GEA: a new finite volume-based open source code for the numerical simulation of atmospheric and ocean flows

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Introduction

Numerical simulations of geophysical flows are not only an essential tool for ocean and weather forecast, but they could also provide insights on the mechanisms governing climate change. We introduce GEA (Geophysical and Environmental Applications)[1], a new open-source atmosphere and ocean modeling framework within the finite volume C++ library OpenFOAM. We present two computational pipelines to reduce the computational cost of the numerical simulations, which could also be used simultaneously: (i) Reduced Order Models (ROMs) that enable fast computations without a significant loss in terms of accuracy and (ii) Large Eddy Simulation (LES) models that allow to use coarser meshes than those required by a DNS thanks to a model for the effect of the small scales that do not get resolved.

Quasi-geostrophic equations: methodology [2, 3]

The BV-α model reads: find potential vorticity q, *filtered vorticity* $\overline{q}$, and stream function ψ such that

$$
\begin{aligned}
\partial_t q + \nabla \cdot (\boldsymbol{u} q) - \frac{1}{\mathrm{Re}} \Delta q = F & \quad \text{in } \Omega \times (t_0, T), \\
-\alpha^2 \nabla \cdot (a(q) \nabla \overline{q}) + \overline{q} = q & \quad \text{in } \Omega \times (t_0, T), \\
-\mathrm{Ro} \Delta \psi + y = \overline{q} & \quad \text{in } \Omega \times (t_0, T),
\end{aligned}
$$

where

$$
q = Ro\, \omega + y, \quad Ro = U/(\beta L^2),
$$

α can be interpreted as the *filtering radius* and $a(\cdot)$ is a scalar function such that:

$a(q) \simeq 0$ where the flow field does not need regularization;

$a(q) \simeq 1$ where the flow field does need regularization.

Taking inspiration from the large body of work on the Leray-α model, we propose the following indicator function:

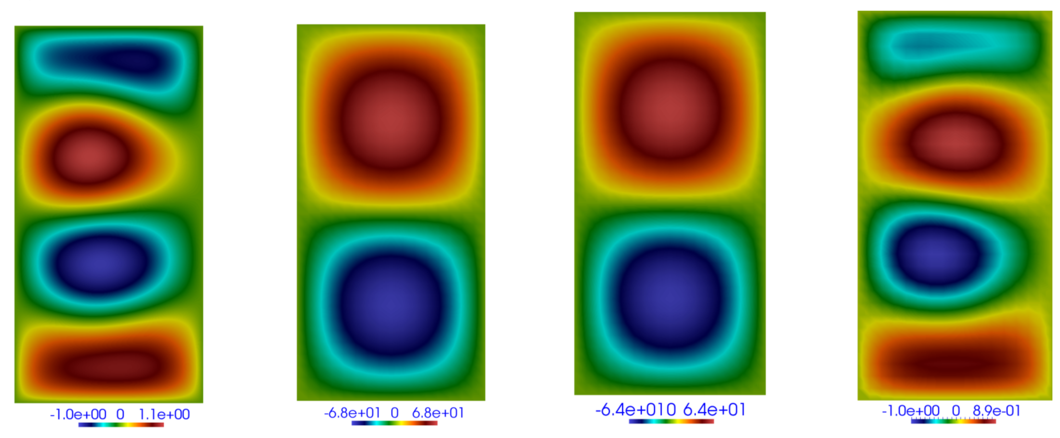
$$
a(q) = \frac{|\nabla q|}{\max\left(1, ||\nabla q||_\infty\right)}.
$$

We consider two approaches for ROM, which differ at the online stage:

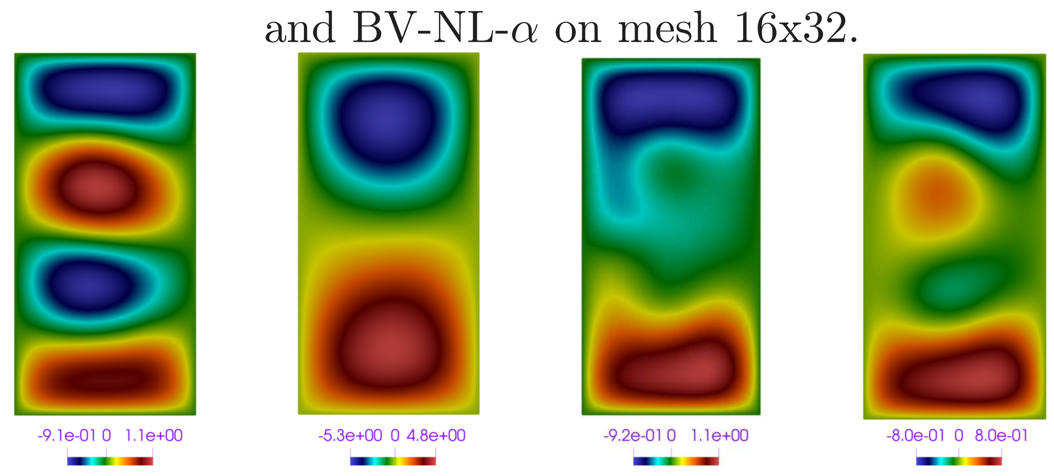
- QGE-QGE ROM: the system to be solved online results from Galerkin projection of the QGE on the reduced (POD) space;
- QGE-BV-α ROM: Galerkin projection of the BV-α model on the POD space provides the system that has to be solved online.

Quasi-geostrophic equations: main results [2, 3]

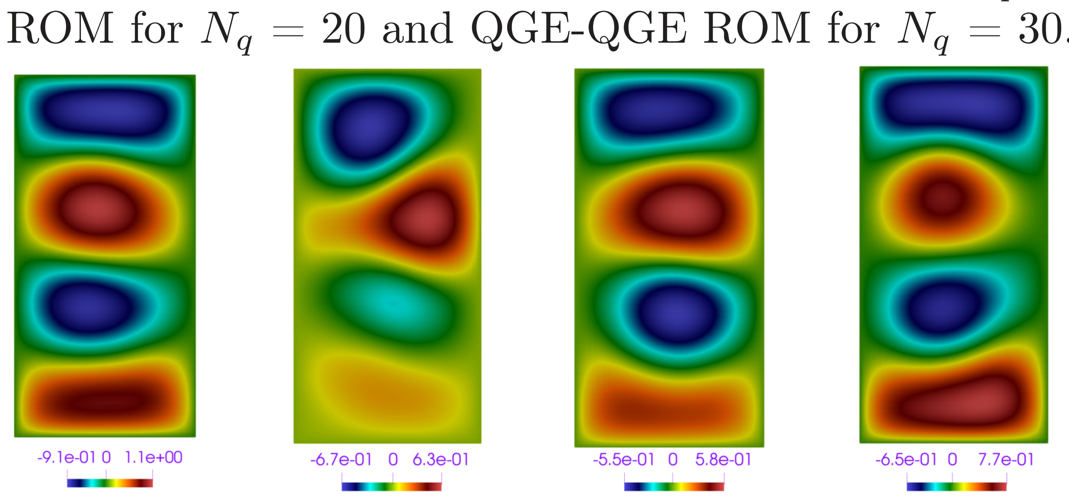
We set $F = sin(\pi y)$ and consider $Ro = 0.008$ and $Re = 1000$.



From left to right: DNS on mesh 256x512, QGE on mesh 16x32, BV-α on mesh 16x32 and BV-NL-α on mesh 16x32.



From left to right: FOM on mesh 32x64, QGE-QGE ROM for $N_q = 10$, QGE-QGE ROM for $N_q = 20$ and QGE-QGE ROM for $N_q = 30$.



From left to right: FOM on mesh 32x64, QGE-BV-α ROM for $N_q = 10$, QGE-BV-α ROM for $N_q = 20$ and QGE-BV-α ROM for $N_q = 30$.

Compressible Euler equations: methodology [4, 5, 6]

$$
\begin{aligned}
&\frac{\partial \rho}{\partial t} + \nabla \cdot (\rho \boldsymbol{u}) = 0 && \text{in } \Omega \times (0, t_f], \\
&\frac{\partial (\rho \boldsymbol{u})}{\partial t} + \nabla \cdot (\rho \boldsymbol{u} \otimes \boldsymbol{u}) + \nabla p' + g z \nabla \rho - \nabla \cdot (2 \mu_a \boldsymbol{\epsilon}(\boldsymbol{u})) = 0 && \text{in } \Omega \times (0, t_f], \\
&\frac{\partial (\rho e)}{\partial t} + \nabla \cdot (\rho e \boldsymbol{u}) + \nabla \cdot (p \boldsymbol{u}) + \rho g \boldsymbol{u} \cdot \widehat{\boldsymbol{k}} - \nabla \cdot \left(\frac{\mu_a c_p}{Pr} \nabla T \right) = 0 && \text{in } \Omega \times (0, t_f].
\end{aligned}
$$

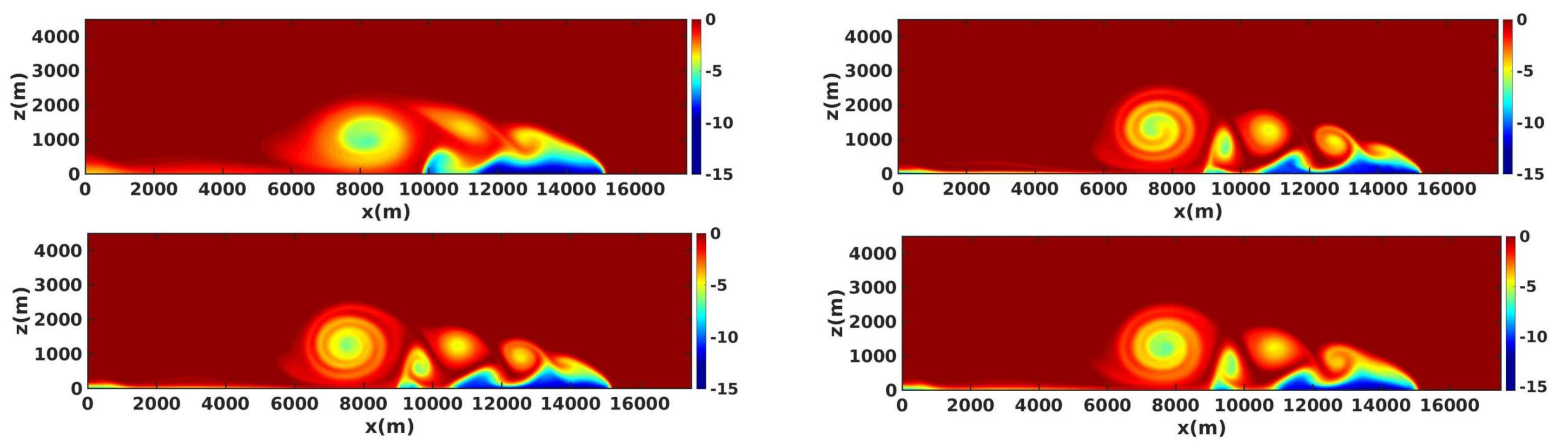
Main features:

- The pressure is expressed as the sum of a fluctuation p' with respect to a background state: $p = p' + \rho g z$.
- We adopt the Leray-α model with three different indicator functions:

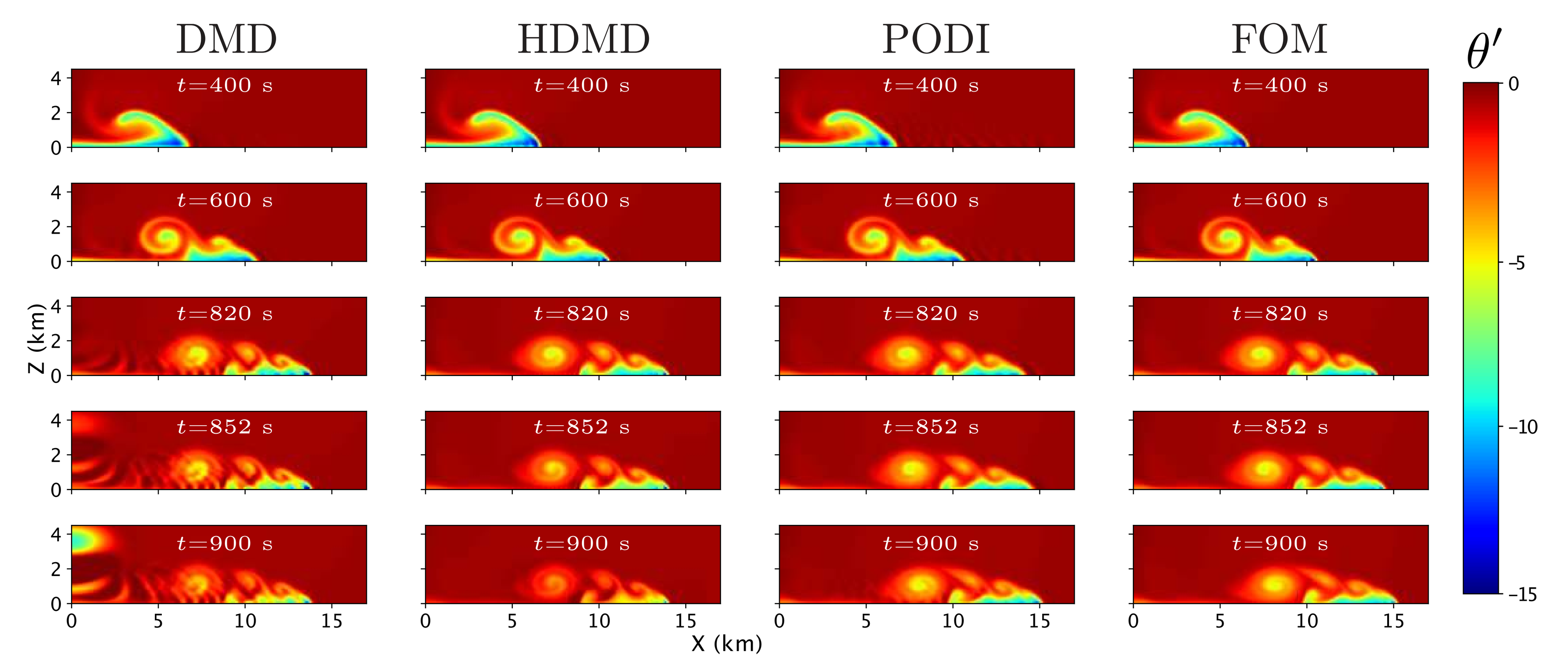
$$
a(\boldsymbol{u}) = a_L(\boldsymbol{u}) = 1, \quad a(\boldsymbol{u}) = a_S(\boldsymbol{u}) = \frac{|\nabla \boldsymbol{u}|}{\|\nabla \boldsymbol{u}\|_\infty}, \quad a(\boldsymbol{u}) = a_D(\boldsymbol{u}) = |\boldsymbol{u} - D(F(\boldsymbol{u}))| \,.
$$

- We consider three approaches for ROM relying on the SVD algorithm as a main tool to compute the basis functions:
  - DMD extracts the dominant dynamic flow structure from a unsteady flow field. It forecasts future states of a non-linear time-dependent system through a linear combination of few main structures evolving linearly.
  - HDMD is a variation of the standard DMD algorithm based on the idea to combine the DMD algorithm with time delay embedding.
  - PODI differs from DMD and HDMD in that it is not designed to forecast the system evolution, but rather to interpolate solutions in a parameter space, where time is one of possibly many parameters of interest.

Compressible Euler equations: main results [4, 5, 6]



Spatial distribution of the potential temperature perturbation θ' at $t = 900$ s with mesh $h = 25$ m: a_L and $\alpha = 2.7$ (top left), a_S and $\alpha = 8$ (top right), a_D and $\alpha = 10$ (bottom left), a_D and $\alpha = 12$ (bottom right).



Comparison of time evolution of θ' given by the ROMs (first 3 columns) and the FOM (last column) for the 99% of the cumulative energy.

References

[1] GEA - Geophysical and Environmental Applications. `https://github.com/GEA-Geophysical-and-Environmental-Apps/GEA`.

[2] Girfoglio, M. and Quaini, A. and Rozza, G. *J. Comput. Appl. Math.*, 2023.

[3] Girfoglio, M. and Quaini, A. and Rozza, G. *CR Mécanique*, 2023.

[4] Girfoglio, M. and Quaini, A. and Rozza, G. *AIP Advances*, 2023.

[5] Clinco, N. and Girfoglio, M. and Quaini, A. and Rozza, G. *Comput. Fluids*, 2023.

[6] Hajisharifi, A. and Girfoglio, M. and Quaini, A. and Rozza, G. *Finite Elem. Anal. Des.*, 2024.