Introduction

In this work, we introduce a reduced order model (ROM) to describe the evolution of urban air pollutants. The underlying model is the transport-diffusion equation, where the convective field is given by the solution of the Navier-Stokes equation, and the source term is an empirical time series.

We developed a hybrid technique based on POD with interpolation (POD-I) coupled with Galerkin Projection (POD-G) in order to preserve the advantages of both approaches. Our data-driven method exploits a feedforward neural network to recover nonintrusively the convective reduced-order operator needed for the online evaluation.

Problem Formulation

The transport-diffusion equation is a linear partial differential equation, which takes the form:

$$
\frac{\partial c}{\partial t} - \nu \Delta c + \nabla \cdot (u c) = f;
$$

where $c(x,t) : \mathbb{R}^n \times [0, +\infty) \rightarrow \mathbb{R}$ is the unknown function, which can be thought of as the concentration of a pollutant such as NO$_2$.

Specifically, the quantity $c(x,t) \Delta V$ represents the mass present at time $t$ in an infinitesimal neighborhood of the point $x$. Consistently, the mass of pollutant present in volume $V$ at time $t$ is given by:

$$
\int_V c(x,t) \, dV.
$$

The term "$\nabla \cdot (u c)$" models the convective transport effect, that is, the transport of the pollutant due to the motion of the fluid in which it is immersed.

Reduced order model

We employed the Reduced Basis (RB) method. The POD modes are used to approximate the solution $c(t, \mu)$ for any new value of the parameter with a linear combination:

$$
c(t, \mu) \approx \sum_{i \in I} a_i(\mu, t) \phi_i(x),
$$

where $a_i(\mu, t)$ are the parameter dependent coefficients and $\phi_i(x)$ are the parameter independent basis functions.

The coefficients of Eq. 5 are then obtained solving:

$$
M_r \dot{a} - \nu_T B_r a + C_r a = f_r(t),
$$

where each term inside Eq. 6 is obtained by Galerkin projection:

$$
\begin{align*}
(M_r)_{ij} &= (\phi_i, \phi_j)_{L_2(\Omega)}, \\
(B_r)_{ij} &= (\phi_i, \Delta \phi_j)_{L_2(\Omega)}, \\
(C_r)_{ij} &= (\phi_i, \nabla \cdot (u \mu) \phi_j)_{L_2(\Omega)}, \\
(f_r)_{ij} &= (\phi_i, f(t))_{L_2(\Omega)}.
\end{align*}
$$

Numerical Results

- **Mesh**: main campus of the University of Bologna, with 40k cells.
- **Numerical Discretization**: The offline phase was conducted using the finite volume method. The libraries employed are OpenFoam and ITAHACA-FV [3].
- **Dataset**: Syntetic emission data using the fastrace traffic model and 1 year long empirical measurements for the inlet velocity condition.

POD-NN [1] and POD-DEIM [4]

The complexity in the treatment of the Eq. 6 concerns the convective term and the empirical source term, for which we employed the following strategies:

- **The reduced order convective matrix $C_r$ is obtained using the POD-NN approach, that is:**

$$
(C_r)_{ij} = \sum_{k=1}^{N_u}(\phi_i, \nabla \cdot (u_k(\mu)\phi_j))_{L_2(\Omega)};
$$

where the coefficients $u_k$ are the output of a feedforward NN.

- **The DEIM is employed for the source term $f(t)$, which is approximated as:**

$$
f(t) \approx \sum_{i=1}^{N_{DEIM}} \sum_{j=1}^{N_u} p_i(t) c_j(x);
$$

where the coefficients are determined with a point-wise evaluation of the function $f$ in judiciously chosen magic points $q_i$.

References


