SISSA mathLab work on mathematical modelling for environmental applications

Prof. G. Rozza's Group

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Mathematical modelling for **environmental applications** represents a research area of significant importance. **Numerical simulations** of **geophysical flows** are not only an essential tool for **ocean** and **weather forecast**, but they could also provide insights on the mechanisms governing climate change.

A **Direct Numerical Simulation (DNS)** computes the evolution of all the significant flow structures (eddies and vortices) by resolving them with a properly refined mesh. For example, the typical resolution required by a DNS for atmospheric problems is of the order of 0.1 mm. Such a resolution is beyond reach even with modern supercomputers. When a DNS for environment flows is possible, it is usually expensive in terms of computational time and memory demand due to the large amount of degrees of freedom to be considered for a proper description of the flow system. In addition, often long time intervals have to be simulated.

We are working on two ways to reduce the computational cost, which could also be used simultaneously. These are: (i) **Large Eddy Simulation (LES)** models that allow to use a coarser mesh by modeling the effect of the small scales that do not get resolved and (ii) **Reduced Order Models (ROMs)** that enable fast computations without a significant loss in terms of accuracy.

In the following, we provide a comprehensive list (together with a brief content description) of the publications by Prof. Rozza's group on mathematical and computational modelling for environmental applications. Then, we introduce the open source software libraries developed by the group which have been used in these works and will be used in the future. A list of the universities we collaborate with concludes the document.

Ocean modelling

A novel **LES** model for **quasi-geostrophic equations**, which describe the dynamics of certain **ocean flows,** has been developed in

- M. Girfoglio, A. Quaini, and G. Rozza, *"A novel Large Eddy Simulation model for the Quasi-Geostrophic Equations in a Finite Volume setting"*, Journal of Computational and Applied Mathematics, 2023. [preprint] [doi]

With the aim to further reduce the computational of the numerical simulations, we will develop a **Reduced Ordel Model** based on our LES model.
Towards this goal, we proposed a **POD-Galerkin** approach for the **incompressible Navier-Stokes equations** in a **stream function-vorticity formulation** as an intermediate step:

- M. Girfoglio, A. Quaini, and G. Rozza, *"A POD-Galerkin reduced order model for the Navier-Stokes equations in stream function-vorticity formulation"*, Computers & Fluids, 2022. [preprint] [doi]

We are currently using ideas from our works on **LES** and **ROM** to devise a **regularized ROM** for the **quasi-geostrophic equations**:

- M. Girfoglio, A. Quaini, and G. Rozza, *"A linear filter regularization for POD-based reduced order models of the quasi-geostrophic equations"*, accepted on Comptes Rendus Mècanique, 2023. [preprint]

This is a collaboration with Prof. Annalisa Quaini from Houston University.

We have also worked on the implementation of reduced order models starting from **embedded computations** for the **shallow water equations**:

- X. Zeng, G. Stabile, E. N. Karatzas, G. Scovazzi, G. Rozza, "*Embedded domain Reduced Basis Models for the shallow water hyperbolic equations with the Shifted Boundary Method",* accepted in Computer Methods in Applied Mechanics and Engineering, 2022. [preprint][doi]

This is a collaboration with the team of Prof. Scovazzi at the Duke University (US), Dr. E. Karatzas from University of Athens (Greece) and Dr. X. Zeng from University of Texas (US).

Atmospheric modelling

We are currently working on a pressure-based solver for the **dry atmosphere** modelled by **compressible Euler equation**:

- M. Girfoglio, A. Quaini, and G. Rozza, *"Validation of an OpenFOAM-based solver for the Euler equations with benchmarks for mesoscale atmospheric modeling",* 2023. [preprint]

This is a collaboration with Prof. Annalisa Quaini from the University of Houston.

Another line of research is related to the application of reduced order modelling for the development of **digital twins** in the context of **simulation of pollutant dispersion** in **urban environments**. A work is in progress:

- M. Khamlich, G. Stabile, G. Rozza, L. Környei, Z. Horváth, "*ROM for Large-Scale Modelling of Urban air Pollution*", in preparation, 2022

This is a collaboration with the Széchenyi István University in Hungary

We are also working on the application of ROMs exploiting **deep learning** techniques for **fast transient dynamics simulating explosion scenarios**:

- M. Cracco, A. Lario, G. Stabile, M. Larcher, F. Casadei, G. Valsamos, G. Rozza, "*Deep-learning based ROMs for fast transient dynamics*", 2022. [preprint]

This is a collaboration with the Joint Research Center of the European Commission based in Ispra (Italy).

SISSA mathLab Open Source Software and Tools

ITHACA-FV (In real Time Highly Advanced Computational Applications for Finite Volumes) is C++ library based on the finite volume solver OpenFOAM . It consists of the implementation of several reduced order modeling techniques for parametrized problems.[github] [SISSA mathLab].

EZyRB is a python library for the Model Order Reduction based on **baricentric triangulation** for the selection of the parameter points and on **Proper Orthogonal Decomposition** for the selection of the modes. [github] [SISSA mathLab].

GEA (Geophysical and Environmental Applications) is a C++ library based on the finite volume solver OpenFOAM. It consists of the implementation of several models for atmospheric and ocean flows. [github].

Collaborations

UNIVERSITY OF HOUSTON

Methodological development of reduced order modelling techniques and large eddy simulation models for ocean and atmospheric flows.

DUKE UNIVERSITY
Methodological development of reduced order modelling techniques for ocean flows

UNIVERSITY OF ATHENS
Methodological development of reduced order modelling techniques for ocean flows

UNIVERSITY OF TEXAS
Methodological development of reduced order modelling techniques for ocean flows

SZÉCHENYI ISTVÀN UNIVERSITY
Methodological development of reduced order modelling techniques for atmospheric flows.

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