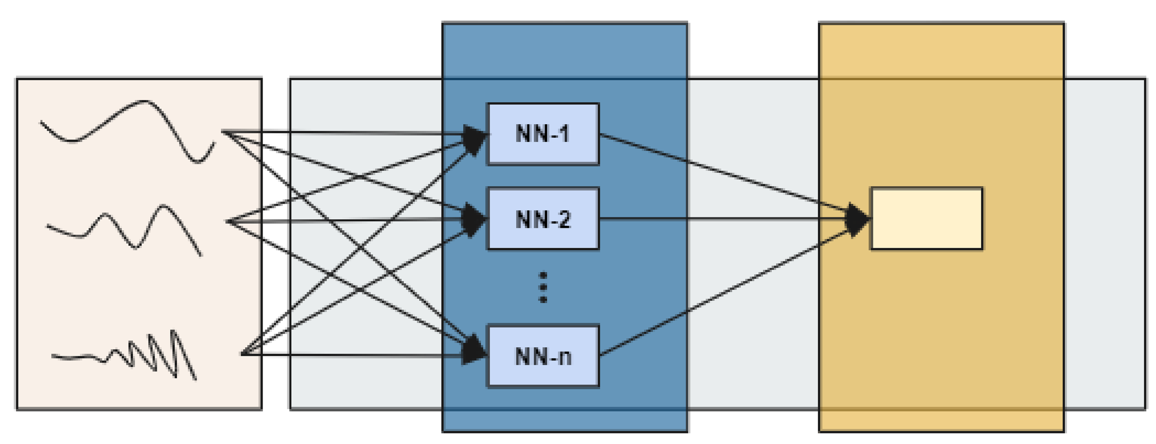
$$g(t_i, a_j) = F_j(g(t_{i-W}, a_j), ..., g(t_{i-1}, a_j), a_j) \quad \forall t_i > W, \quad \forall j \in \{1, ..., n\} \ s.t. \ a_j \in P_{\text{training}}$$



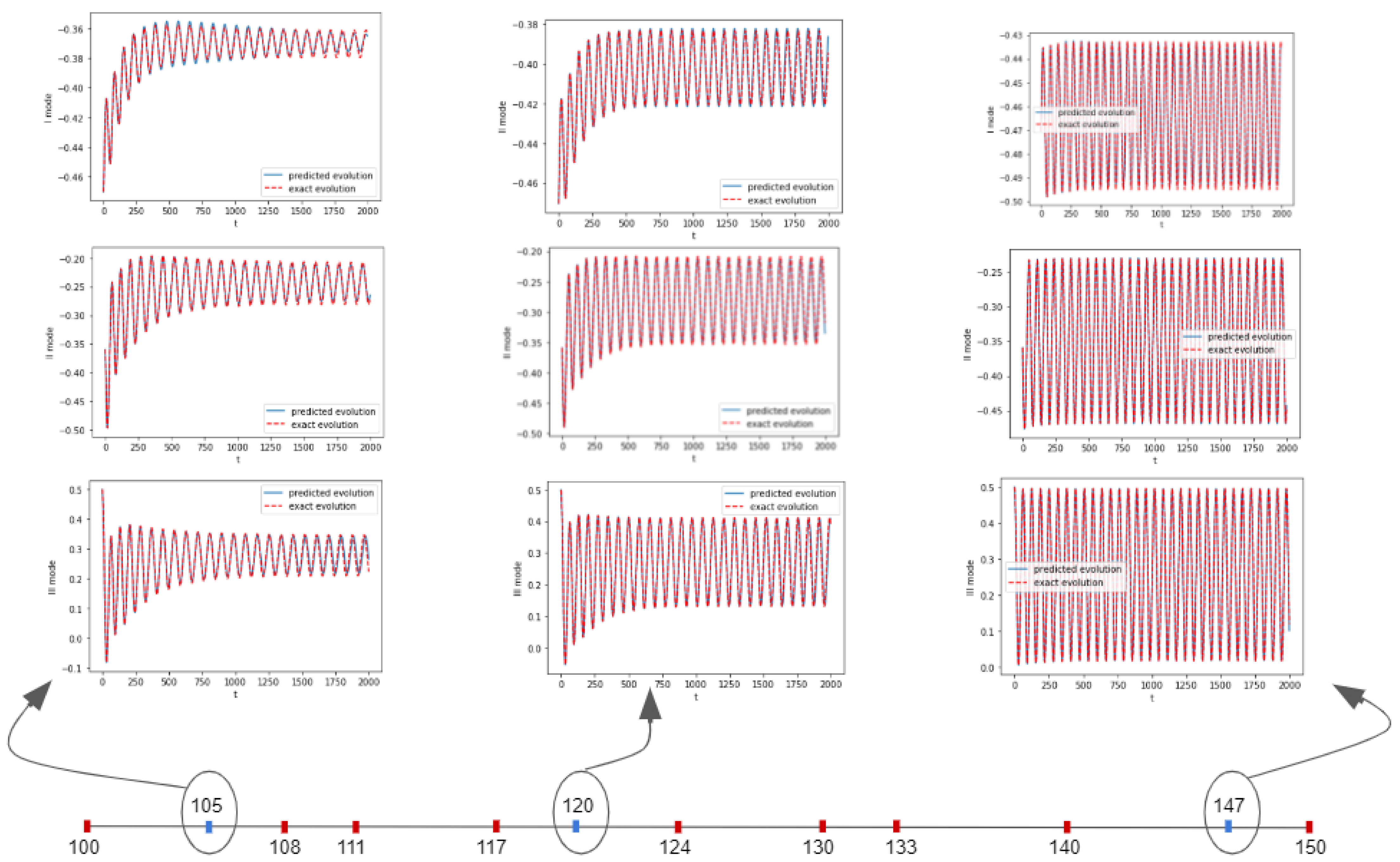
2B - Training, phase II

The second-step Neural Network is trained a posteriori to the first-step predictions. Indeed, they correspond to its inputs and are appropriately combined to result in a proper forecast for the next temporal step and for a general parameter.

$$g(t_i, p) = G(F_1(\mathbf{t}, a_1), ..., F_n(\mathbf{t}, a_n), p) \quad \forall t_i > W, \quad \forall p \in P_{\text{space}}$$



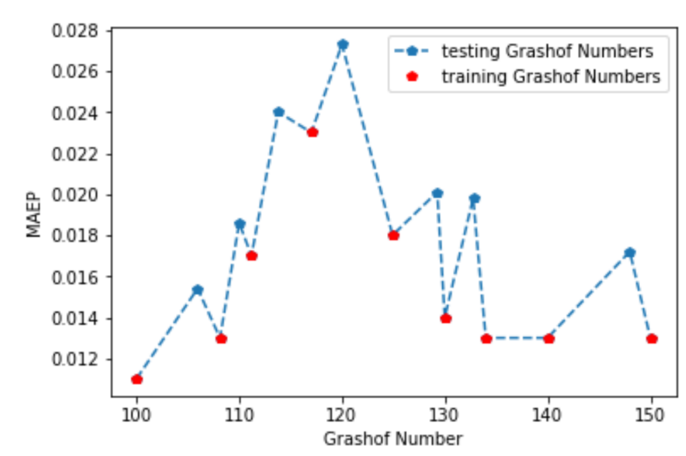
3 - Application to incompressible Navier-Stokes equation in a heated cavity



Incompressible Navier-Stokes equation:

$$\frac{\delta \mathbf{u}}{\delta t} + (\mathbf{u} \cdot \nabla)\mathbf{u} - \nu \nabla^2 \mathbf{u} = -\frac{1}{\rho}\nabla p + \mathbf{g}$$

Choosing the case of a laterally heated cavity, the parameter space now represents the Grashof Number's values. After a POD decomposition and after having trained the framework with the exactly computed modes related to 9 different parameter values, the modes' evolution in time is predicted for general Grashof Numbers.



The modes' forecasts are performed in a mean time of 12 minutes, improving of a factor of 10 the time taken to extract them exactly by simulation. The predictions, tested for the first 4 modes of 7 random Grashof numbers' values, have shown a mean absolute error percentage of 1.7%, and a maximum absolute error percentage of 2.8% on 10 times bigger periods than the initial time-window width given to the framework.

References and Acknowledgements

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