ROM for Large-scale Modelling of Urban Air Pollution

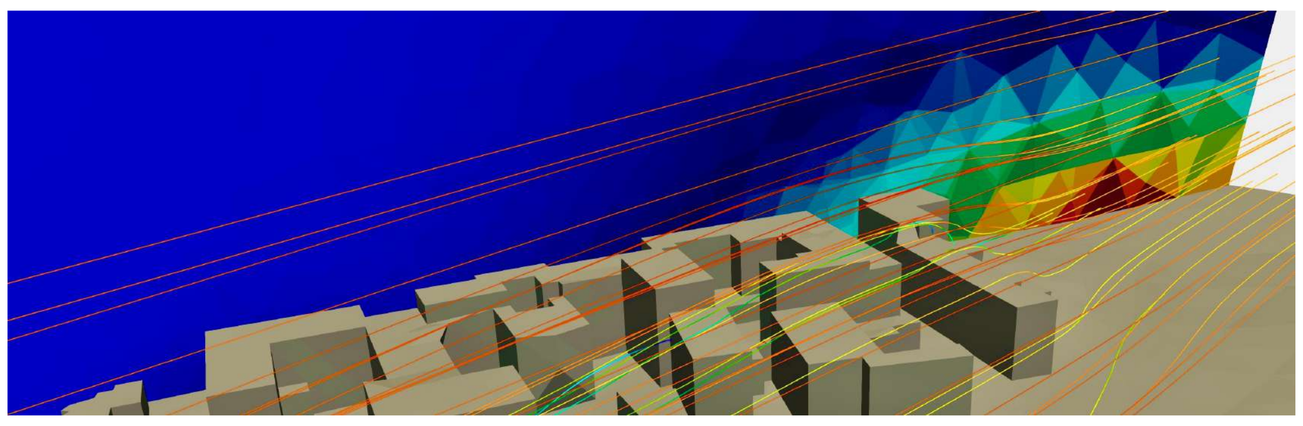
Moaad Khamlich [1], Giovanni Stabile [1], Gianluigi Rozza [1], László Környei[2], Zoltán Horváth[2]

[1] Mathlab, Mathematics Area, International School of Advanced Studies (SISSA), Trieste, Italy

[2] Széchenyi István University, Department of Mathematics and Computational Sciences, Gyor, Hungary

Introduction

In this work, we introduce a **reduced order model (ROM)** to describe the evolution of urban air pollutants. The underlying model is the transport-diffusion equation, where the convective field is given by the solution of the Navier-Stokes equation, and the source term is an empirical time series. We developed a hybrid technique based on **POD with interpolation (POD-I)** coupled with Galerkin Projection (POD-G) to preserve the advantages of both approaches. Our data-driven method exploits a feedforward neural network to recover nonintrusively the convective reduced-order operator for the online evaluation.



Streamlines of the velocity and a cross section of the concentration field.

Problem Formulation

The **transport-diffusion equation** is a linear partial differential equation, which takes the form:

$$\frac{\partial c}{\partial t} - \nu \Delta c + \nabla \cdot (\mathbf{u} c) = f; \tag{1}$$

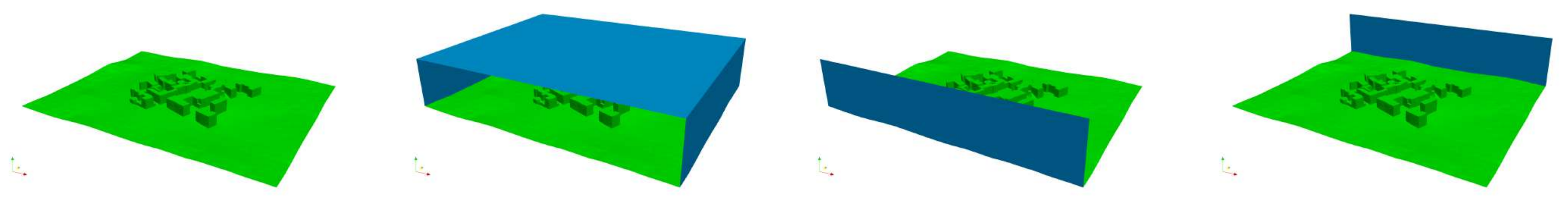
where $c(\mathbf{x}, t) : \mathbb{R}^n \times [0, +\infty) \to \mathbb{R}$ is the unknown function, which can be thought of as the concentration of a pollutant such as NO_2.
In particular, since we are working within the turbulent regime, we considered the steady **Reynolds Averaged Navier-Stokes** (RANS) equations:

$$\begin{cases} \nabla \cdot (\bar{\mathbf{u}} \otimes \bar{\mathbf{u}}) - \nabla \cdot 2(\mu_L + \mu_T)\nabla^{\mathbf{s}} \bar{\mathbf{u}} = -\nabla \bar{p} & \text{in } \Omega \times [0, T] \, , \\ \nabla \cdot \bar{\mathbf{u}} = \mathbf{0} & \text{in } \Omega \times [0, T] \, . \end{cases} \tag{2}$$

The eddy viscosity μ_T needs then an appropriate turbulence model, for which we have used the **k-ϵ model**.
In addition, we consider the following boundary conditions:

$$\begin{cases} \bar{\mathbf{u}} = \mathbf{0} & \text{on } \Gamma_{FrontAndBack} \cup \Gamma_{Ground} \times [0, T], \\ \bar{\mathbf{u}} = (\mu_1 \cos(\mu_2), \mu_1 \sin(\mu_2), 0) & \text{on } \Gamma_{In} \times [0, T], \\ (\nu \nabla \bar{\mathbf{u}} - p\boldsymbol{I})\boldsymbol{n} = \mathbf{0} & \text{on } \Gamma_{Out} \times [0, T]. \end{cases} \tag{3}$$

The parameter under consideration is $\boldsymbol{\mu} = (\mu_1, \mu_2)$, which codifies the inlet velocity condition.



Ground *Front and back* *Inlet* *Outlet*

Reduced order model

We employed the **Reduced Basis** (RB) method. The POD modes are used to approximate the solution $c(t, \boldsymbol{\mu})$ for any new value of the parameter with a linear combination:

$$c(t, \boldsymbol{\mu}) \approx \sum_{i=1}^{N_s} a_i(\boldsymbol{\mu}, t) \phi_i(x), \tag{4}$$

where $a_i(\boldsymbol{\mu}, t)$ are the parameter dependent coefficients and $\phi_i(x)$ are the parameter independent basis functions.
The coefficients of Eq. 4 are then obtained solving:

$$\boldsymbol{M_r} \dot{\boldsymbol{a}} - \nu_T \boldsymbol{B_r} \boldsymbol{a} + \boldsymbol{C_r} \boldsymbol{a} = \boldsymbol{f_r}(t), \tag{5}$$

where each term inside Eq. 5 is obtained by Galerkin projection:

$$\begin{cases} (\boldsymbol{M_r})_{ij} = (\phi_i, \phi_j)_{L_2(\Omega)}, & (\boldsymbol{B_r})_{ij} = (\phi_i, \Delta \phi_j)_{L_2(\Omega)}, \\ (\boldsymbol{C_r})_{ij} = (\phi_i, \nabla \cdot (\mathbf{u}(\boldsymbol{\mu}) \phi_j))_{L_2(\Omega)}, & (\boldsymbol{f_r})_i(t) = (\phi_i, f(t))_{L_2(\Omega)}. \end{cases} \tag{6}$$

POD-NN [1] and POD-DEIM [4]

The complexity in the treatment of the Eq. 5 concerns the convective term and the empirical source term, for which we employed the following strategies:

- The reduced order convective matrix C_r is obtained using the POD-NN approach, that is:

$$(\boldsymbol{C_r})_{ij}(\mu) = \sum_{i=1}^{N_\phi} (\phi_i, \nabla \cdot (u_k \Psi_k \phi_j))_{L_2(\Omega)}; \tag{7}$$

  where the coefficients u_k are the output of a feedforward NN.

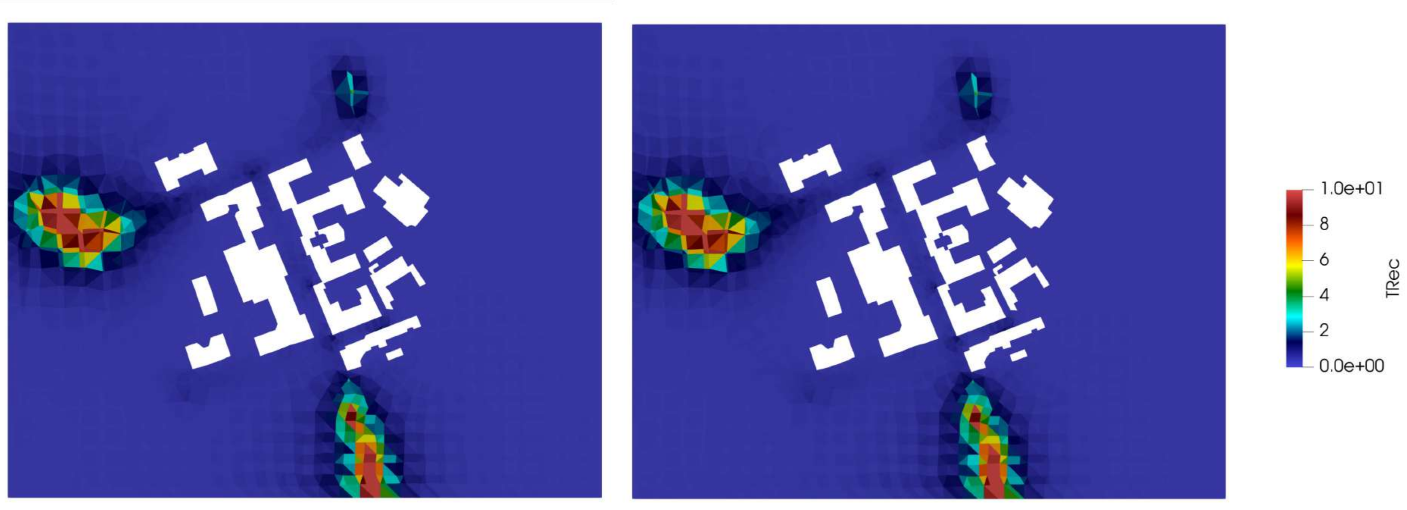
- The DEIM is employed for the source term $f(t)$, which is approximated as:

$$f(t) \approx \sum_{i=1}^{N_{DEIM}} p_i(t) \chi_i(\boldsymbol{x}). \tag{8}$$

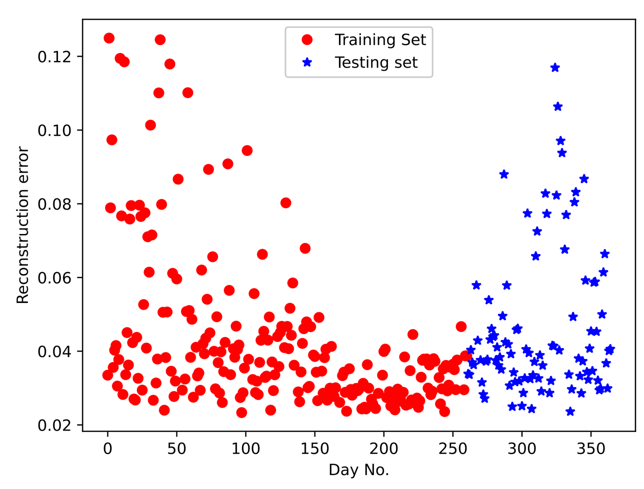
Numerical Results [3]

Test case: main campus of the University of Bologna.

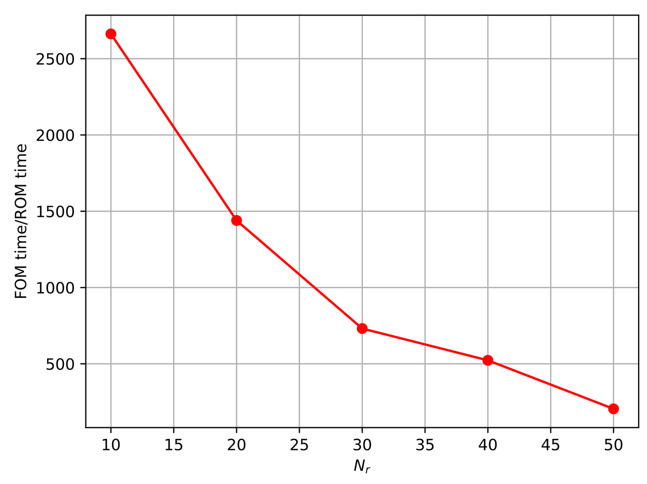
Dataset: Syntetic emission data using the fastrace traffic model and 1 year long empirical measurements for the inlet velocity condition.



Offline (left) and Online (right) solution for Day 20, $t = 7500$s.



Average daily reconstruction errors.



Speed-up w.r.t the number of basis functions.

References

[1] J. S. Hesthaven and S. Ubbiali. Non-intrusive reduced order modeling of nonlinear problems using neural networks. *J. Comput. Phys.*, 2018.

[2] M. Khamlich, G. Stabile, G. Rozza, L. Kornyei, and Z. Horváth. ROM for large-scale modeling of urban air pollution. *In preparation*, 2022.

[3] G. Stabile and G. Rozza. ITHACA-FV - In real Time Highly Advanced Computational Applications for Finite Volumes. `http://www.mathlab.sissa.it/ithaca-fv`.

[4] G. Stabile, M. Zancanaro, and G. Rozza. Efficient geometrical parametrization for finite-volume-based reduced order methods. *International Journal for Numerical Methods in Engineering*, 2020.