Dimension reduction in heterogeneous parametric spaces: shape and drag optimization in naval engineering

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Introduction

Development of a new framework for parameters space reduction in naval engineering obtained by coupling

- Active Subspace Method to identify lower dimensional structure in the parameters space [1].
- Free Form Deformation to morph the geometry.
- Boundary Element Method as an high fidelity solver for the wave resistance [2].

The Active Subspace Method

Consider a function, its gradient vector and a sampling density

$$f = f(\mathbf{x}), \quad \mathbf{x} \in \mathbb{R}^m, \quad \nabla f(\mathbf{x}) \in \mathbb{R}^m, \quad \rho : \mathbb{R}^m \to \mathbb{R}_+$$

Take the average outer product of the gradient and its eigendecomposition,

$$\mathbf{C} = \mathbb{E}\left[\nabla_{\mathbf{x}} f \, \nabla_{\mathbf{x}} f^T\right] = \int (\nabla_{\mathbf{x}} f)(\nabla_{\mathbf{x}} f)^T \rho \, d\mathbf{x} = \mathbf{W} \mathbf{\Lambda} \mathbf{W}^T$$

Partition the eigendecomposition,

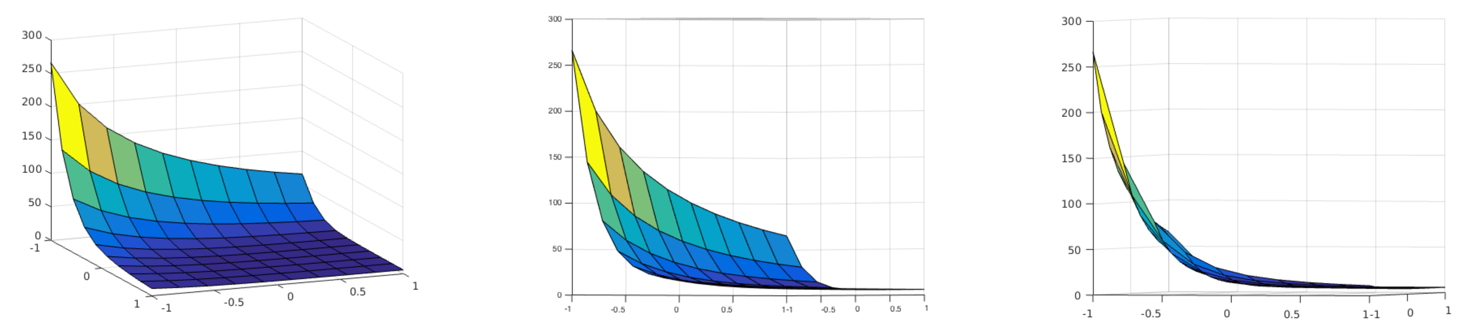
$$\mathbf{\Lambda} = \begin{bmatrix} \mathbf{\Lambda}_1 & \\ & \mathbf{\Lambda}_2 \end{bmatrix}, \qquad \mathbf{W} = [\mathbf{W}_1 \quad \mathbf{W}_2], \qquad \mathbf{W}_1 \in \mathbb{R}^{m \times n}$$

Rotate and separate the coordinates,

$$\mathbf{x} = \mathbf{W}\mathbf{W}^T\mathbf{x} = \mathbf{W}_1\mathbf{W}_1^T\mathbf{x} + \mathbf{W}_2\mathbf{W}_2^T\mathbf{x} = \mathbf{W}_1\mathbf{y} + \mathbf{W}_2\mathbf{z}.$$

$\mathbf{y}$ is the active variable and $\mathbf{z}$ the inactive one:

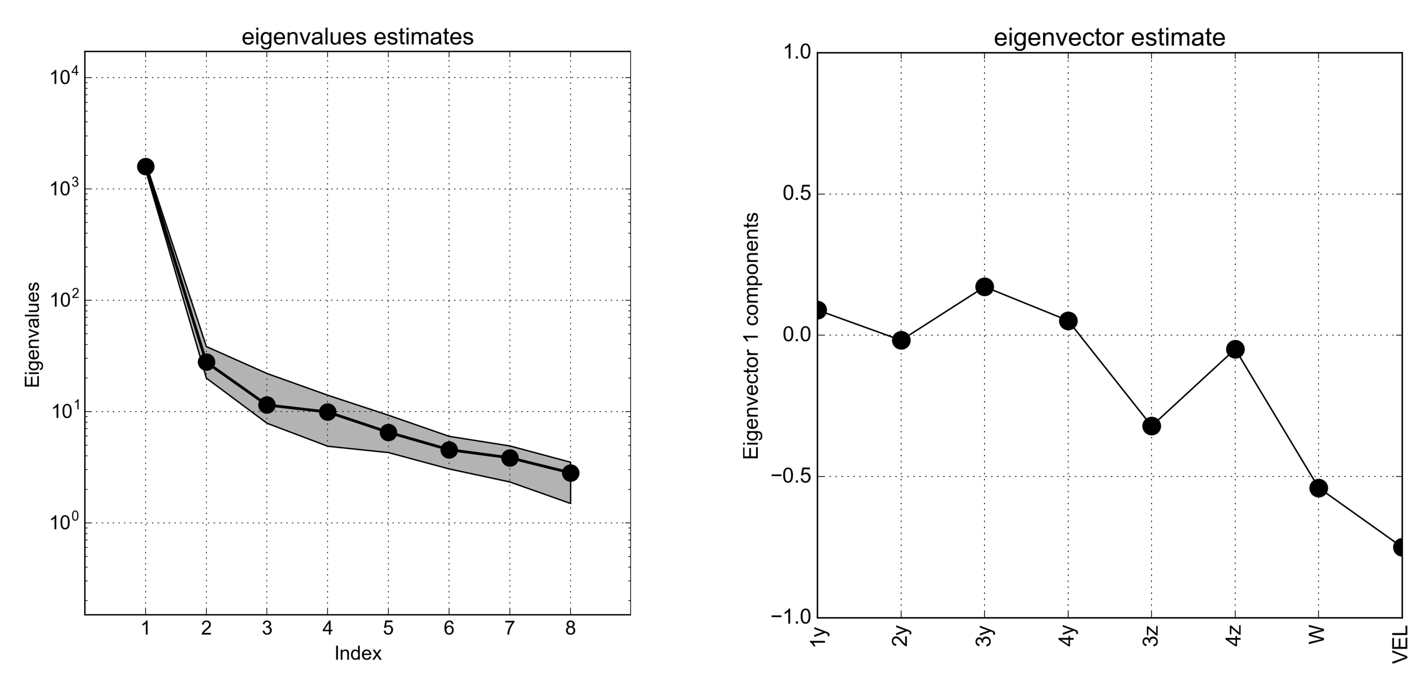
$$\mathbf{y} = \mathbf{W}_1^T\mathbf{x} \in \mathbb{R}^n, \qquad \mathbf{z} = \mathbf{W}_2^T\mathbf{x} \in \mathbb{R}^{m-n}$$



Bivariate output f, rotation of the domain, and f along the active variable.

Eigenvalues analysis

- To calculate the gradients of the wave resistance with respect to the parameters we use a local linear model that approximates them with the best linear approximation using 13 nearest neighbours.
- Presence of a major gap between the first and the second eigenvalue and a minor one between the second and the third.

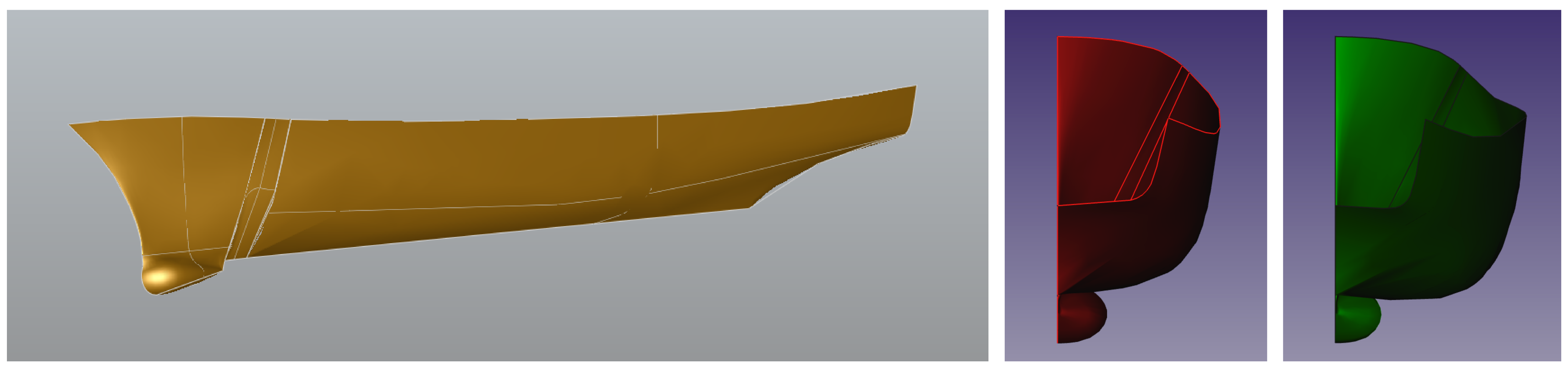


Free Form Deformation: the PyGeM library

- PyGeM is a python library using Free Form Deformation and RBF interpolation to parametrize and morph complex geometries.
- It interacts with the major industrial file formats used for CAD management (.iges, .stl), mesh files (.unv and OpenFOAM), and output files (.vtk).
- Available at: github.com/mathLab/PyGeM & mathlab.sissa.it/cse-software

The DTMB 5415 hull and the parameters

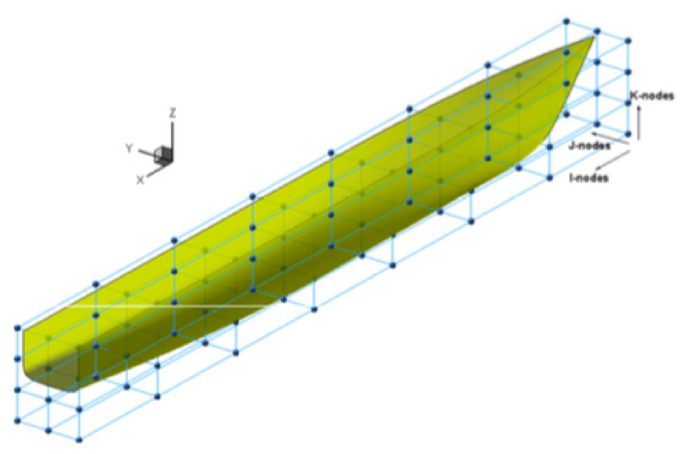
- The DTMB 5415 is a hull conceived for preliminary design of a US Navy Combatant in 1980 and it is a very common benchmark for the validation of CFD models. The hull geometry includes both a sonar dome and transom stern.



- As geometrical parameters we select 6 components of 4 different control points of a FFD lattice over one side wall of the hull. Then we apply the same deformation to the other side.
- The structural parameter is the displacement of the hull and the physical one is the velocity.

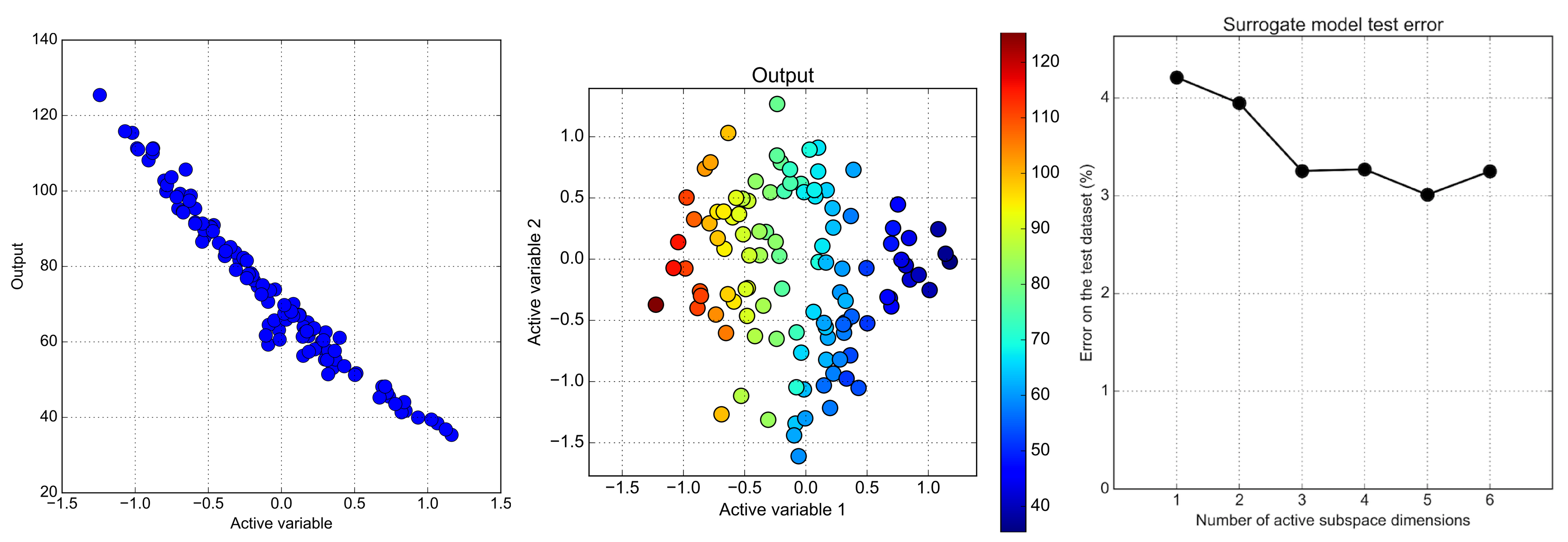
Parameter	Nature	Lower bound	Upper bound
u_1	FFD Point 1 y	-0.2	0.3
u_2	FFD Point 2 y	-0.2	0.3
u_3	FFD Point 3 y	-0.2	0.3
u_4	FFD Point 4 y	-0.2	0.3
u_5	FFD Point 3 z	-0.2	0.5
u_6	FFD Point 4 z	-0.2	0.5
u_7	weight (kg)	500	800
u_8	velocity (m/s)	1.87	2.70

Credits: CMAME 283:1525-1544 2015



Sufficient summary plot and error analysis

- The sufficient summary plot confirms the presence of an active subspace of dimension 1 and 2.



- Using a response surface of order 1 ensures an error on the test dataset equal to 4.2%. Doing so we reduced the parameter space from dimension 8 to 1.
- Good starting point to perform further optimization.

References

[1] P. G. Constantine. *Active subspaces: Emerging ideas for dimension reduction in parameter studies*, volume 2. SIAM, 2015.

[2] A. Mola, L. Heltai, A. De Simone, et al. Ship sinkage and trim predictions based on a cad interfaced fully nonlinear potential model. In *The 26th International Ocean and Polar Engineering Conference*. International Society of Offshore and Polar Engineers, 2016.

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